AAPT UNITED STATES PHYSICS TEAM
AIP 2020

2020 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. The only scratch paper you may use is scratch paper provided by the proctor. You may not use your own.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones cannot be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
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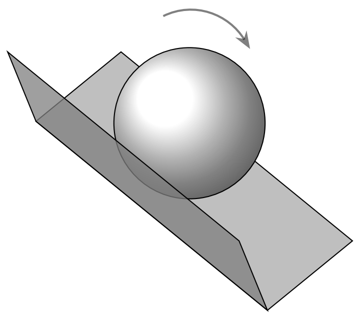
We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Ariel Amir, JiaJia Dong, Mark Eichenlaub, Abijith Krishnan, Daniel Longenecker, Kye Shi, Brian Skinner, Paul Stanley, Mike Winer, and Kevin Zhou.

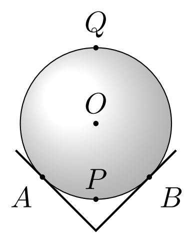
1. A ball is launched straight toward the ground from height h. When it bounces off the ground, it loses half of its kinetic energy. It reaches a maximum height of $2h$ before falling back to the ground again. What was the initial speed of the ball?

   (A) $\sqrt{gh}$
   (B) $\sqrt{2gh}$
   (C) $\sqrt{3gh}$
   (D) $\sqrt{4gh}$
   (E) $\sqrt{6gh}$

2. A rigid ball of radius R is rolling without slipping along the rib of a right-angle chute, as shown at left. A cross section of the ball, taken perpendicular to the ball's direction of travel, is shown at right. Which of the marked point(s) of the ball have the highest speed?



side (3d) view



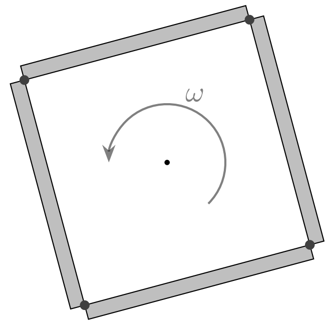
cross-section (2d) view

   (A) All the marked points have the same speed.
   (B) The contact points A and B.
   (C) The center O.
   (D) The point P.
   (E) The point Q.

3. When an axe is swung with kinetic energy E directly at a piece of wood, the edge of the axe is buried a depth L into the wood. If the axe is swung with kinetic energy $2E$, how deep will it be buried into the wood? Assume that the axe is wedge-shaped with a constant angle and that the force per unit contact area between the axe and the wood during the impact is proportional to the depth.

   (A) $2^{1/4}L$
   (B) $2^{1/3}L$
   (C) $\sqrt{2}L$
   (D) $2L$
   (E) $4L$

4. Four identical rods, each of mass m and length $2d$, are joined together to form a square. The square is then spun around its center, as shown in the figure, at an angular frequency of ω. What is the magnitude of the force that the joints between the rods (at the corners of the square) must bear?



(A) $m\omega^2 d/2$

(B) $m\omega^2 d/\sqrt{2}$

(C) $m\omega^2 d$

(D) $\sqrt{2}m\omega^2 d$

(E) $2m\omega^2 d$

5. A pendulum of length L oscillates inside a box. A person picks up the box and gently shakes it horizontally with frequency ω and a fixed amplitude for a fixed time. The final amplitude can be maximized if ω satisfies

(A) $\omega = \sqrt{g/L}$

(B) $\omega = 2\sqrt{g/L}$

(C) $\omega = (1/2)\sqrt{g/L}$

(D) There will be no effect on the amplitude for any value of ω.

(E) None of the above

6. A planet is orbiting a star in a circular orbit of radius r_0. Over a very long period of time, much greater than the period of the orbit, the star slowly and steadily loses 1% of its mass. Throughout the process, the planet's orbit remains approximately circular. The final orbit radius is closest to

(A) $1.02r_0$

(B) $1.01r_0$

(C) r_0

(D) $0.99r_0$

(E) $0.98r_0$

7. An astronaut standing on the exterior of the international space station wants to dispose of three pieces of trash. They face the station's direction of travel with the Earth to their left. From the astronaut's perspective, the three pieces are thrown (I) left, (II) right, and (III) up. To the astronaut's frustration, some of the pieces of trash return to the space station after several hours. They are

(A) II only

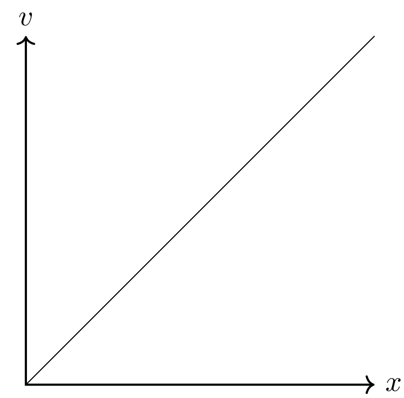
(B) III only

(C) I and II

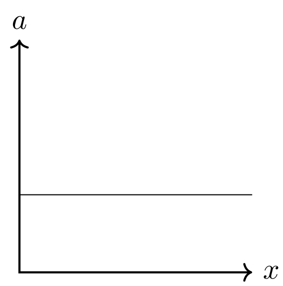
(D) II and III

(E) I, II, and III

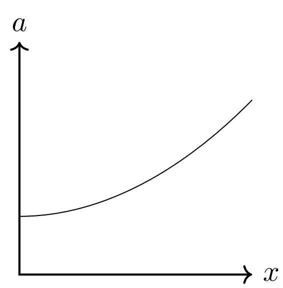
8. The velocity versus position plot of a particle is shown below. Which following choices is the correct acceleration vs. position plot of the particle?



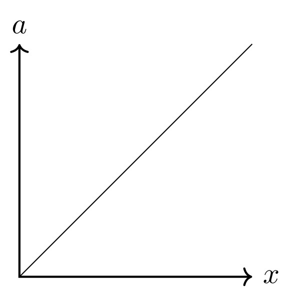
(A)



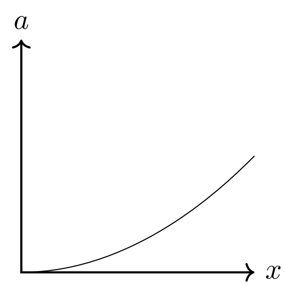
(B)



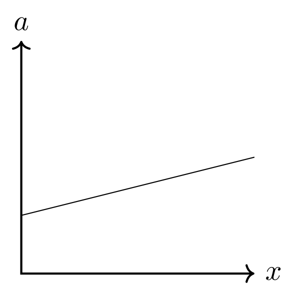
(C)



(D)

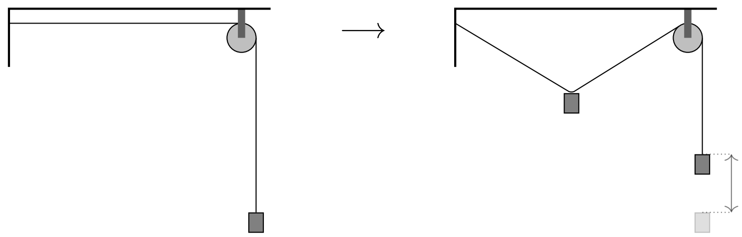


(E)



The following information is relevant to problems 9 and 10.

9. A block of mass m is attached to a massless string. The string is passed over a massless pulley and the end of the string is fixed in place. The horizontal part of the string has length L. Now a small mass m is hung from the horizontal part of the string, and the system comes to equilibrium. (Diagram not necessarily to scale.)



Neglecting friction everywhere, the tension at the end of the string is

(A) $mg/2$
(B) mg
(C) $3mg/2$
(D) $2mg$
(E) $3mg$

10. During this process, the block has been raised by approximately a height

(A) $0.15L$
(B) $0.23L$
(C) $0.31L$
(D) $0.37L$
(E) $0.40L$

11. The maximal tension per area a material can sustain without failure is called its *tensile strength.* Plain steel has a tensile strength of 415 MPa. What is the maximal mass one can hang on a vertical steel rod of negligible mass and a diameter of 2 cm?

(A) 1300 kg
(B) 5200 kg
(C) 13 000 kg
(D) 52 000 kg
(E) The answer depends on the length of the steel rod.

12. A point mass m is glued inside a massless hollow rod of length L at an unknown location. When the rod is pivoted at one end, the period of small oscillations is T. When the rod is pivoted at the other end, the period of small oscillations is $2T$. How far is the mass from the center?

(A) $L/8$
(B) $L/6$
(C) $L/4$
(D) $3L/10$
(E) $2L/5$

13. A ballerina with moment of inertia I is quickly twirling with angular velocity ω. In her hand she has a pen of mass m at a radius R from her axis of rotation. The ballerina releases the pen. Afterward, what happens to the vertical component of the angular momentum of the system consisting of the ballerina and the pen? You may ignore all friction, but not gravity or normal forces.

(A) It decreases until the pen hits the floor.

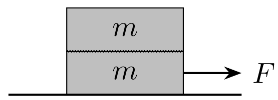
(B) It increases until the pen hits the floor.

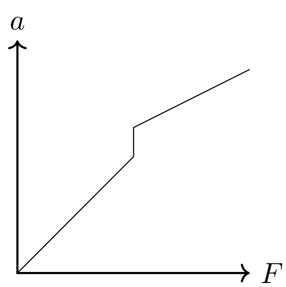
(C) It always stays the same.

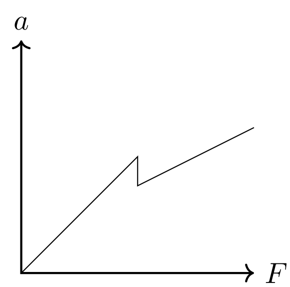
(D) It initially stays the same, but decreases when the pen hits the floor.

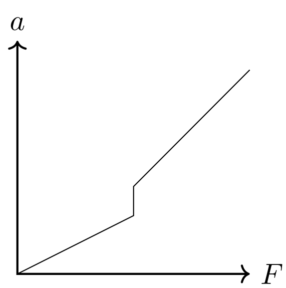
(E) It initially stays the same, but increases when the pen hits the floor.

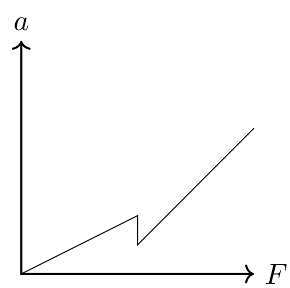
14. Two blocks of mass m are placed on top of each other, and the bottom block is placed on the ground. The ground is frictionless. The static and kinetic coefficients of friction between the two blocks are μ_s and μ_k, with $\mu_s < \mu_k$. The blocks are at rest initially. When a constant horizontal force F is then applied to the bottom block, which of the following graphs could show its acceleration as a function of F?

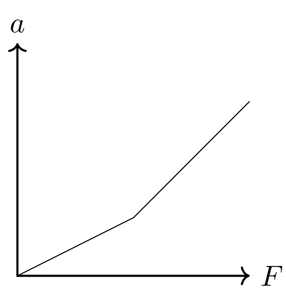


(A) 

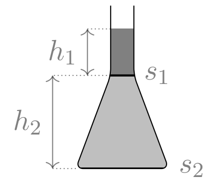
(D) 

(B) 

(E) 

(C) 

15. As shown in the figure, a vessel contains two types of liquid: the liquid with density ρ_1 on top and ρ_2 on the bottom. The depth of the top liquid is h_1, and the interface area between the top and the bottom liquid is s_1. The bottom liquid has a depth of h_2. The area of the bottom of the vessel is s_2. What is the gauge pressure (i.e. pressure in excess of atmospheric pressure) at the bottom of the vessel?



(A) $(\rho_1 h_1 + \rho_2 h_2)g$

(B) $\dfrac{\rho_1 s_1 h_1 + \rho_2 s_2 h_2}{s_2} g$

(C) $\dfrac{1}{2}(\rho_1 + \rho_2)(h_1 + h_2)g$

(D) $(\rho_1 + \rho_2)(h_1 + h_2)g$

(E) $\rho_2(h_1 + h_2)g$

16. Liquid droplets store a given amount of potential energy per unit surface area, due to their surface tension. When two identical, nearly spherical liquid droplets coalesce on a certain type of surface, part of this energy can be converted into upward kinetic energy, causing the coalesced droplet to jump. Assuming the conversion is 100% efficient, how does the maximum height h depend on the radius r of the initial droplets?

(A) $h \propto r$

(B) $h \propto r^{1/2}$

(C) $h \propto r^{-1/2}$

(D) $h \propto r^{-1}$

(E) $h \propto r^{-2}$

17. Paul the Giant stands outside on a force-meter calibrated in Newtons, which reads 5000 N. Paul is wearing a large cowboy hat, which has horizontal cross-sectional area $A = 1\,\mathrm{m}^2$ and completely covers both him and the scale when seen from directly above. At time $t = 0$, rain begins to fall vertically downward on Paul, and any rain that hits his hat is collected in the hat's brim. The raindrops have a constant downward speed of 1 m/s, and the rain accumulates on the ground at a rate of 1 mm/s. What is the reading (in N) on the scale as a function of the time $t > 0$ (in s)? The density of water is $1000\,\mathrm{kg/m^3}$.

(A) $5001 + 11t$

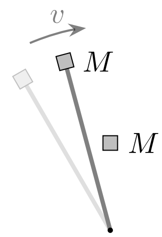
(B) $5001 + 10t$

(C) $5000 + 11t$

(D) $5001 + 1.1t$

(E) $5001 + t$

18. A massless rigid rod is pivoted at one end, and a mass M is at the other end. Originally, the rod rotates frictionlessly about the pivot with a uniform angular velocity such that the mass M has speed v. The rotating rod collides with another mass M at its midpoint, which then sticks to the rod. After the collision, what is the kinetic energy of the system?



(A) $\frac{1}{4}Mv^2$

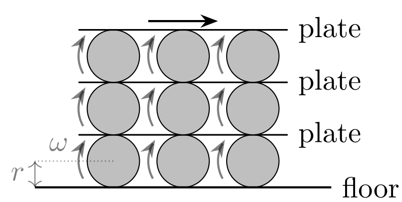
(B) $\frac{1}{3}Mv^2$

(C) $\frac{7}{18}Mv^2$

(D) $\frac{2}{5}Mv^2$

(E) $\frac{1}{2}Mv^2$

19. A system of cylinders and plates is set up as shown. The cylinders all have radius r, and roll without slipping to the right with angular velocity ω. What is the speed of the top plate?



(A) ωr

(B) $2\omega r$

(C) $3\omega r$

(D) $4\omega r$

(E) $6\omega r$

20. A car is driving against the wind at a constant speed v_0 relative to the ground. The wind direction is always opposite to the car's velocity, but its speed fluctuates about an average speed of v relative to the ground. The air drag force is Av_{rel}^2, where A is a constant and v_{rel} is the relative speed between the car and the wind. What is the average rate $\overline{P}$ of energy dissipation due to the air resistance?

(A) $\overline{P} = Av_0(v_0 + v)^2$

(B) $\overline{P} > Av_0(v_0 + v)^2$

(C) $\overline{P} < Av_0(v_0 + v)^2$

(D) Both (B) and (C) are possible depending on how v fluctuates.

(E) Both (A) and (C) are possible depending on how v fluctuates.

21. A circular table has radius R and $N > 2$ equally spaced legs of length h attached to its perimeter. Suppose the table has a uniform mass density with total mass m, and neglect the mass of the legs. Assuming the table does not slip, the minimum horizontal force needed to tip over the table is

(A) $\dfrac{mgR}{h}$

(B) $\dfrac{mgR}{h} \sin\left(\dfrac{N-2}{2N}\,\pi\right)$

(C) $\dfrac{mgR}{h} \cos\left(\dfrac{\pi}{N}\right)$

(D) $\dfrac{mgR}{h} \tan\left(\dfrac{N-2}{2N}\,\pi\right)$

(E) $\dfrac{mgR}{h} \sin\left(\dfrac{\pi}{2N}\right)$

22. A collision occurs between two masses. In each inertial reference frame, one can compute the change in total momentum $\Delta\mathbf{P}$ and the change in total kinetic energy ΔK due to the collision. Which of the following is true?

(A) $\Delta\mathbf{P}$ and ΔK do not depend on the frame.

(B) $\Delta\mathbf{P}$ and ΔK do not depend on the frame for perfectly elastic collisions, but $\Delta\mathbf{P}$ may depend on the frame for inelastic collisions.

(C) $\Delta\mathbf{P}$ and ΔK do not depend on the frame for perfectly elastic collisions, but ΔK may depend on the frame for inelastic collisions.

(D) $\Delta\mathbf{P}$ and ΔK do not depend on the frame for perfectly elastic collisions, but both may depend on the frame for inelastic collisions.

(E) $\Delta\mathbf{P}$ and ΔK may both depend on the frame, for both perfectly elastic and inelastic collisions.

23. Steve determines the spring constant k of a spring by applying a force F to it and measuring the change in length Δx. The tools he uses to measure F and Δx both have a constant absolute uncertainty, leading to an uncertainty in k of δk_S. If Tiffany measures the same spring constant with the same tools but by using a force that is five times larger, what will her uncertainty in k be in terms of δk_S?

(A) $\delta k_T = 0.04\,\delta k_S$

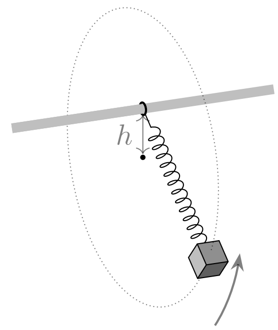
(B) $\delta k_T = 0.08\,\delta k_S$

(C) $\delta k_T = 0.2\,\delta k_S$

(D) $\delta k_T = 0.4\,\delta k_S$

(E) $\delta k_T = 0.5\,\delta k_S$

24. A mass m is connected to one end of a zero-length spring with spring constant k. The other end of the spring is connected to a frictionless bearing mounted around a horizontal pole so that the mass can swing in a vertical circle of radius R around the pole. The setup is shown in the figure below. What is the vertical distance h between the center of the circular orbit and the axis of the pole? Assume that both the diameter of the pole and the rest length of the spring are negligible compared to R.



(A) $\sqrt{mgR/k}$

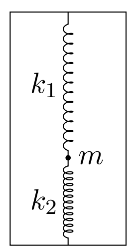
(B) $R\sqrt{(R + mg/k)/(R - mg/k)}$

(C) $R - mg/k$

(D) mg/k

(E) $\sqrt{R^2 - (mg/k)^2}$

25. A ball of negligible radius and mass m is connected to two ideal springs. Each spring has rest length ℓ_0. The springs are connected to the ball inside a box of height $2\ell_0$, and the ball is allowed to come to equilibrium, as shown. Under what condition is this equilibrium point stable with respect to small horizontal displacements?



(A) $k_1 > k_2$

(B) $k_2 > k_1$

(C) $k_1 - k_2 > mg/\ell_0$

(D) $k_1k_2/(k_1 + k_2) > mg/\ell_0$

(E) $k_1k_2/(k_1 - k_2) > mg/\ell_0$

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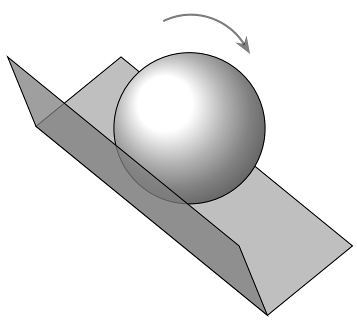
1. A ball is launched straight toward the ground from height h. When it bounces off the ground, it loses half of its kinetic energy. It reaches a maximum height of $2h$ before falling back to the ground again. What was the initial speed of the ball?

   (A) $\sqrt{gh}$
   (B) $\sqrt{2gh}$
   (C) $\sqrt{3gh}$
   (D) $\sqrt{4gh}$
   (E) $\sqrt{6gh}$ $\leftarrow$ **CORRECT**

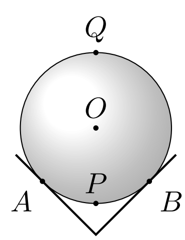
Solution

The calculation is easiest in the framework of energy: $E_0 = mgh + \frac{1}{2}mv_0^2 = 2E_f = 2mg(2h)$. So solving for initial velocity we get $v_0 = \sqrt{6gh}$.

2. A rigid ball of radius R is rolling without slipping along the rib of a right-angle chute, as shown at left. A cross section of the ball, taken perpendicular to the ball's direction of travel, is shown at right. Which of the marked point(s) of the ball have the highest speed?



side (3d) view



cross-section (2d) view

   (A) All the marked points have the same speed.
   (B) The contact points A and B.
   (C) The center O.
   (D) The point P.
   (E) The point Q. $\leftarrow$ **CORRECT**

Solution

The points A and B are stationary because they are in contact with the chute, and the ball is rolling without slipping. The speed of a point in the ball is therefore proportional to its distance from the axis AB, which is maximized for point Q. Incidentally, it's not too hard to show that if the center has speed v, then point Q has speed $(1+\sqrt{2})v$.

3. When an axe is swung with kinetic energy E directly at a piece of wood, the edge of the axe is buried a depth L into the wood. If the axe is swung with kinetic energy $2E$, how deep will it be buried into the wood? Assume that the axe is wedge-shaped with a constant angle and that the force per unit contact area between the axe and the wood during the impact is proportional to the depth.

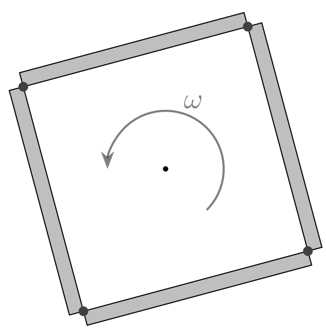
   (A) $2^{1/4}L$
   (B) $2^{1/3}L$ ← **CORRECT**
   (C) $\sqrt{2}L$
   (D) $2L$
   (E) $4L$

Solution

The energy required to enter the wood scales as L^3. This is because the energy to displace a harmonic oscillator scales as L^2, and the amount of contact with the wood gives another factor of d. Thus, doubling the energy will increase the distance into the wood by a factor of $2^{1/3}$.

Calculus is not necessary here, but may aid understanding this solution. We can define a spring constant density $\frac{k}{\ell}$ so that the energy per unit length is $\frac{ky^2}{2\ell}$, where $y = rL$ is the perpendicular displacement, and r is the wedge ratio. We can then integrate the energy as a function of distance into the wood: $\int_0^L dL' \frac{kL'^2}{2r^2\ell} = \frac{k}{6r^2\ell}L^3$.

4. Four identical rods, each of mass m and length $2d$, are joined together to form a square. The square is then spun around its center, as shown in the figure, at an angular frequency of ω. What is the magnitude of the force that the joints between the rods (at the corners of the square) must bear?



   (A) $m\omega^2 d/2$
   (B) $m\omega^2 d/\sqrt{2}$ ← **CORRECT**
   (C) $m\omega^2 d$
   (D) $\sqrt{2}m\omega^2 d$
   (E) $2m\omega^2 d$

Solution

Consider the rightmost rod, and consider an arbitrary point (d, h) on this rod. The centripetal acceleration at that point is given by $\omega^2\mathbf{r} = \omega^2(d, h)$.

Thus, the rightward component of centripetal acceleration is $\omega^2 d$, which is constant along the rod. Thus, the rightward force on the rod is $F = ma = m\omega^2 d$.

Using Newton's third law and symmetry, we see that the tension in each joint must be in the direction perpendicular to the corresponding diagonal. Thus, we can set up a free-body diagram equation for each rod, with the two forces at the joints and the centripetal force.

Since these forces must cancel, we get $2F_{\text{joint}}\cos(\pi/4) = F_{\text{cent}} = m\omega^2 d$, so $F_{\text{joint}} = m\omega^2 d/\sqrt{2}$

5. A pendulum of length L oscillates inside a box. A person picks up the box and gently shakes it horizontally with frequency ω and a fixed amplitude for a fixed time. The final amplitude can be maximized if ω satisfies

   (A) $\omega = \sqrt{g/L}$ ← **CORRECT**

   (B) $\omega = 2\sqrt{g/L}$

   (C) $\omega = (1/2)\sqrt{g/L}$

   (D) There will be no effect on the amplitude for any value of ω.

   (E) None of the above

Solution

In the frame of the box, this is equivalent to a horizontal driving force with frequency ω. To increase the amplitude, the driving should be at resonance, so $\omega = \sqrt{g/L}$.

6. A planet is orbiting a star in a circular orbit of radius r_0. Over a very long period of time, much greater than the period of the orbit, the star slowly and steadily loses 1% of its mass. Throughout the process, the planet's orbit remains approximately circular. The final orbit radius is closest to

   (A) $1.02r_0$

   (B) $1.01r_0$ ← **CORRECT**

   (C) r_0

   (D) $0.99r_0$

   (E) $0.98r_0$

Solution

The angular momentum $L = mvr$ is conserved. Moreover, since the orbit is always circular, the kinetic energy and potential energy are related by $2K = -U$, giving $mv^2 = GMm/r$, where M is the mass of the star. Therefore $M \propto v^2 r \propto L^2/r$, so a 1% decrease in the star's mass gives a 1% increase in the orbital radius.

7. An astronaut standing on the exterior of the international space station wants to dispose of three pieces of trash. They face the station's direction of travel with the Earth to their left. From the astronaut's perspective, the three pieces are thrown (I) left, (II) right, and (III) up. To the astronaut's frustration, some of the pieces of trash return to the space station after several hours. They are

   (A) II only

   (B) III only

   (C) I and II

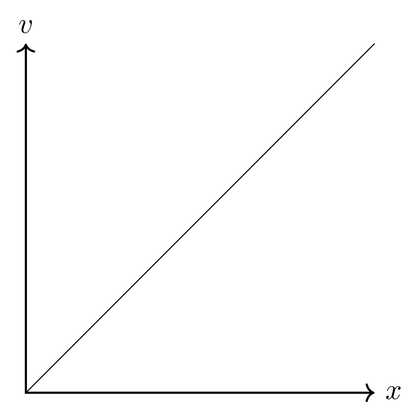
   (D) II and III

   (E) I, II, and III ← **CORRECT**

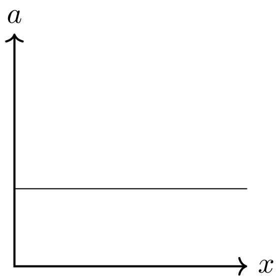
Solution

By Kepler's third law, $T^2 \propto a^3$ where a is the semimajor axis, and $E = -GMm/2a$. The astronaut's throw has a negligible impact on the total energy of the trash, and hence a negligible impact on a and hence T. Thus, after one full orbit, which takes about 1.5 hours, all three pieces of trash simply return to the space station.

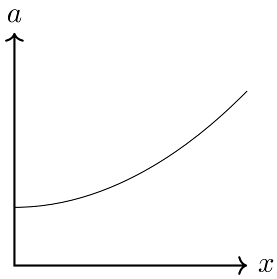
8. The velocity versus position plot of a particle is shown below. Which following choices is the correct acceleration vs. position plot of the particle?



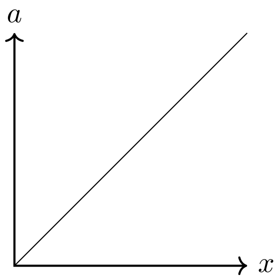
(A)



(B)

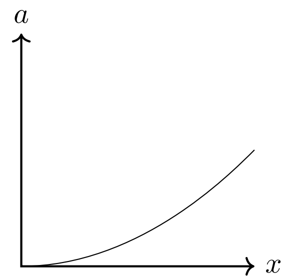


(C)

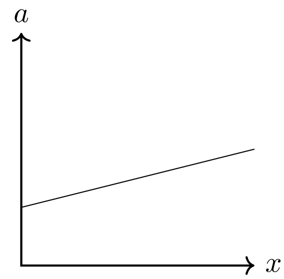


← **CORRECT**

(D)



(E)



Solution

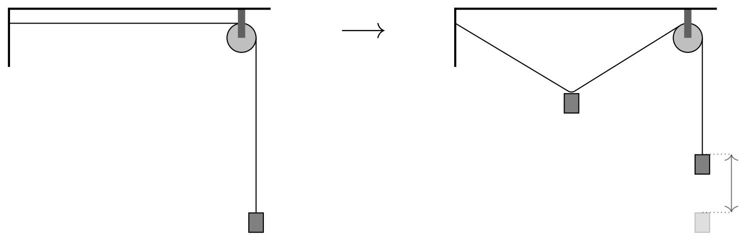
Suppose the particle starts at $x = 0$. Then its velocity is zero, meaning it remains at 0, and its acceleration is zero. So the correct acceleration plot must start at (0,0). It remains to determine whether the plot of acceleration vs time curves upward or is a straight line.

Suppose it is a straight line, as shown in choice (c). Then the average force on the particle traveling from some very small value to some x is proportional to x. The energy of the particle, which is the average force multiplied by x, is then proportional to x^2. Because the energy is the square of the velocity, we have $v^2 \propto x^2$, or simply $v \propto x$, as desired, so choice (c) is the one consistent with the shown v vs x plot.

For a calculus-based solution, the velocity is given by $v = kx$ for some k. Differentiating both sides with respect to time, we find $a = kv = k^2 x$, so again we find that the acceleration is a straight line starting at zero and sloping upward.

The following information is relevant to problems 9 and 10.

9. A block of mass m is attached to a massless string. The string is passed over a massless pulley and the end of the string is fixed in place. The horizontal part of the string has length L. Now a small mass m is hung from the horizontal part of the string, and the system comes to equilibrium. (Diagram not necessarily to scale.)



Neglecting friction everywhere, the tension at the end of the string is

(A) $mg/2$

(B) mg $\leftarrow$ **CORRECT**

(C) $3mg/2$

(D) $2mg$

(E) $3mg$

Solution

In equilibrium, the tensions on both sides of any pulley must be equal, for torque balance on the pulley. The same logic applies to the mass; one can think of it as just another pulley. Hence the tension everywhere in the string is mg.

10. During this process, the block has been raised by approximately a height

(A) $0.15L$ $\leftarrow$ **CORRECT**

(B) $0.23L$

(C) $0.31L$

(D) $0.37L$

(E) $0.40L$

Solution

For force balance on the disc, the strings must attach to it at an angle of $30°$ to the horizontal; this also implies the mass must be placed at the middle of the horizontal section. Hence the horizontal section of string, originally of length L, becomes two tilted sections, each with length $L/\sqrt{3}$. The change in length is $L(2/\sqrt{3} - 1)$, which is equal to the increase in height of the block.

11. The maximal tension per area a material can sustain without failure is called its *tensile strength*. Plain steel has a tensile strength of 415 MPa. What is the maximal mass one can hang on a vertical steel rod of negligible mass and a diameter of 2 cm?

(A) 1300 kg

(B) 5200 kg

(C) 13 000 kg $\leftarrow$ **CORRECT**

(D) 52 000 kg

(E) The answer depends on the length of the steel rod.

Solution

Balancing the forces, we have:

$$m_{\text{max}} g = \sigma \pi r^2$$

$$\rightarrow m_{\text{max}} = 13000\text{kg}$$

12. A point mass m is glued inside a massless hollow rod of length L at an unknown location. When the rod is pivoted at one end, the period of small oscillations is T. When the rod is pivoted at the other end, the period of small oscillations is $2T$. How far is the mass from the center?

(A) $L/8$

(B) $L/6$

(C) $L/4$

(D) $3L/10$ $\leftarrow$ **CORRECT**

(E) $2L/5$

Solution

We have $T \propto \sqrt{\Delta L}$, where ΔL is the distance from the pivot point to the mass. Then the ratio of distances is 4, so the mass is $0.2L$ from one of the ends, or $0.3L$ from the center.

13. A ballerina with moment of inertia I is quickly twirling with angular velocity ω. In her hand she has a pen of mass m at a radius R from her axis of rotation. The ballerina releases the pen. Afterward, what happens to the vertical component of the angular momentum of the system consisting of the ballerina and the pen? You may ignore all friction, but not gravity or normal forces.

(A) It decreases until the pen hits the floor.

(B) It increases until the pen hits the floor.

(C) It always stays the same. $\leftarrow$ **CORRECT**

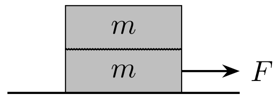
(D) It initially stays the same, but decreases when the pen hits the floor.

(E) It initially stays the same, but increases when the pen hits the floor.

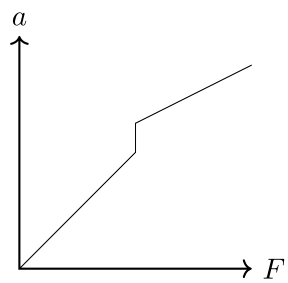
Solution

There are no external vertical torques in this problem, as both the gravity and normal forces are vertical, so the vertical component of angular momentum is conserved.

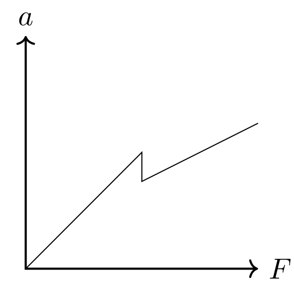
14. Two blocks of mass m are placed on top of each other, and the bottom block is placed on the ground. The ground is frictionless. The static and kinetic coefficients of friction between the two blocks are μ_s and μ_k, with $\mu_s < \mu_k$. The blocks are at rest initially. When a constant horizontal force F is then applied to the bottom block, which of the following graphs could show its acceleration as a function of F?



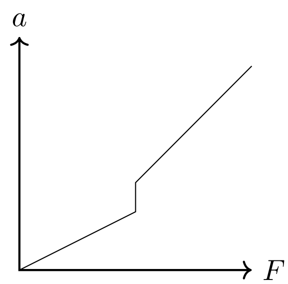
(A)



(D)

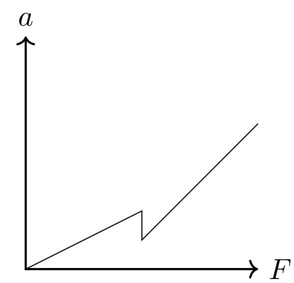


(B)

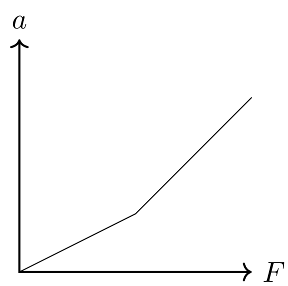


← **CORRECT**

(E)



(C)

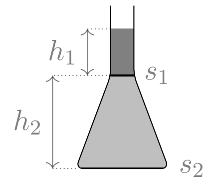


Solution

The statement $\mu_s < \mu_k$ above is a typo, which makes the setup not make physical sense. As a result, credit was given for all answers. The intended solution (for $\mu_k < \mu_s$) is as follows.

For small forces, the blocks move together, and the acceleration is linear in the force. At a certain threshold, the top block begins to slip. Because $\mu_k < \mu_s$, the friction force decreases suddenly at this threshold, so the acceleration of the bottom block increases suddenly. For higher forces, the acceleration is again linear in the force, but with a greater slope, since the top block is not moving with the bottom block.

15. As shown in the figure, a vessel contains two types of liquid: the liquid with density ρ_1 on top and ρ_2 on the bottom. The depth of the top liquid is h_1, and the interface area between the top and the bottom liquid is s_1. The bottom liquid has a depth of h_2. The area of the bottom of the vessel is s_2. What is the gauge pressure (i.e. pressure in excess of atmospheric pressure) at the bottom of the vessel?



(A) $(\rho_1 h_1 + \rho_2 h_2)g$ ← **CORRECT**

(B) $\dfrac{\rho_1 s_1 h_1 + \rho_2 s_2 h_2}{s_2} g$

(C) $\dfrac{1}{2}(\rho_1 + \rho_2)(h_1 + h_2)g$

(D) $(\rho_1 + \rho_2)(h_1 + h_2)g$

(E) $\rho_2(h_1 + h_2)g$

Solution

The gauge pressure is zero by definition at the top, so it is $\rho_1 h_1 g$ at the liquid interface. The pressure then increases by $\rho_2 h_2 g$ from here to the bottom of the vessel, giving the answer, $(\rho_1 h_1 + \rho_2 h_2)g$.

16. Liquid droplets store a given amount of potential energy per unit surface area, due to their surface tension. When two identical, nearly spherical liquid droplets coalesce on a certain type of surface, part of this energy can be converted into upward kinetic energy, causing the coalesced droplet to jump. Assuming the conversion is 100% efficient, how does the maximum height h depend on the radius r of the initial droplets?

(A) $h \propto r$

(B) $h \propto r^{1/2}$

(C) $h \propto r^{-1/2}$

(D) $h \propto r^{-1}$ ← **CORRECT**

(E) $h \propto r^{-2}$

Solution

The change in surface area, and hence the energy released, is proportional to r^2. The mass is proportional to r^3. Since the energy released is equal to mgh, we have $h \propto r^{-1}$.

17. Paul the Giant stands outside on a force-meter calibrated in Newtons, which reads 5000 N. Paul is wearing a large cowboy hat, which has horizontal cross-sectional area $A = 1\,\text{m}^2$ and completely covers both him and the scale when seen from directly above. At time $t = 0$, rain begins to fall vertically downward on Paul, and any rain that hits his hat is collected in the hat's brim. The raindrops have a constant downward speed of 1 m/s, and the rain accumulates on the ground at a rate of 1 mm/s. What is the reading (in N) on the scale as a function of the time $t > 0$ (in s)? The density of water is $1000\,\text{kg/m}^3$.

(A) $5001 + 11t$

(B) $5001 + 10t$ ← **CORRECT**

(C) $5000 + 11t$

(D) $5001 + 1.1t$

(E) $5001 + t$

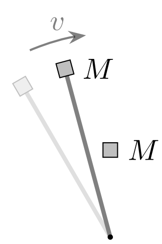
Solution

The force on the scale has three components: (1) Shrek's weight, (2) the weight of the water collected in the ogre hat, and (3) the pressure associated with changing the momentum of the downward-falling rain. Component (1) gives a constant weight 5000 N. Component (2) is equal to the weight of the water collected in the hat. The total volume of water collected is $V = (1\,\text{mm/s}) \times A \times t$, and the weight of the water is $V \times (1000\,\text{kg/m}^3) \times g = 10 \times t$.

Component (3) can be found by noting that in a time t, the total mass $M = \rho V$ of water that hits the hat has its momentum changed from $p = Mv$ to zero. Here $\rho = 1000\,\text{kg/m}^3$ is the density of water. The force applied by the hat on the water is therefore $F = \Delta p/\Delta t = Mv/t = 1$ N.

Adding these three contributions together gives a total weight $W = 5000 + 1 + 10t$.

18. A massless rigid rod is pivoted at one end, and a mass M is at the other end. Originally, the rod rotates frictionlessly about the pivot with a uniform angular velocity such that the mass M has speed v. The rotating rod collides with another mass M at its midpoint, which then sticks to the rod. After the collision, what is the kinetic energy of the system?



(A) $\frac{1}{4}Mv^2$

(B) $\frac{1}{3}Mv^2$

(C) $\frac{7}{18}Mv^2$

(D) $\frac{2}{5}Mv^2$ ← **CORRECT**

(E) $\frac{1}{2}Mv^2$

Solution

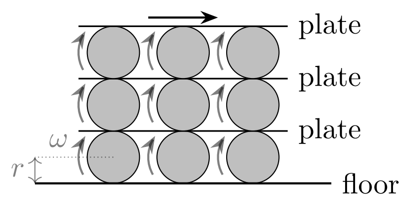
Angular momentum about the pivot is conserved in the collision. Let the final speed of the original mass M be v'. Then the final speed of the second mass is $v'/2$. Balancing the initial and final angular momentum,

$$Mvr = Mv'r + M(v'/2)(r/2)$$

which gives $v' = (4/5)v$. The final kinetic energy is

$$\frac{1}{2}M(v')^2 + \frac{1}{2}M(v'/2)^2 = \frac{2}{5}Mv^2.$$

19. A system of cylinders and plates is set up as shown. The cylinders all have radius r, and roll without slipping to the right with angular velocity ω. What is the speed of the top plate?



(A) ωr

(B) $2\omega r$

(C) $3\omega r$

(D) $4\omega r$

(E) $6\omega r$ ← **CORRECT**

Solution

Instantaneously, each cylinder can be thought of as rotating about its point of contact with the ground or plate below. The rotational angular velocity is ω. This means that, relative to the bottom of the cylinder, the top is moving at speed $2\omega r$. So the first plate is moving to the right at $2\omega r$. The second plate moves to the right at $2\omega r$ faster than the first, and the top plate moves to the right at $6\omega r$.

20. A car is driving against the wind at a constant speed v_0 relative to the ground. The wind direction is always opposite to the car's velocity, but its speed fluctuates about an average speed of v relative to the ground. The air drag force is Av_{rel}^2, where A is a constant and v_{rel} is the relative speed between the car and the wind. What is the average rate $\overline{P}$ of energy dissipation due to the air resistance?

(A) $\overline{P} = Av_0(v_0 + v)^2$

(B) $\overline{P} > Av_0(v_0 + v)^2$ ← **CORRECT**

(C) $\overline{P} < Av_0(v_0 + v)^2$

(D) Both (B) and (C) are possible depending on how v fluctuates.

(E) Both (A) and (C) are possible depending on how v fluctuates.

Solution

Define $u = v_0 + v$. Then the relative speed of the car and wind is $u + \delta u$, where δu averages to zero. The instantaneous energy dissipation rate is

$$P = Fv_0 = Av_0(u + \delta u)^2 = Av_0(u^2 + 2u\,\delta u + (\delta u)^2).$$

On average, the second term in parentheses will be zero, because δu averages to zero. But the third term is positive. So if the wind fluctuates at all, then on average,

$$\overline{P} > Av_0 u^2 = Av_0(v_0 + v)^2.$$

21. A circular table has radius R and $N > 2$ equally spaced legs of length h attached to its perimeter. Suppose the table has a uniform mass density with total mass m, and neglect the mass of the legs. Assuming the table does not slip, the minimum horizontal force needed to tip over the table is

(A) $\dfrac{mgR}{h}$

(B) $\dfrac{mgR}{h}\sin\left(\dfrac{N-2}{2N}\pi\right)$ ← **CORRECT**

(C) $\dfrac{mgR}{h}\cos\left(\dfrac{\pi}{N}\right)$ ← **CORRECT**

(D) $\dfrac{mgR}{h}\tan\left(\dfrac{N-2}{2N}\pi\right)$

(E) $\dfrac{mgR}{h}\sin\left(\dfrac{\pi}{2N}\right)$

Solution

Label the tops of two adjacent legs by A and B. The best way to push is to push directly opposite the table from the midpoint of AB, directly towards this midpoint.

When the table is about to tip over, these two legs will bear all of the weight of the table, and hence provide all of the frictional force. Balancing torques on the table about the axis AB, we have $Fh = mgR'$ where $R' = R\cos(\pi/N)$ is the distance from the center ofthe table to the midpoint of AB. Solving for F gives the answer. Note that credit was given for both choices (B) and (C), since they are equivalent.

22. A collision occurs between two masses. In each inertial reference frame, one can compute the change in total momentum $\Delta\mathbf{P}$ and the change in total kinetic energy ΔK due to the collision. Which of the following is true?

(A) $\Delta\mathbf{P}$ and ΔK do not depend on the frame. ← **CORRECT**

(B) $\Delta\mathbf{P}$ and ΔK do not depend on the frame for perfectly elastic collisions, but $\Delta\mathbf{P}$ may depend on the frame for inelastic collisions.

(C) $\Delta\mathbf{P}$ and ΔK do not depend on the frame for perfectly elastic collisions, but ΔK may depend on the frame for inelastic collisions.

(D) $\Delta\mathbf{P}$ and ΔK do not depend on the frame for perfectly elastic collisions, but both may depend on the frame for inelastic collisions.

(E) $\Delta\mathbf{P}$ and ΔK may both depend on the frame, for both perfectly elastic and inelastic collisions.

Solution

In a collision, momentum is conserved whether the collision is elastic or inelastic. This means $\Delta\mathbf{P} = 0$ regardless of reference frame, so $\Delta\mathbf{P}$ does not depend on frame.

In an elastic collision, $\Delta K = 0$, which is again independent of frame. In an inelastic collision, whatever kinetic energy is dissipated will go to heat up the masses (assuming they are isolated, and energy can only go into thermal energy). The heat capacity and temperature change of the masses are both frame-invariant, so the kinetic energy dissipated is again independent of reference frame. (This can also be shown more straightforwardly by direct calculation.)

23. Steve determines the spring constant k of a spring by applying a force F to it and measuring the change in length Δx. The tools he uses to measure F and Δx both have a constant absolute uncertainty, leading to an uncertainty in k of δk_S. If Tiffany measures the same spring constant with the same tools but by using a force that is five times larger, what will her uncertainty in k be in terms of δk_S?

(A) $\delta k_T = 0.04\,\delta k_S$

(B) $\delta k_T = 0.08\,\delta k_S$

(C) $\delta k_T = 0.2\,\delta k_S$ ← **CORRECT**

(D) $\delta k_T = 0.4\,\delta k_S$

(E) $\delta k_T = 0.5\,\delta k_S$

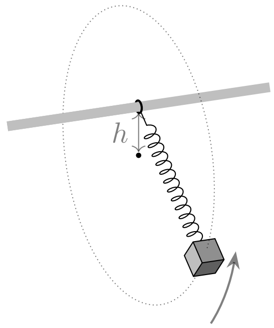
Solution

Conceptually, quintupling all forces, and consequently Δx (including the errors) would leave the error in k the same. So quintupling the values but not the errors multiplies the error in k by 0.2. Quantitatively, we use the equation for relative errors:

$$\left(\frac{\delta k}{k}\right)^2 = \left(\frac{\delta F}{F}\right)^2 + \left(\frac{\delta x}{\Delta x}\right)^2$$

The denominators on the right hand side will quintuple when Tiffany does the measurement, hence, the new uncertainty is $\frac{1}{5}\delta k_S$.

24. A mass m is connected to one end of a zero-length spring with spring constant k. The other end of the spring is connected to a frictionless bearing mounted around a horizontal pole so that the mass can swing in a vertical circle of radius R around the pole. The setup is shown in the figure below. What is the vertical distance h between the center of the circular orbit and the axis of the pole? Assume that both the diameter of the pole and the rest length of the spring are negligible compared to R.



(A) $\sqrt{mgR/k}$

(B) $R\sqrt{(R+mg/k)/(R-mg/k)}$

(C) $R - mg/k$

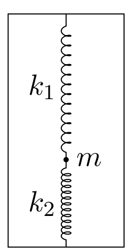
(D) mg/k ← **CORRECT**

(E) $\sqrt{R^2 - (mg/k)^2}$

Solution

Consider the instant when the mass is moving vertically upward. In this instant the mass's acceleration is perfectly horizontal, which means that the vertical component of the force from the spring must be equal and opposite to the force of gravity. The angle θ that the spring makes with the horizontal satisfies $\tan\theta = h/R$, where h is the desired vertical distance. The length of the spring in this instant is $\sqrt{h^2 + R^2}$, so the upward force is $k\sqrt{h^2+R^2}\sin\theta = kh$. This should be set equal to mg, which means $h = mg/k$.

25. A ball of negligible radius and mass m is connected to two ideal springs. Each spring has rest length ℓ_0. The springs are connected to the ball inside a box of height $2\ell_0$, and the ball is allowed to come to equilibrium, as shown. Under what condition is this equilibrium point stable with respect to small horizontal displacements?



(A) $k_1 > k_2$

(B) $k_2 > k_1$

(C) $k_1 - k_2 > mg/\ell_0$ ← **CORRECT**

(D) $k_1k_2/(k_1+k_2) > mg/\ell_0$

(E) $k_1k_2/(k_1-k_2) > mg/\ell_0$

Solution

Suppose we call z the height measured from the middle of the box and x the horizontal displacement, also measured from the middle of the box. When the ball is at equilibrium at some z_0, the springs obey

$$-k_1 z_0 - k_2 z_0 = mg.$$

We can use this to find the equilibrium position,

$$z_0 = \frac{-mg}{k_1 + k_2}.$$

Imagine a horizontal displacement of size x. If x is small, the tension in the top spring is not changed from the tension at equilibrium (to first order in x). So the restoring force is

$$F_1 = T_1 \sin\theta \approx T_1 \theta \approx \frac{-k_1 x z_0}{\ell_0 - z_0}$$

where $\theta \approx \frac{x}{\ell_0 - z_0}$ is the angle the spring makes with the vertical.

The bottom spring similarly exerts a force

$$F_2 \approx \frac{k_2 x z_0}{\ell_0 + z_0}$$

pushing the ball away from equilibrium.

For equilibrium to be stable, for positive x, we must have

$$F_1 + F_2 < 0$$

Putting in F_1 and F_2, we have

$$-\frac{k_1 x z_0}{\ell_0 - z_0} + \frac{k_2 x z_0}{\ell_0 + z_0} < 0$$

Dividing by x and z_0, which we recall is negative,

$$\frac{k_1}{\ell_0 - z_0} < \frac{k_2}{\ell_0 + z_0}.$$

Plugging in our expression for z_0 and rearranging this, we derive

$$k_1 - k_2 < \frac{mg}{\ell_0}.$$

AAPT UNITED STATES PHYSICS TEAM
AIP 2020

2020 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. The only scratch paper you may use is scratch paper provided by the proctor. You may not use your own.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones cannot be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
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Ariel Amir, JiaJia Dong, Mark Eichenlaub, Abijith Krishnan, Daniel Longenecker, Kye Shi, Brian Skinner, Paul Stanley, Mike Winer, and Kevin Zhou.

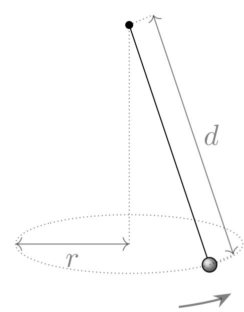
1. A ball is bouncing vertically between a floor and ceiling, which are both horizontal and separated by 4 m. All collisions are perfectly elastic, and when the ball hits the floor, it has a speed of 12 m/s. How long does a complete up-down cycle take?

   (A) 0.3 s
   (B) 0.4 s
   (C) 0.6 s
   (D) 0.8 s
   (E) 2.4 s

2. A uniform rod of mass m and length ℓ has moment of inertia $m\ell^2/12$ about an axis that is perpendicular to the rod and passes through its center. What is the moment of inertia of a uniform square plate with mass M and side length L about the axis along its diagonal?

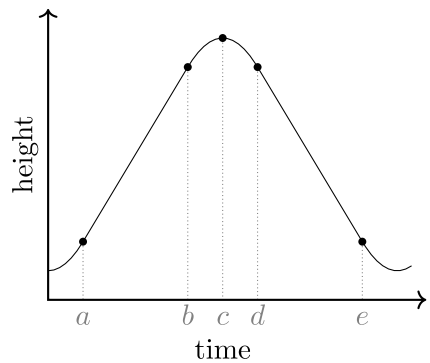
   (A) $ML^2/12$
   (B) $\sqrt{2}\,ML^2/12$
   (C) $ML^2/6$
   (D) $ML^2/4$
   (E) $ML^2/3$

3. A conical pendulum of length d swings in a horizontal circle of radius r, as shown. If ω is the angular frequency of this motion, what is ω^2?



   (A) g/d
   (B) g/r
   (C) $g/\sqrt{d^2 + r^2}$
   (D) $g/\sqrt{d^2 - r^2}$
   (E) $g/\sqrt{dr}$

4. In "zero-g" airplane rides, passengers can float around the cabin, as if they were weightless. A flight trajectory for such a ride is shown below, with a few points during the journey labelled. Which of the following statements is correct? Take $\mathbf{a}$ to be the acceleration of the plane during the "zero-g" part of the flight.



(A) The "zero-g" flight begins at a and ends at c, during which $|\mathbf{a}| = g$ and $\mathbf{a}$ points up.

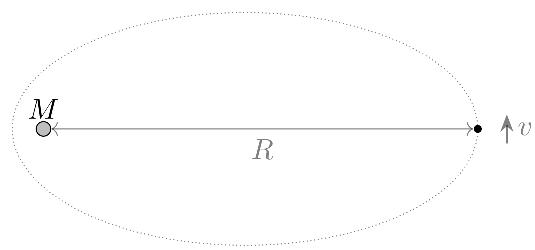
(B) The "zero-g" flight begins at a and ends at e, during which $|\mathbf{a}| = 0$.

(C) The "zero-g" flight begins at b and ends at d, during which $|\mathbf{a}| = g$ and $\mathbf{a}$ points down.

(D) The "zero-g" flight begins at c and ends at e, during which $|\mathbf{a}| = g$ and $\mathbf{a}$ points down.

(E) The "zero-g" flight begins at d and ends at e, during which $|\mathbf{a}| = 0$.

5. A mote of dust is initially located at distance R from the sun, which has mass M. At this point, the mote has a small tangential velocity v. Which of the following is a good approximation for the distance of closest subsequent approach between the mote and the sun?



(A) $\dfrac{R^3 v^4}{G^2 M^2}$

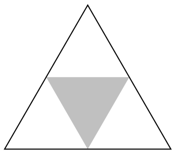
(B) $\dfrac{R^3 v^4}{2G^2 M^2}$

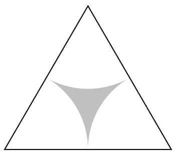
(C) $\dfrac{R^2 v^2}{2GM}$

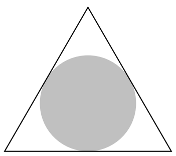
(D) $\dfrac{R^2 v^2}{GM}$

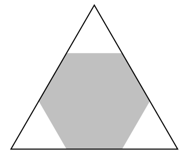
(E) $\dfrac{2R^2 v^2}{GM}$

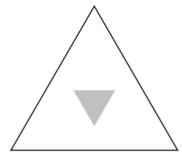
6. A three-legged table is shaped like a uniform equilateral triangle, and has identical legs at each corner. When the mass of an object placed on the center of the table exceeds $m_{\max}$, the table's legs will all simultaneously break. Which of the following shaded regions shows the area within which an object of $2m_{\max}/3$ can be placed without breaking any legs of the table?

(A) 

(B) 

(C) 

(D) 

(E) 

7. Two satellites are initially in identical circular orbits around the Sun, with orbital speed $v = 1 \times 10^4\,\mathrm{m/s}$. The first satellite fires its thrusters toward the Sun, and quickly obtains a radial velocity of $\Delta v_r = 1\,\mathrm{m/s}$. The second satellite instead fires its thrusters behind it, and quickly increases its tangential velocity by Δv_t. If the two satellites subsequently perform orbits with the same period, approximately what was Δv_t?

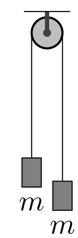
(A) $0.000\,05\,\mathrm{m/s}$
(B) $0.005\,\mathrm{m/s}$
(C) $0.5\,\mathrm{m/s}$
(D) $1\,\mathrm{m/s}$
(E) $50\,\mathrm{m/s}$

8. A conveyor belt is moving with velocity v to the east. A block with velocity v to the south slides from the ground onto the conveyor belt. The coefficient of friction between the block and the belt is μ. The block stops slipping after a time

(A) $\dfrac{v}{\sqrt{2}\,\mu g}$
(B) $\dfrac{v}{\mu g}$
(C) $\dfrac{\sqrt{2}\,v}{\mu g}$
(D) $\dfrac{2v}{\mu g}$
(E) The block never stops slipping.

The following information is relevant to problems 9 and 10.

9. Two equal masses m are connected by an elastic string that acts like an ideal spring with spring constant k and unstretched length l. The two masses are hung over a frictionless pulley. What is the total length of the string at equilibrium? (Diagram not necessarily to scale.)



(A) $2mg/k$
(B) $l + mg/k$
(C) $l + mg/2k$
(D) $l + 2mg/k$
(E) There is not enough information to decide.

10. The two masses are both displaced downward by a small vertical distance x and simultaneously released from rest. What is the period of oscillation?

(A) $2\pi\sqrt{l/g}$
(B) $\pi\sqrt{2m/k}$
(C) $2\pi\sqrt{m/k}$
(D) $2\pi\sqrt{m/k + l/g}$
(E) There is not enough information to decide.

11. An escalator can carry passengers up a vertical distance of 10 m in 30 s. A mischievous person of mass 50 kg walks down the up-escalator so that they stay in place with respect to the building. If the child does this for 30 s, the total work the child performs on the escalator, in the frame of the building, is

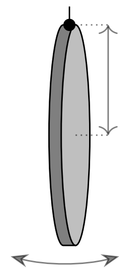
(A) -10^4 J
(B) -5×10^3 J
(C) 0 J
(D) 5×10^3 J
(E) 10^4 J

12. A platform juts out horizontally from the edge of a building. If the platform is modeled as a uniform metal rod, which one of the following statements about the tensile and compressive stress is correct?

(A) There is a horizontal compression throughout the rod.
(B) There is a horizontal tension throughout the rod.
(C) There is horizontal tension in the top of the rod and a compression in the bottom.
(D) There is a horizontal tension near the middle of the rod and a compression near the end.

(E) There is a horizontal compression near the middle of the rod and a tension near the end.

13. A uniform disk of mass m and radius r is attached at its edge to a flexible pivot on the ceiling. It is given a small displacement *perpendicular* to the plane of the disk, so that it begins to oscillate perpendicular to the plane of the disk. What is the period of oscillation? The moment of inertia of a disk about the axis going through its center and perpendicular to the plane it's in is $I_{\text{disk}} = \frac{1}{2}mr^2$

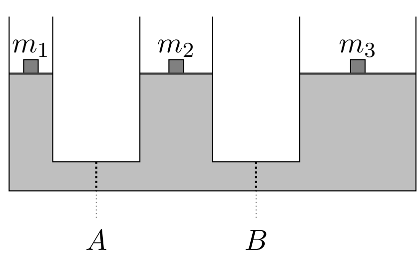


(A) $\pi\sqrt{2r/5g}$
(B) $\pi\sqrt{5r/g}$
(C) $\pi\sqrt{6r/g}$
(D) $2\pi\sqrt{r/g}$
(E) $2\pi\sqrt{2r/g}$

14. A large block of mass 5 kg is moving to the right at a velocity of 10 m/s. Spaced out every meter are smaller, initially stationary blocks of mass 1 kg. All collisions are perfectly inelastic. Neglecting friction, how far will the large block travel before its velocity has decreased to below 3 m/s?

(A) 5 m
(B) 8 m
(C) 12 m
(D) 17 m
(E) 50 m

15. In the device shown, blocks of various masses are placed on pistons so that the device is in equilibrium. (The fluid in the drawing is to scale.) There are valves that are both initially open at locations A and B. One of the valves is closed, and the system is allowed to come to equilibrium. How will this affect the height of the mass m_2?



(A) Closing either valve will have no effect on the height of m_2.
(B) Closing either valve causes m_2 to rise.
(C) Closing either valve causes m_2 to fall.
(D) Closing valve A causes m_2 to rise, and closing valve B causes m_2 to fall.
(E) Closing valve A causes m_2 to fall, and closing valve B causes m_2 to rise.

The following information is relevant to problems 16 and 17.

16. A mass M sits on top of a vertical spring of spring constant k, in equilibrium. A mass m is held a height h above it. The mass M is then pushed downward by a distance Δx, and both masses are released from rest simultaneously. For what value of h will the two masses first collide when M first returns to its equilibrium position?

(A) $\dfrac{\pi M^2 g^2}{4k^2 \Delta x}$

(B) $\dfrac{8Mg}{\pi^2 k}$

(C) $\dfrac{Mg}{k}$

(D) $\dfrac{\pi^2 Mg}{8k}$

(E) $\dfrac{\pi k \Delta x^2}{4Mg}$

17. Assume that h takes the value found in the previous question, and that the collision between the two masses is perfectly elastic. For what value of Δx will m rebound to a maximum height that is exactly equal to its original height?

(A) $\dfrac{\pi g}{2k}\sqrt{\dfrac{m^3}{M}}$

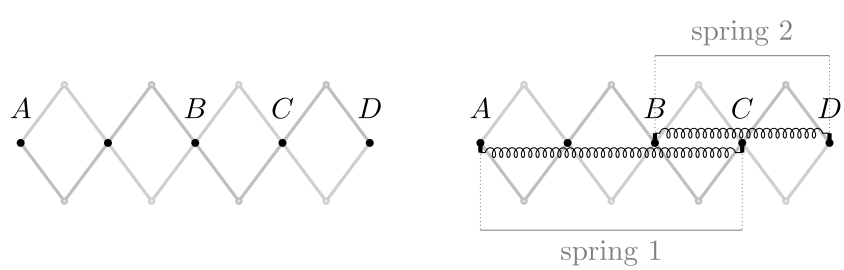
(B) $\dfrac{\pi g}{k}\sqrt{\dfrac{m^3}{2M}}$

(C) $\dfrac{2g}{\pi k}\sqrt{\dfrac{2m^3}{M}}$

(D) $\dfrac{4mg}{\pi k}$

(E) $\dfrac{\pi mg}{2k}$

18. An extendable arm is made from rigid beams free to pivot around the dots shown. Spring 1, with equilibrium length L_1, is attached between points A and C while spring 2, with equilibrium length L_2 is attached between B and D. The system is allowed to come to equilibrium. In equilibrium, what is the ratio of the tension in spring 1 to the tension in spring 2?



(A) $-2/3$

(B) $-3/4$

(C) -1

(D) $-3/2$

(E) There is not enough information to determine the ratio.

19. Consider an axially symmetric object that experiences no external forces and initially rotates about its symmetry axis. The object then changes its shape while remaining axially symmetric. Afterward, it is found that its moment of inertia about the symmetry axis has increased. How have its kinetic energy T and angular velocity ω changed?

    (A) T has decreased, and ω has decreased.
    (B) T has decreased, and ω stays the same.
    (C) T stays the same, and ω has decreased.
    (D) T has increased, and ω has increased.
    (E) T has increased, and ω stays the same.

20. A well-calibrated scale reads zero when nothing is placed on it. When a deflated balloon is placed on the scale, it reads mg. When a non-airtight box is placed on the scale, it reads Mg, where $M > m$. The balloon is inflated with helium, so that it floats upward in air. The balloon is placed in the box, and the box is placed on the scale. If the scale reads W, which of the following is true?

    (A) $W \leq (M - m)g$
    (B) $(M - m)g < W < (M + m)g$
    (C) $W = (M + m)g$
    (D) $W > (M + m)g$
    (E) None of the above are necessarily true.

21. A person stands on the seat of a swing and squats down, so that the distance between their center of mass (CM) and the swing's pivot is ℓ. As the swing gets to the lowest point, the speed of their CM is v. At this moment, they quickly stand up, and thus decrease the distance from their CM to the swing's pivot to ℓ'. Immediately after they finish standing up, their CM speed is v'. Which of the following statements is correct? You may neglect friction, the change in moment of inertia of the person about their CM, and the time taken to stand up.

    (A) $v/\ell = v'/\ell'$
    (B) $v = v'$
    (C) $v\ell = v'\ell'$
    (D) $\frac{1}{2}v^2 = \frac{1}{2}v'^2 + g(\ell - \ell')$
    (E) Multiple statements are correct.

22. A point mass m sits on a long block, also of mass m, which rests on the floor. The coefficient of static and kinetic friction between the mass and the block is μ, and the coefficient of static and kinetic friction between the block and the floor is $\mu/3$. An impulse gives a horizontal momentum p to the point mass. After a long time, how far has the point mass moved relative to the block? Assume the mass does not fall off the block.

    (A) $\dfrac{3p^2}{8m^2\mu g}$
    (B) $\dfrac{15p^2}{32m^2\mu g}$
    (C) $\dfrac{9p^2}{16m^2\mu g}$

(D) $\dfrac{3p^2}{10m^2\mu g}$

(E) $\dfrac{3p^2}{4m^2\mu g}$

23. Assume that the drag force for a fish in water depends only on the typical length scale of the fish R, its velocity v, and the density of water ρ. A pufferfish is about 10 cm in length and swims at about 5 m/s. How fast does a clown fish, about 1 cm in length, need to swim such that it experiences the same drag force as the pufferfish?

(A) 5 m/s

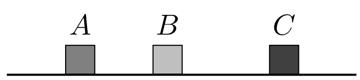
(B) 16 m/s

(C) 50 m/s

(D) 500 m/s

(E) 2500 m/s

24. Three boxes A, B and C lie along a straight line on a horizontal frictionless surface, as shown. Box A is initially moving to the right with speed v while the other two boxes are initially at rest. If all collisions are elastic, and the masses of the boxes can be chosen freely, which of the following is closest to the maximum possible final speed of box C?



(A) v

(B) $2v$

(C) $3v$

(D) $4v$

(E) $5v$

25. If a certain radioactive decay process happens n times per hour on average, then in any given hour, one expects to observe n decays with an uncertainty of $\sqrt{n}$. Each hour can be assumed independent of the previous one, and n can be assumed constant over time. How many hours do you need to conduct the observation so that you can determine n within an uncertainty of 1%?

(A) $n/10^2$

(B) $n/10^4$

(C) $10^2/n$

(D) $10^4/n$

(E) $10^8/n^2$

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1. A ball is bouncing vertically between a floor and ceiling, which are both horizontal and separated by 4 m. All collisions are perfectly elastic, and when the ball hits the floor, it has a speed of 12 m/s. How long does a complete up-down cycle take?

   (A) 0.3 s

   (B) 0.4 s

   (C) 0.6 s

   (D) 0.8 s ← **CORRECT**

   (E) 2.4 s

Solution

If the ball begins at the floor and travels towards, the ceiling, its height h above the floor at a time t is given by

$$h = (12\ \mathrm{m/s})t - \frac{1}{2}gt^2.$$

To find the time for the ball to reach the ceiling, we substitute in $h = 4$ m and $g = 10$ m/s^2. Solving the quadratic equation for t, we obtain

$$t_{\mathrm{top}} = 0.4\,\mathrm{s}.$$

By symmetry, the time for the ball to go down from the ceiling to the floor is the same as the time for the ball to go from the floor to the ceiling. This means the answer is 2×0.4 s $= 0.8$ s.

2. A uniform rod of mass m and length ℓ has moment of inertia $m\ell^2/12$ about an axis that is perpendicular to the rod and passes through its center. What is the moment of inertia of a uniform square plate with mass M and side length L about the axis along its diagonal?

   (A) $ML^2/12$ ← **CORRECT**

   (B) $\sqrt{2}\,ML^2/12$

   (C) $ML^2/6$

   (D) $ML^2/4$

   (E) $ML^2/3$

Solution

Place the origin at the center of mass of the square. The perpendicular axis theorem states that for a two-dimensional mass distribution in the x-y plane,

$$I_z = I_x + I_y.$$

If the x and y axes are oriented parallel to the square's sides, then the moments of inertia I_x and I_y are both those of a uniform rod of mass M and length L,

$$I_x = I_y = \frac{ML^2}{12}.$$

This implies

$$I_z = \frac{ML^2}{6}.$$

Now consider rotating these axes by 45° about the z axis, giving axes x' and y' along the diagonals of the square. The perpendicular axis theorem now gives

$$I_z = I'_x + I'_y$$

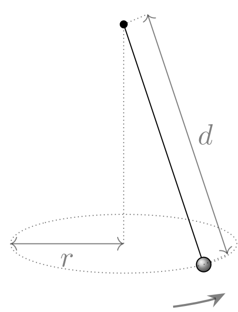
where I'_x is the desired answer. By symmetry, $I'_x = I'_y$, so

$$\frac{ML^2}{6} = 2I'_x$$

and

$$I'_x = \frac{ML^2}{12}.$$

3. A conical pendulum of length d swings in a horizontal circle of radius r, as shown. If ω is the angular frequency of this motion, what is ω^2?



(A) g/d

(B) g/r

(C) $g/\sqrt{d^2 + r^2}$

(D) $g/\sqrt{d^2 - r^2}$ ← **CORRECT**

(E) $g/\sqrt{dr}$

Solution

The vertical component of tension must cancel the force of gravity. The horizontal component of tension must provide the centripetal force.

Let θ be the angle of the pendulum from vertical, satisfying

$$\tan\theta = \frac{r}{\sqrt{r^2 - d^2}}.$$

For the vertical component of tension to cancel gravity, we must have

$$mg = T\cos\theta.$$

For the horizontal component of tension to equal the centripetal force, we have

$$m\omega^2 r = T\sin\theta.$$

Dividing these equations gives

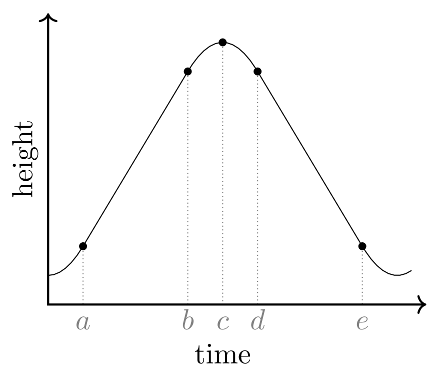
$$\frac{\omega^2 r}{g} = \tan\theta.$$

Substituting in our result for $\tan\theta$ and solving for ω^2, we obtain

$$\omega^2 = \frac{g\tan\theta}{r} = \frac{g}{\sqrt{d^2 - r^2}}.$$

Notice that the in the limit $r \to d$ the pendulum is horizontal, which requires and infinite angular frequency. In the limit $r \to 0$ we obtain the ordinary formula for the angular frequency of a pendulum in a plane. Thus, the problem could also have been solved using just these limiting cases.

4. In "zero-g" airplane rides, passengers can float around the cabin, as if they were weightless. A flight trajectory for such a ride is shown below, with a few points during the journey labelled. Which of the following statements is correct? Take **a** to be the acceleration of the plane during the "zero-g" part of the flight.



(A) The "zero-g" flight begins at a and ends at c, during which $|\mathbf{a}| = g$ and $\mathbf{a}$ points up.

(B) The "zero-g" flight begins at a and ends at e, during which $|\mathbf{a}| = 0$.

(C) The "zero-g" flight begins at b and ends at d, during which $|\mathbf{a}| = g$ and $\mathbf{a}$ points down. ← **CORRECT**

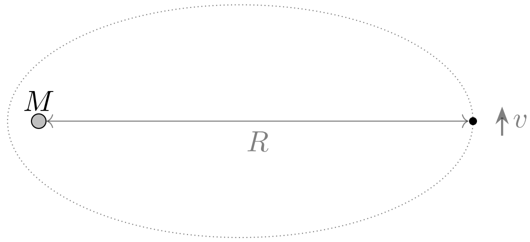
(D) The "zero-g" flight begins at c and ends at e, during which $|\mathbf{a}| = g$ and $\mathbf{a}$ points down.

(E) The "zero-g" flight begins at d and ends at e, during which $|\mathbf{a}| = 0$.

Solution

The zero-g segment corresponds to the part where the airplane falls freely with the passengers inside. That is, during the zero-g segment the acceleration is g downward, and hence a concave down parabola, corresponding to the segment from b to d.

5. A mote of dust is initially located at distance R from the sun, which has mass M. At this point, the mote has a small tangential velocity v. Which of the following is a good approximation for the distance of closest subsequent approach between the mote and the sun?



(A) $\dfrac{R^3v^4}{G^2M^2}$

(B) $\dfrac{R^3v^4}{2G^2M^2}$

(C) $\dfrac{R^2v^2}{2GM}$ ← **CORRECT**

(D) $\dfrac{R^2v^2}{GM}$

(E) $\dfrac{2R^2v^2}{GM}$

Solution

If the particle's closest approach to the sun is R' and its speed there is v', then angular momentum conservation tells us

$$mvR = mv'R'.$$

The particle's initial kinetic energy is negligible because v is small, and its initial potential energy is negligible because it is very far from the sun compared to its closest approach. So we can set the particle's initial energy to zero, and by energy conservation,

$$0 = -\frac{GMm}{R'} + \frac{1}{2}mv'^2.$$

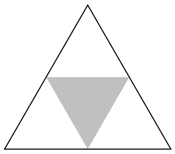
Plugging in our expression for v' from angular momentum conservation gives

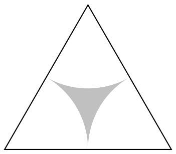
$$\frac{GMm}{R'} = \frac{1}{2}\frac{mv^2R^2}{R'^2}.$$

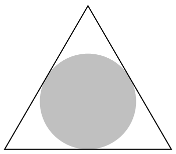
Solving for R' gives the answer,

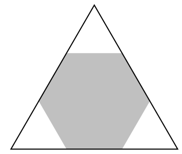
$$R' = \frac{R^2v^2}{2GM}.$$

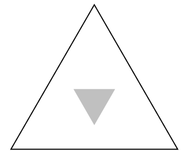
6. A three-legged table is shaped like a uniform equilateral triangle, and has identical legs at each corner. When the mass of an object placed on the center of the table exceeds m_{max}, the table's legs will all simultaneously break. Which of the following shaded regions shows the area within which an object of $2m_{\text{max}}/3$ can be placed without breaking any legs of the table?

(A)  ← **CORRECT**

(B) 

(C) 

(D) 

(E) 

Solution

A leg will break if there is a force of at least $m_{\text{max}}g/3$ on it, i.e. if it bears more the half of the weight of the mass $2m_{\text{max}}/3$.

Label the vertices of the triangle A, B, and C, and consider torque balance about an axis parallel to BC and passing through the mass.

The torque from legs B and C is $y(F_B + F_c)$, with y the distance of the mass from the segment BC. The torque from leg A is $(h - y)F_A$, with h the height of the triangle. Balancing these torques,

$$y(F_B + F_C) = (h - y)F_A.$$

Additionally,

$$F_B + F_C = \frac{2}{3}m_{\text{max}}g - F_A.$$

Combining the previous two equations and solving for F_A,

$$F_A = \frac{2}{3}mg\frac{y}{h}.$$

This shows that the leg at A supports more than half the weight if the distance from the mass to BC is more than half the height of the triangle. The same logic holds for the other two sides. Combining these constraints gives choice (A).

7. Two satellites are initially in identical circular orbits around the Sun, with orbital speed $v = 1 \times 10^4\,\text{m/s}$. The first satellite fires its thrusters toward the Sun, and quickly obtains a radial velocity of $\Delta v_r = 1\,\text{m/s}$. The second satellite instead fires its thrusters behind it, and quickly increases its tangential velocity by Δv_t. If the two satellites subsequently perform orbits with the same period, approximately what was Δv_t?

   (A) $0.000\,05\,\text{m/s}$ ← **CORRECT**

   (B) $0.005\,\text{m/s}$

   (C) $0.5\,\text{m/s}$

   (D) $1\,\text{m/s}$

(E) $50\,\mathrm{m/s}$

Solution

By Kepler's third law, the period only depends on the semimajor axis a. However, the total energy of an elliptic orbit is determined by the semimajor axis by $E = -GMm/2a$. Hence the two satellites must have the same energy, and hence the same final speed, so

$$(v + \Delta v_t)^2 = v^2 + (\Delta v_r)^2$$

which gives

$$\Delta v_t \approx \frac{(\Delta v_r)^2}{2v} = 5 \times 10^{-5}\,\mathrm{m/s}.$$

8. A conveyor belt is moving with velocity v to the east. A block with velocity v to the south slides from the ground onto the conveyor belt. The coefficient of friction between the block and the belt is μ. The block stops slipping after a time

(A) $\dfrac{v}{\sqrt{2}\,\mu g}$

(B) $\dfrac{v}{\mu g}$

(C) $\dfrac{\sqrt{2}\,v}{\mu g}$ ← **CORRECT**

(D) $\dfrac{2v}{\mu g}$

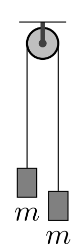
(E) The block never stops slipping.

Solution

Work in the reference frame of the conveyor belt. In this frame, the block's initial velocity perpendicular to the edge of the belt is v and its initial velocity along the belt is also v. Because these are at right angles, the initial speed of the block is $\sqrt{v^2 + v^2} = \sqrt{2}v$. The frictional acceleration is μg, so the time to come to a stop is $\sqrt{2}v/\mu g$.

The following information is relevant to problems 9 and 10.

9. Two equal masses m are connected by an elastic string that acts like an ideal spring with spring constant k and unstretched length l. The two masses are hung over a frictionless pulley. What is the total length of the string at equilibrium? (Diagram not necessarily to scale.)



(A) $2mg/k$

(B) $l + mg/k$ ← **CORRECT**

(C) $l + mg/2k$

(D) $l + 2mg/k$

(E) There is not enough information to decide.

Solution

Balancing the string tension with gravitational force on either of the masses gives

$$kx = mg,$$

so the total length is

$$l + x = l + \frac{mg}{k}.$$

10. The two masses are both displaced downward by a small vertical distance x and simultaneously released from rest. What is the period of oscillation?

(A) $2\pi\sqrt{l/g}$

(B) $\pi\sqrt{2m/k}$ ← **CORRECT**

(C) $2\pi\sqrt{m/k}$

(D) $2\pi\sqrt{m/k + l/g}$

(E) There is not enough information to decide.

Solution

Gravity does not affect the oscillation frequency, so we may treat the two-body spring-mass system as though it is isolated (we can also write down equations of motion to confirm this assumption by considering displacement from the equilibrium positions).

The reduced mass is $\mu = \frac{m}{2}$, so the oscillation frequency is

$$2\pi\sqrt{\frac{\mu}{k}} = \pi\sqrt{\frac{2m}{k}}.$$

Equivalently, one can notice that the length of the string is changed by $2x$ when each mass is displaced downward by x, so that each mass experiences a restoring force

$$F = -2kx.$$

In other words, each mass experiences an effective spring constant $k_{\text{eff}} = 2k$. The oscillation period

$$T = 2\pi\sqrt{\frac{m}{k_{\text{eff}}}} = \pi\sqrt{\frac{2m}{k}}.$$

11. An escalator can carry passengers up a vertical distance of 10 m in 30 s. A mischievous person of mass 50 kg walks down the up-escalator so that they stay in place with respect to the building. If the child does this for 30 s, the total work the child performs on the escalator, in the frame of the building, is

    (A) -10^4 J
    (B) -5×10^3 J $\leftarrow$ **CORRECT**
    (C) 0 J
    (D) 5×10^3 J
    (E) 10^4 J

Solution

The average power is $\bar{F}v_y$ where $\bar{F} = -mg$ is the average force exerted by the child on the escalator, and v_y is the vertical velocity of the point of contact, i.e. one or both of the child's shoes, when they are touching the escalator. This velocity is precisely the speed of the escalator, $v_y = 10/30$ m/s. The result is hence $(-500)(1/3)(30)$ J $= -5 \times 10^3$ J. Notice that the work done by the child on the escalator is the negative of the work done by the escalator on the child, which has the same value $W = mgh$ as it would have if the child had remained stationary. (Here h is the height of the escalator.)

One might think that the answer is 0 J because the center of mass of the child isn't moving. This is incorrect because work depends on the motion of the part of a system the force is applied on, not the motion of the system's center of mass. Another incorrect way to get at an answer of 0 J is to work in the reference frame moving upward with the escalator. In this frame the escalator behaves like a static set of stairs, so it can do no work on the child. That is correct, but the amount of work done depends on the reference frame, and the problem asked for the work done in the frame of the building.

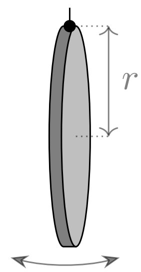
12. A platform juts out horizontally from the edge of a building. If the platform is modeled as a uniform metal rod, which one of the following statements about the tensile and compressive stress is correct?

    (A) There is a horizontal compression throughout the rod.
    (B) There is a horizontal tension throughout the rod.
    (C) There is horizontal tension in the top of the rod and a compression in the bottom. $\leftarrow$ **CORRECT**
    (D) There is a horizontal tension near the middle of the rod and a compression near the end.
    (E) There is a horizontal compression near the middle of the rod and a tension near the end.

Solution

There must be tensive and compressive forces at the top and bottom of the rod, respectively, to balance the torque on the rod from gravity; ultimately these forces come from the wall the rod is attached to.

13. A uniform disk of mass m and radius r is attached at its edge to a flexible pivot on the ceiling. It is given a small displacement *perpendicular* to the plane of the disk, so that it begins to oscillate perpendicular to the plane of the disk. What is the period of oscillation? The moment of inertia of a disk about the axis going through its center and perpendicular to the plane it's in is $I_{\text{disk}} = \frac{1}{2}mr^2$



(A) $\pi\sqrt{2r/5g}$
(B) $\pi\sqrt{5r/g}$ ← **CORRECT**
(C) $\pi\sqrt{6r/g}$
(D) $2\pi\sqrt{r/g}$
(E) $2\pi\sqrt{2r/g}$

Solution

Let $I_\perp = \frac{1}{2}mr^2$ be the moment of inertia about the central axis of the disk, let I_{cm} be the moment of inertia of the disk about an axis through the center of the disk *parallel* to the plane of the disk, and let I denote the moment of inertia about the axis of oscillation. By the Perpendicular Axis theorem,

$$I_\perp = I_{\text{cm}} + I_{\text{cm}},$$

so

$$I_{\text{cm}} = \frac{1}{2}I_\perp = \frac{1}{4}mr^2.$$

By the Parallel Axis theorem,

$$I = I_{\text{cm}} + mr^2 = \frac{5}{4}mr^2..$$

For some small oscillation displacement by an angle θ, the torque about the pivot is

$$\tau = -mgr\sin\theta \approx -mgr\theta,$$

so applying $\tau = I\ddot{\theta}$ gives

$$\ddot{\theta} = \frac{\tau}{I} = -\frac{4}{5}\frac{g}{r}\cdot\theta = -\omega^2\theta.$$

Thus the period of oscillation is

$$T = \frac{2\pi}{\omega} = \pi\sqrt{5\cdot\frac{r}{g}}.$$

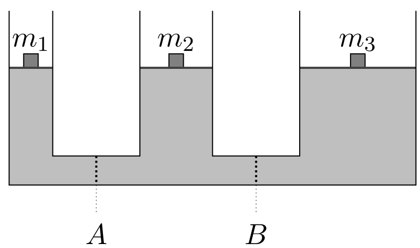
14. A large block of mass 5 kg is moving to the right at a velocity of 10 m/s. Spaced out every meter are smaller, initially stationary blocks of mass 1 kg. All collisions are perfectly inelastic. Neglecting friction, how far will the large block travel before its velocity has decreased to below 3 m/s?

(A) 5 m
(B) 8 m
(C) 12 m ← **CORRECT**
(D) 17 m
(E) 50 m

Solution

Momentum is conserved, and the block starts out with 50 kg m/s of momentum. For its velocity to go below 3 m/s, its mass must exceed 16.7 kg. The first time this happens is after it absorbs 12 blocks and has mass 5 kg + 12 kg = 17 kg

15. In the device shown, blocks of various masses are placed on pistons so that the device is in equilibrium. (The fluid in the drawing is to scale.) There are valves that are both initially open at locations A and B. One of the valves is closed, and the system is allowed to come to equilibrium. How will this affect the height of the mass m_2?



(A) Closing either valve will have no effect on the height of m_2. ← **CORRECT**
(B) Closing either valve causes m_2 to rise.
(C) Closing either valve causes m_2 to fall.
(D) Closing valve A causes m_2 to rise, and closing valve B causes m_2 to fall.
(E) Closing valve A causes m_2 to fall, and closing valve B causes m_2 to rise.

Solution

The gauge pressure of the water directly under block m_1 must be $\frac{m_1 g}{A_1}$, where A_1 is the cross-sectional area of the tube containing block m_1. The pressure in the horizontal section of the tube is therefore $\rho g h + \frac{m_1 g}{A_1}$. But by the same reasoning, the pressure is also $\rho g h + \frac{m_2 g}{A_2}$ and $\rho g h + \frac{m_3 g}{A_3}$. We conclude $\frac{m_1}{A_1} = \frac{m_2}{A_2} = \frac{m_3}{A_3}$. This is the condition for equilibrium in the device. It we close valve A, the condition for equilibrium in the right two tubes is $\frac{m_2}{A_2} = \frac{m_3}{A_3}$, which is already met, so these tubes remain in equilibrium. The same applies to valve B. In other words, the pressure is the same to the left and right of a valve, so closing the valve doesn't affect the equilibrium.

The following information is relevant to problems 16 and 17.

16. A mass M sits on top of a vertical spring of spring constant k, in equilibrium. A mass m is held a height h above it. The mass M is then pushed downward by a distance Δx, and both masses are released from rest simultaneously. For what value of h will the two masses first collide when M first returns to its equilibrium position?

(A) $\dfrac{\pi M^2 g^2}{4k^2 \Delta x}$

(B) $\dfrac{8Mg}{\pi^2 k}$

(C) $\dfrac{Mg}{k}$

(D) $\dfrac{\pi^2 Mg}{8k}$ ← **CORRECT**

(E) $\dfrac{\pi k \Delta x^2}{4Mg}$

Solution

For the masses to collide at the equilibrium position of M, we need for the time during which m is falling to be equal to $\frac{\pi}{2}\sqrt{\frac{M}{k}}$, which is one quarter of the oscillation period of the spring. The time to free fall a distance h, starting from rest, is given by $\sqrt{\frac{2h}{g}}$, so we have

$$h = \frac{\pi^2 Mg}{8k}.$$

17. Assume that h takes the value found in the previous question, and that the collision between the two masses is perfectly elastic. For what value of Δx will m rebound to a maximum height that is exactly equal to its original height?

(A) $\dfrac{\pi g}{2k}\sqrt{\dfrac{m^3}{M}}$

(B) $\dfrac{\pi g}{k}\sqrt{\dfrac{m^3}{2M}}$

(C) $\dfrac{2g}{\pi k}\sqrt{\dfrac{2m^3}{M}}$

(D) $\dfrac{4mg}{\pi k}$

(E) $\dfrac{\pi mg}{2k}$ ← **CORRECT**

Solution

Since the potential energy of the compressed spring is turned into kinetic energy for mass M, such that $\frac{1}{2}Mv_M^2 = \frac{1}{2}k(\Delta x)^2$, we have

$$v_M = \sqrt{\frac{k}{M}}\Delta x,$$

where v_M is the speed of mass M just before colliding. The speed of mass m just before colliding is

$$v_m = \sqrt{2gh} = \frac{\pi g}{2}\sqrt{\frac{M}{k}}.$$

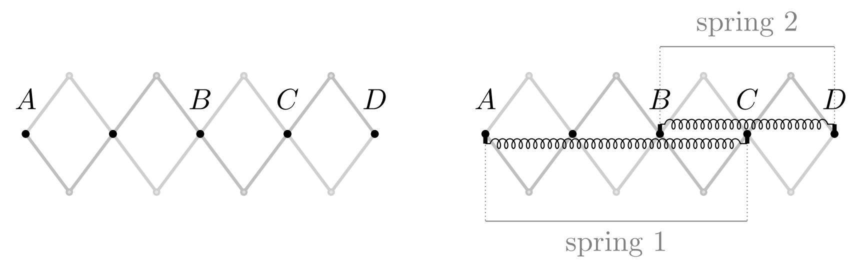
We need $mv_m = Mv_M$ for an elastic collision in which the masses just turn around at the same speed with which they came in. This equation gives

$$\frac{\pi mg}{2}\sqrt{\frac{M}{k}} = M\sqrt{\frac{k}{M}}\Delta x,$$

so that

$$\Delta x = \frac{\pi mg}{2k}.$$

18. An extendable arm is made from rigid beams free to pivot around the dots shown. Spring 1, with equilibrium length L_1, is attached between points A and C while spring 2, with equilibrium length L_2 is attached between B and D. The system is allowed to come to equilibrium. In equilibrium, what is the ratio of the tension in spring 1 to the tension in spring 2?



(A) $-2/3$ ← **CORRECT**
(B) $-3/4$
(C) -1
(D) $-3/2$
(E) There is not enough information to determine the ratio.

Solution

Imagine extending the entire arm by an amount dx. Then spring 1 gets longer by $\frac{3}{4}\mathrm{d}x$ and spring 2 by $\frac{1}{2}\mathrm{d}x$. The change in potential energy of the arm is $T_1\frac{3}{4}\mathrm{d}x + T_2\frac{1}{2}\mathrm{d}x$. Because the arm is in equilibrium, small extensions do not change the potential energy for very small dx, so $T_1\frac{3}{4}\mathrm{d}x + T_2\frac{1}{2}\mathrm{d}x = 0$. Solving for $\frac{T_1}{T_2}$, we get $\frac{T_1}{T_2} = -\frac{2}{3}$.

19. Consider an axially symmetric object that experiences no external forces and initially rotates about its symmetry axis. The object then changes its shape while remaining axially symmetric. Afterward, it is found that its moment of inertia about the symmetry axis has increased. How have its kinetic energy T and angular velocity ω changed?

(A) T has decreased, and ω has decreased. ← **CORRECT**

(B) T has decreased, and ω stays the same.

(C) T stays the same, and ω has decreased.

(D) T has increased, and ω has increased.

(E) T has increased, and ω stays the same.

Solution

At every moment we have $L = I\omega$, and the angular momentum L is conserved. The kinetic energy can be written in terms of L and I,

$$T = \frac{I\omega^2}{2} = \frac{L^2}{2I}.$$

Since I increases, T decreases. Since $L = I\omega$ is conserved, since I increases, ω decreases.

20. A well-calibrated scale reads zero when nothing is placed on it. When a deflated balloon is placed on the scale, it reads mg. When a non-airtight box is placed on the scale, it reads Mg, where $M > m$. The balloon is inflated with helium, so that it floats upward in air. The balloon is placed in the box, and the box is placed on the scale. If the scale reads W, which of the following is true?

(A) $W \leq (M - m)g$

(B) $(M - m)g < W < (M + m)g$

(C) $W = (M + m)g$

(D) $W > (M + m)g$

(E) None of the above are necessarily true. $\leftarrow$ **CORRECT**

Solution

Consider the forces on the box. Before the balloon is in the box, the weight of the box, the buoyant force on the box due to the atmosphere, and the normal force from the scale all balance, and the normal force is Mg. When the balloon is placed in, the only change in the forces on the box is an upward force, as the balloon pushes up on the top of the box. Thus the answer must be less than Mg. However, we don't know whether it is larger or smaller than $(M - m)g$ because this depends on how buoyant the balloon is, so there is not enough information to decide.

21. A person stands on the seat of a swing and squats down, so that the distance between their center of mass (CM) and the swing's pivot is ℓ. As the swing gets to the lowest point, the speed of their CM is v. At this moment, they quickly stand up, and thus decrease the distance from their CM to the swing's pivot to ℓ'. Immediately after they finish standing up, their CM speed is v'. Which of the following statements is correct? You may neglect friction, the change in moment of inertia of the person about their CM, and the time taken to stand up.

(A) $v/\ell = v'/\ell'$

(B) $v = v'$

(C) $v\ell = v'\ell'$ $\leftarrow$ **CORRECT**

(D) $\frac{1}{2}v^2 = \frac{1}{2}v'^2 + g(\ell - \ell')$

(E) Multiple statements are correct.

Solution

Because the person is doing work to stand up, the energy is *not* conserved in this problem. Instead, the angular momentum about the swing pivot is conserved since there is no net torque. The angular momentum about the person's CM is $mv\ell$ while squatting, and $mv'\ell'$ after they stand up. (The linear momentum is not conserved, because internal forces in the swing give the person an impulse as they stand up.)

22. A point mass m sits on a long block, also of mass m, which rests on the floor. The coefficient of static and kinetic friction between the mass and the block is μ, and the coefficient of static and kinetic friction between the block and the floor is $\mu/3$. An impulse gives a horizontal momentum p to the point mass. After a long time, how far has the point mass moved relative to the block? Assume the mass does not fall off the block.

(A) $\dfrac{3p^2}{8m^2\mu g}$ ← **CORRECT**

(B) $\dfrac{15p^2}{32m^2\mu g}$

(C) $\dfrac{9p^2}{16m^2\mu g}$

(D) $\dfrac{3p^2}{10m^2\mu g}$

(E) $\dfrac{3p^2}{4m^2\mu g}$

Solution

Initially, the mass will begin to slip on the block, which in turn will begin to slip on the floor. The frictional force between the point mass and the block is $-\mu mg$, and the frictional force between the block and the floor is $(\mu/3)(2m)g$. Because $\mu > 2\mu/3$, the block will always be moving forward during the point mass's motion. Applying $f = \mu N$, the mass has a backward acceleration of μg, while the block has a forward acceleration of $(\mu - 2\mu/3)g = \mu g/3$. Therefore, the relative acceleration is $a_{\text{rel}} = (4/3)\mu g$.

The initial relative velocity of the blocks is $v_i = p/m$, so slipping between the point mass and block stops after a time $t = v_i/a_{\text{rel}}$. The average relative velocity during this time is $v_i/2$. Therefore, the total relative motion is $v_i^2/2a_{\text{rel}}$, giving an answer of $(3/8)p^2/(m^2\mu g)$.

23. Assume that the drag force for a fish in water depends only on the typical length scale of the fish R, its velocity v, and the density of water ρ. A pufferfish is about 10 cm in length and swims at about 5 m/s. How fast does a clown fish, about 1 cm in length, need to swim such that it experiences the same drag force as the pufferfish?

(A) 5 m/s

(B) 16 m/s

(C) 50 m/s ← **CORRECT**

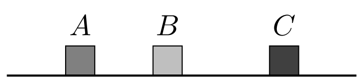
(D) 500 m/s

(E) 2500 m/s

Solution

Dimensional analysis gives $F \propto R^2 v^2 \rho$. For the two fish to have the same drag, $R_{\text{puff}}^2 v_{\text{puff}}^2 = R_{\text{clown}}^2 v_{\text{clown}}^2$, giving the correct answer 50 m/s.

24. Three boxes A, B and C lie along a straight line on a horizontal frictionless surface, as shown. Box A is initially moving to the right with speed v while the other two boxes are initially at rest. If all collisions are elastic, and the masses of the boxes can be chosen freely, which of the following is closest to the maximum possible final speed of box C?



(A) v

(B) $2v$

(C) $3v$

(D) $4v$ ← **CORRECT**

(E) $5v$

Solution

Consider the elastic collision between A and B. After the collision, box B's speed is

$$v_B = \frac{m_A - m_B}{m_A + m_B} v + v \le 2v.$$

So if $m_C \ll m_B \ll m_A$, repeating this reasoning shows that the final speed for box C will be $4v$. This is the maximum possible speed.

This is the intended physical intuition to answer the question, but showing it more rigorously is subtle. It's clearly optimal to have m_A as large as possible. However, box C can be hit multiple times if the ratio m_C/m_B is made higher, with each impact transferring more momentum. Thus, one might suspect that having a larger value of m_C/m_B could yield a higher final speed.

This is in fact not the case, because when m_C/m_B is larger enough to have multiple collisions, the effect of each collision becomes much smaller. For example, consider the case $m_C/m_B = 1$. In this case three collisions happen in total, and boxes B and C end up with the same final speed $2v$. Therefore, if we make m_C/m_B slightly higher than 1, box B will be able to catch up with box C, hitting it a second time. However, this gives a final speed of just over $2v$, which is much worse than $4v$. Moreover, in the limit $m_C/m_B \gg 1$, many collisions happen, but the net effect is like a single elastic collision between m_A and m_C mediated by box B, which has a maximum possible final speed of $2v$. Thus, the optimal setup is indeed $m_C \ll m_B \ll m_A$.

25. If a certain radioactive decay process happens n times per hour on average, then in any given hour, one expects to observe n decays with an uncertainty of $\sqrt{n}$. Each hour can be assumed independent of the previous one, and n can be assumed constant over time. How many hours do you need to conduct the observation so that you can determine n within an uncertainty of 1%?

(A) $n/10^2$

(B) $n/10^4$

(C) $10^2/n$

(D) $10^4/n$ ← **CORRECT**

(E) $10^8/n^2$

Solution

Let the number of hours we observe be h. Then the number of decays observed will be, on average, hn. The standard deviation in the number of decays observed in an hour is $\sqrt{n}$. Each hour is independent of the ones before it. When we add h independent samples of a random variable, the sum has standard deviation of $\sqrt{h}\sigma$, with σ the standard deviation of the underlying distribution. So in this case, we will observe nh decays and the standard deviation in the number of decays observed is $\sqrt{hn}$. Our estimate for n will be

$$\frac{nh \pm \sqrt{nh}}{h} = n \pm \sqrt{\frac{n}{h}}.$$

We want the standard deviation to be about 1% of the correct value, so

$$\sqrt{\frac{n}{h}} \approx 0.01n.$$

Solving for nh, the total number of decays, we find

$$nh \approx 10^4.$$

AAPT UNITED STATES PHYSICS TEAM
AIP 2021

2021 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
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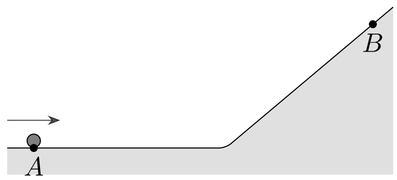
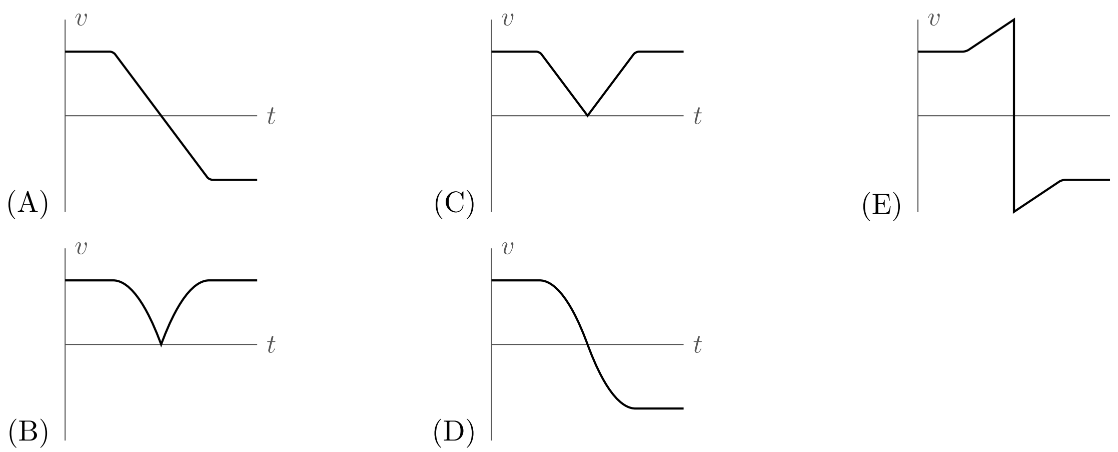
AAPT UNITED STATES PHYSICS TEAM
AIP 2021

2021 $F = ma$ Competition

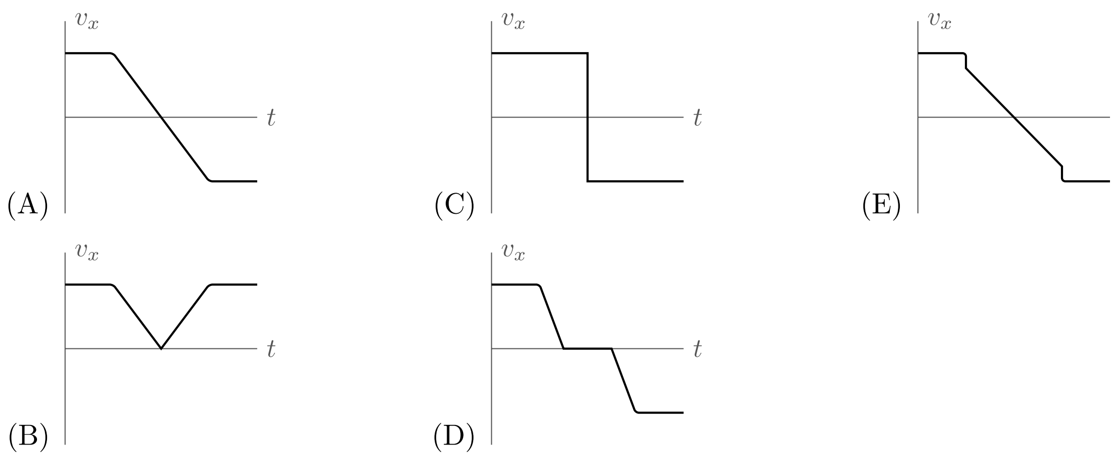
The following information applies to problems 1 and 2.
At time $t = 0$, a small ball is released on the track shown, with an initial rightward velocity. Assume the ball always rolls along the track without slipping.



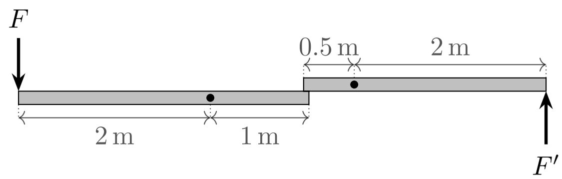
1. The ball starts at point A, turns around at point B, and returns to point A. Which of the following shows the **speed** of the ball as a function of time?



2. Which of the following shows the horizontal **velocity** of the ball as a function of time?



3. Two massless rods are attached to frictionless pivots, with their ends touching. The distances between the pivot points and the endpoints of the rods are shown below.



Neglecting friction between the rods, if a force F is applied at the left end of the left rod, what force F' must be applied at the right end of the right rod to keep the system in equilibrium?

(A) $F/8$
(B) $F/2$
(C) $4F/7$
(D) $6F/5$
(E) $2F$

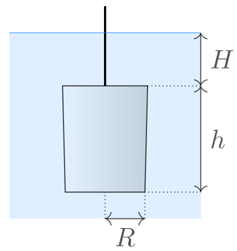
4. Alice and Bethany stand side by side on the Earth's equator. If Alice jumps directly upward, in her frame of reference, to a small height h much less than the radius of the Earth, she will land a distance D to the west of Bethany. If Alice had instead jumped to a height $2h$, how far to the west of Bethany would she land? Neglect air resistance.

(A) $D/\sqrt{2}$
(B) D
(C) $\sqrt{2}\,D$
(D) $2D$
(E) $2^{3/2}D$

5. A train starts from city A and stops in city B. The distance between the cities is s. The train's maximal acceleration is a_1 and its maximal deceleration is a_2 (in absolute value). What is the shortest time in which the train can travel between A and B?

(A) $2\sqrt{\dfrac{s}{a_1+a_2}}$
(B) $2\sqrt{\dfrac{s}{\sqrt{a_1a_2}}}$
(C) $\sqrt{2\dfrac{s(a_1+a_2)}{a_1a_2}}$
(D) $\sqrt{\dfrac{2sa_2}{a_1(a_1+a_2)}}$
(E) $\sqrt{\dfrac{s\sqrt{a_1a_2}}{(a_1+a_2)^2}}$

6. A cylindrical bucket of negligible mass has radius R and height h, and is open at the top. It is submerged in water of density ρ, with its top a distance H below the surface. How much work is needed to pull the bucket slowly up so that its bottom is just above the lake surface?



(A) $\rho g \pi R^2 h(H-h)$

(B) $\rho g \pi R^2 h^2$

(C) $\rho g \pi R^2 (H-h)^2$

(D) $\rho g \pi R^2 h^2/2$

(E) $\rho g \pi R^2 h(H-h)/2$

7. A point mass slides with speed v on a frictionless horizontal surface between two fixed parallel walls, initially a distance L apart. It bounces between the walls perfectly elastically. You move one of the walls towards the other by a distance $0.01L$, with speed $0.0001v$. What is the final speed of the point mass?

(A) $1.001v$

(B) $1.002v$

(C) $1.005v$

(D) $1.01v$

(E) $1.02v$

8. A mint produces 100,000 coins. Upon weighing some of them with a precise scale, officials find that the coins vary slightly in weight, with an independent uncertainty of 1%. About how many coins must be randomly sampled and weighed in order to determine the total weight of the coins to within 0.1% uncertainty? Assume there are no sources of systematic uncertainty.

(A) Almost all of the coins must be weighed.

(B) 10,000

(C) 1,000

(D) 100

(E) 10

9. NASA trains astronauts to experience weightlessness with an airplane which flies in a parabolic arc with constant acceleration g toward the ground. The plane can remain on this trajectory for at most 25 seconds, due to the large change in altitude required. If instead of simulating weightlessness, NASA wanted to fly a trajectory that would simulate the gravitational acceleration of Mars 3.7 m/s^2, for what length of time can the plane simulate Mars gravity? Assume that the maximum change in altitude is the same for both trajectories.

(A) 25 s

(B) 31 s

(C) 41 s

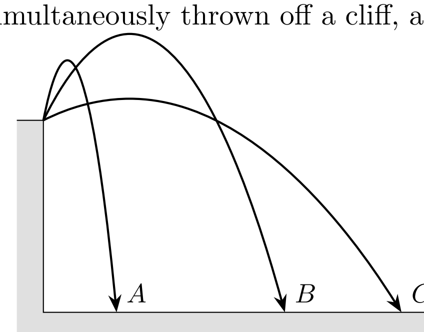
(D) 68 s

(E) 183 s

10. A uniform solid circular disk of mass m is on a flat, frictionless horizontal table. The center of mass of the disk is at rest and the disk is spinning with angular frequency ω_0. A stone, modeled as a point object also of mass m, is placed on the edge of the disk, with zero initial velocity relative to the table. A rim built into the disk constrains the stone to slide, with friction, along the disk's edge. After the stone stops sliding with respect to the disk, what is the angular frequency of rotation of the disk and stone together?

(A) ω_0
(B) $2\omega_0/3$
(C) $\omega_0/2$
(D) $\omega_0/3$
(E) $\omega_0/4$

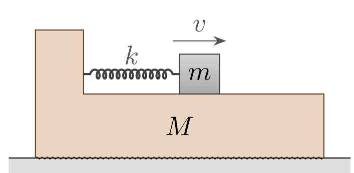
11. Projectiles A, B, and C are simultaneously thrown off a cliff, and take the trajectories shown.



Neglecting air resistance, rank the times t_A, t_B, and t_C they take to hit the ground.

(A) $t_A < t_B < t_C$
(B) $t_A < t_C < t_B$
(C) $t_C < t_B < t_A$
(D) $t_C < t_A < t_B$
(E) There is not enough information to decide.

12. An object of mass $m = 1\,\text{kg}$ is attached to a platform of mass $M = 4\,\text{kg}$ with a spring of spring constant $k = 400\,\text{N/m}$. There is no friction between the object and the platform, and the coefficient of static friction between the platform and the ground is $\mu = 0.1$. The object is placed at its equilibrium position, and then given a horizontal velocity v.



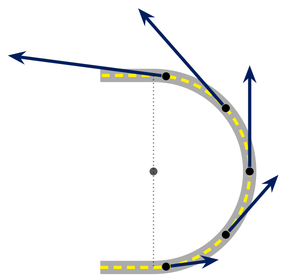
For what v will the platform never slip on the ground?

(A) $v \leq 0.1\,\text{m/s}$
(B) $v \leq 0.2\,\text{m/s}$
(C) $v \leq 0.25\,\text{m/s}$

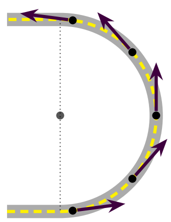
(D) $v \leq 0.4\,\mathrm{m/s}$

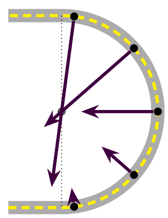
(E) $v \leq 0.5\,\mathrm{m/s}$

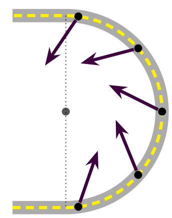
13. A car is driving on a semicircular racetrack. Its velocity at several points along the track is shown below.

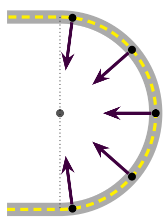


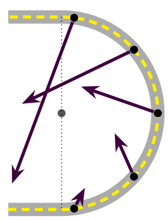
Which of the following could depict its acceleration at the corresponding points?

(A) 

(C) 

(E) 

(B) 

(D) 

14. A bacterium cell swims by rotating its bundle of flagella to counter a viscous drag force in the medium. The drag force $F(R, v, \eta)$ only depends on the typical length scale of the cell R, its speed v, and the viscosity of the fluid η, which has units of $\mathrm{kg/(m \cdot s)}$. It is observed under a microscope that a cell of length 1 μm swims at about 20 μm/s. Estimate the speed of a cell of length 0.5 μm, assuming cells of all sizes generate the same amount of force from their flagella.

(A) 5 μm/s

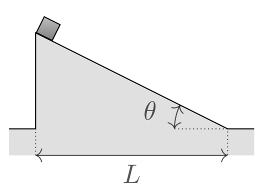
(B) 10 μm/s

(C) 40 μm/s

(D) 80 μm/s

(E) There is not enough information to decide.

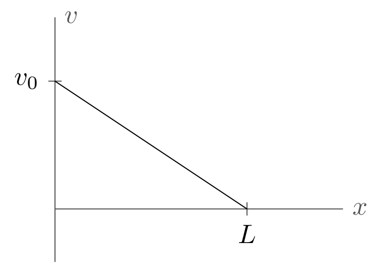
15. A block is released from rest at the top of a fixed, frictionless ramp with horizontal length L and inclination θ.



For a fixed value of L, which value of θ minimizes the time needed for the block to reach the bottom of the ramp?

(A) 30°
(B) 45°
(C) 60°
(D) 75°
(E) 80°

16. A particle begins at the point $x = 0$ and moves along the x-axis with initial rightward velocity v_0. Later, the particle reaches the point $x = L$. The velocity as a function of position during this time interval is shown below.



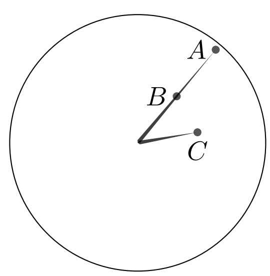
Consider the following three statements.

I. The particle instantaneously stops at $x = L$.
II. The particle is uniformly accelerated.
III. The particle could be performing simple harmonic motion.

Which of these statements are true?

(A) Only I.
(B) Only II.
(C) Both I and II.
(D) Both I and III.
(E) None of the above.

17. In a certain country, the short hand of a clock is exactly half as long as the long hand, and rotates twice for each rotation of the long hand.



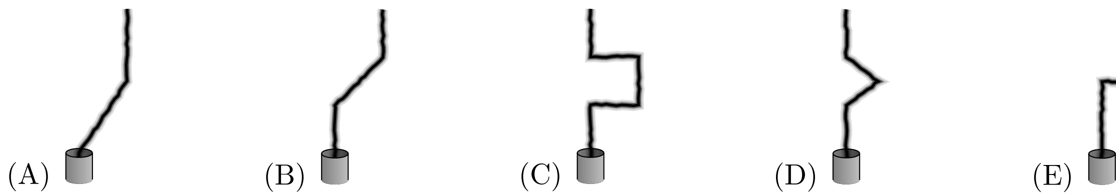
The three points shown on the clock hands have accelerations of magnitude a_A, a_B, and a_C. The point B is at the midpoint of the long hand. Which of the following is true?

(A) $a_A < a_B = a_C$
(B) $a_A = a_B < a_C$
(C) $a_B < a_A < a_C$

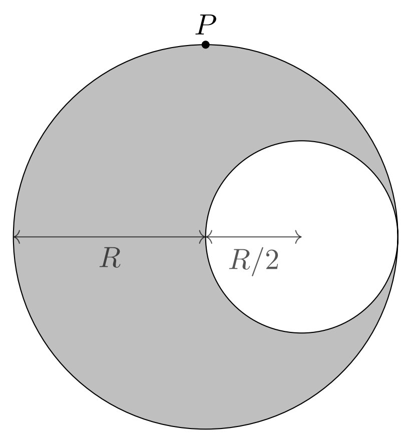
(D) $a_B < a_A = a_C$

(E) $a_B < a_C < a_A$

18. A factory's smokestack continually produces smoke. The smoke always rises with a constant speed relative to the air around it. Suppose that the air was initially still, then abruptly started blowing to the right, then abruptly became still again for some time. Which of the following could show the resulting shape of the smoke plume?



19. A cavity of radius $R/2$ is dug out of a spherical planet with uniform mass density of mass M and radius R. What is the magnitude of the gravitational field at point P in the diagram below?



(A) $(0.200)GM/R^2$

(B) $(0.457)GM/R^2$

(C) $(0.829)GM/R^2$

(D) $(0.900)GM/R^2$

(E) $(0.912)GM/R^2$

20. The line between day and night on a planet or moon is called the *terminator*. How fast does the terminator of Earth's moon move across its surface at the equator? The radius of the moon is 1.74×10^6 m.

(A) 0 m/s

(B) 4.5 m/s

(C) 83 m/s

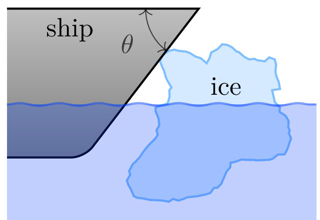
(D) 465 m/s

(E) 2201 m/s

21. A solid sphere sits at the top of a ramp of height h inclined at angle θ to the horizontal. Both the static and kinetic coefficients of friction between the sphere and incline are $\mu_k = \mu_s = 0.2$. The sphere is released from rest at the top of the incline. For which of the following values of θ is the total translational plus rotational kinetic energy of the sphere greatest when it reaches the bottom of the incline?

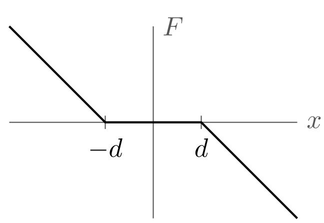
(A) $10°$
(B) $45°$
(C) $60°$
(D) $80°$
(E) The mechanical energy is the same for all choices.

22. A ship rams into a hunk of ice floating in the sea. "Icebreakers" are ships designed so that when this happens, the ice is pushed down beneath the ship. If the coefficient of static friction between the ice and the ship is μ_s, what condition applies to the angle θ, as shown in the figure, so that the icebreaker functions as intended?



(A) $\cot\theta > \mu$
(B) $\cos\theta > \mu$
(C) $\cot\theta < \mu$
(D) $\cos\theta < \mu$
(E) It depends on the curvature of the ice.

23. An imperfect spring has a restoring force F that depends on the displacement x from equilibrium as in the graph shown below.



The slope of the curve for $x < -d$ and $x > d$ is a constant $-k$. A mass m is attached to the spring and released from rest at the position $x = A$, where $A > d$. What is the period of the subsequent motion?

(A) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{2d}{A}\right)$
(B) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{2d}{A-d}\right)$
(C) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{4d}{A-d}\right)$

(D) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{\pi d}{A-d}\right)$

(E) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{2\pi d}{A-d}\right)$

24. Two satellites are in circular orbits around a star with equal radius r, speed v, and period T. The satellites are initially diametrically opposite each other. In order to meet the second satellite in time $T/2$, the first satellite should decrease its speed to approximately

(A) $0.50v$

(B) $0.64v$

(C) $0.71v$

(D) $0.76v$

(E) $0.82v$

25. Spaceman Fred's trusty pellet sprayer is held at rest a distance h away from the center of Planet Orb, which has radius $R \ll h$. The pellet sprayer ejects pellets radially outward, uniformly in the plane of the page. These pellets are all launched with the same speed v, so that a pellet launched directly away from Orb by the pellet sprayer can just barely escape it. What fraction of the pellets eventually lands on Orb? (Hint: you may use the small angle approximation, $\sin\theta \approx \theta$ for $\theta \ll 1$, where θ is in radians.)

Left: the pellet sprayer relative to Orb Right: a close-up of the pellet sprayer

(A) $\frac{1}{2\pi}\frac{R}{h}$

(B) $\frac{1}{\pi}\frac{R}{h}$

(C) $\frac{2}{\pi}\frac{R}{h}$

(D) $\frac{1}{2\pi}\sqrt{\frac{R}{h}}$

(E) $\frac{1}{\pi}\sqrt{\frac{R}{h}}$

AAPT UNITED STATES PHYSICS TEAM
AIP 2021

2021 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

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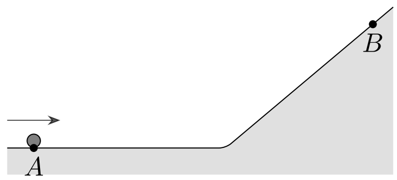
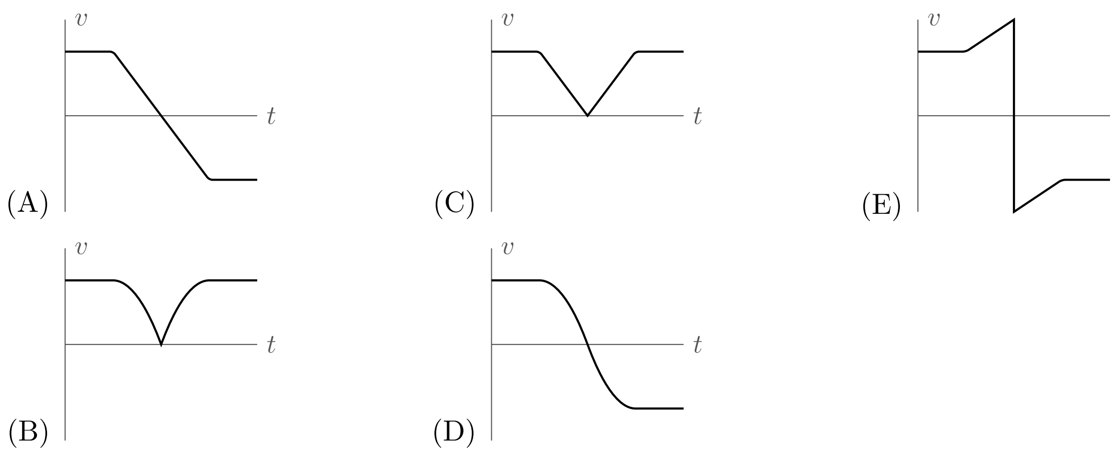


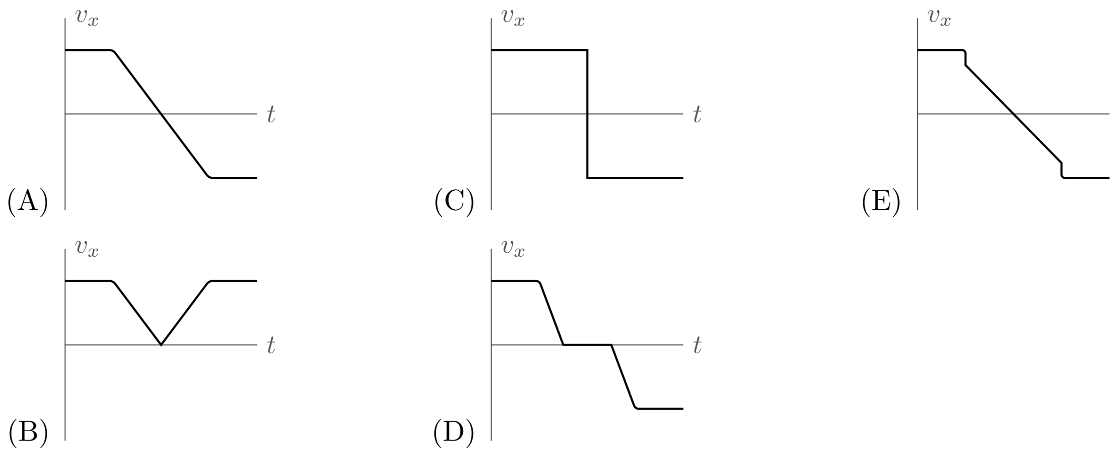
1. The ball starts at point A, turns around at point B, and returns to point A. Which of the following shows the **speed** of the ball as a function of time?



Solution

The speed starts uniform, then uniformly decelerates to zero as the ball rises up the slope, and is instantaneously zero when the ball is at point B. Then the ball speeds up again as it goes down the slope, and returns to point A with the same speed it started with. Thus, the answer is (C).

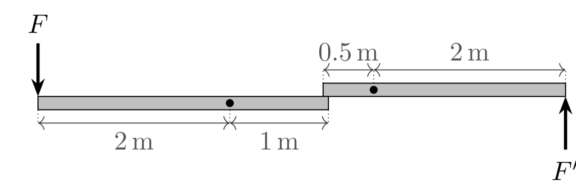
2. Which of the following shows the horizontal **velocity** of the ball as a function of time?



Solution

The horizontal velocity is initially constant. As in the previous question, when the ball goes through the bend in the track, its speed is unchanged. However, the direction of its velocity quickly changes, so that only part of it is directed horizontally. Thus, the horizontal velocity very quickly drops. As the ball climbs up the hill, its horizontal velocity decreases uniformly, then very quickly increases in magnitude when it goes through the curve again. Therefore, the answer is (E).

3. Two massless rods are attached to frictionless pivots, with their ends touching. The distances between the pivot points and the endpoints of the rods are shown below.



Neglecting friction between the rods, if a force F is applied at the left end of the left rod, what force F' must be applied at the right end of the right rod to keep the system in equilibrium?

(A) $F/8$

(B) $F/2$ ← **CORRECT**

(C) $4F/7$

(D) $6F/5$

(E) $2F$

Solution

By balancing torques on the first rod, we see that the normal force on the right end is twice the force exerted on the left end; that is, the first rod is acting as a lever with mechanical advantage 2. By similar logic, the second rod acts as a lever with mechanical advantage $1/4$. Thus, the combination of the two has mechanical advantage $2(1/4) = 1/2$, so $F' = F/2$.

4. Alice and Bethany stand side by side on the Earth's equator. If Alice jumps directly upward, in her frame of reference, to a small height h much less than the radius of the Earth, she will land a distance D to the west of Bethany. If Alice had instead jumped to a height $2h$, how far to the west of Bethany would she land? Neglect air resistance.

(A) $D/\sqrt{2}$

(B) D

(C) $\sqrt{2}\,D$

(D) $2D$

(E) $2^{3/2}D$ ← **CORRECT**

Solution

The distance to the left of the starting position is given by $\Delta v\, t$, where $\Delta v = v_{\text{ave}} - v_0$ is the average change in tangential velocity due to the jump and t is the total time of the jump. We can compute the proportionality between Δv and h using conservation of angular momentum,

$$v_{\text{ave}}(R + h_{\text{ave}}) \approx v_0 R \implies v_{\text{ave}} \approx \frac{v_0 R}{R + h_{\text{ave}}}.$$

We use an $\approx$ because $[v \cdot (R+h)]_{\text{ave}} \neq v_{\text{ave}} \cdot (R + h_{\text{ave}})$, but they both exhibit the same scaling. Then,

$$\Delta v = v_0 - v_{\text{ave}} = \frac{v_0 h}{R + h_{\text{ave}}} \implies \Delta v \propto h.$$

From basic kinematics, $t \propto h^{1/2}$, so $\Delta v\, t \propto h^{3/2}$, so the answer is (E).

5. A train starts from city A and stops in city B. The distance between the cities is s. The train's maximal acceleration is a_1 and its maximal deceleration is a_2 (in absolute value). What is the shortest time in which the train can travel between A and B?

(A) $2\sqrt{\dfrac{s}{a_1 + a_2}}$

(B) $2\sqrt{\dfrac{s}{\sqrt{a_1 a_2}}}$

(C) $\sqrt{2\dfrac{s(a_1 + a_2)}{a_1 a_2}}$ ← **CORRECT**

(D) $\sqrt{\dfrac{2sa_2}{a_1(a_1 + a_2)}}$

(E) $\sqrt{\dfrac{s\sqrt{a_1 a_2}}{(a_1 + a_2)^2}}$

Solution

When going from city to city as quickly as possible, the train accelerates for some time t_1, reaches a maximum speed v, then immediately decelerates for a time t_2, coming to rest right when it arrives at the new city. The speed v obeys $v = a_1 t_1$ and $v = a_2 t_2$, and so

$$a_1 t_1 = a_2 t_2 = v.$$

The average speed of the train is $v/2$, so we also have

$$(t_1 + t_2)\frac{v}{2} = s.$$

If we solve these equations for t_1, we find

$$t_1 = \sqrt{\frac{2a_2 s}{a_1(a_1 + a_2)}}.$$

Then we can find that

$$t_2 = \frac{a_1 t_1}{a_2},$$

and therefore

$$t = t_1 + t_2 = t_1\left(\frac{a_1 + a_2}{a_2}\right).$$

Substituting in our equation for t_1, we have

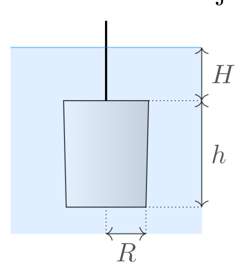
$$t = \sqrt{\frac{2s(a_1 + a_2)}{a_1 a_2}}.$$

Alternatively, we can consider the limit $a_2 \to \infty$. In this case, the train accelerates all the way to the next city, and the time is

$$t_{a_2 \to \infty} = \sqrt{\frac{2s}{a_1}}.$$

Only the correct answer simplifies appropriately.

6. A cylindrical bucket of negligible mass has radius R and height h, and is open at the top. It is submerged in water of density ρ, with its top a distance H below the surface. How much work is needed to pull the bucket slowly up so that its bottom is just above the lake surface?



(A) $\rho g \pi R^2 h(H - h)$

(B) $\rho g \pi R^2 h^2$

(C) $\rho g \pi R^2 (H - h)^2$

(D) $\rho g \pi R^2 h^2/2$ ← **CORRECT**

(E) $\rho g \pi R^2 h(H-h)/2$

Solution

While the bucket is in the lake, it is full of water, and the weight of the water is exactly cancelled by the buoyant force. Thus, no work is required to pull the bucket until its top is at the lake surface. From this point onward, the force linearly increases. The average force required to pull the bucket out of the water is $mg/2$ where $m = \pi \rho R^2 h$ is the mass of the water in the bucket, and the distance is h, giving total work $\rho g \pi R^2 h^2/2$.

The problem can also be solved using energy conservation. After the bucket has been lifted, a mass m of water has been raised from the water surface to height $h/2$ above it, giving work $mgh/2$ and the same result.

7. A point mass slides with speed v on a frictionless horizontal surface between two fixed parallel walls, initially a distance L apart. It bounces between the walls perfectly elastically. You move one of the walls towards the other by a distance $0.01L$, with speed $0.0001v$. What is the final speed of the point mass?

(A) $1.001v$

(B) $1.002v$

(C) $1.005v$

(D) $1.01v$ ← **CORRECT**

(E) $1.02v$

Solution

The total time is $100L/v$, during which 50 round-trips occur. Because the collisions are elastic, each collision with the moving wall increases the speed of the block by $0.0002v$. Thus, the final speed is increased by $(50)(0.0002v) = 0.01v$.

8. A mint produces 100,000 coins. Upon weighing some of them with a precise scale, officials find that the coins vary slightly in weight, with an independent uncertainty of 1%. About how many coins must be randomly sampled and weighed in order to determine the total weight of the coins to within 0.1% uncertainty? Assume there are no sources of systematic uncertainty.

(A) Almost all of the coins must be weighed.

(B) 10,000

(C) 1,000

(D) 100 ← **CORRECT**

(E) 10

Solution

Since the uncertainty comes from the independent 1% of each coin, to reduce the uncertainty by a factor of 10, one needs to take $10^2 = 100$ samples.

9. NASA trains astronauts to experience weightlessness with an airplane which flies in a parabolic arc with constant acceleration g toward the ground. The plane can remain on this trajectory for at most 25 seconds, due to the large change in altitude required. If instead of simulating weightlessness, NASA wanted to fly a trajectory that would simulate the gravitational acceleration of Mars 3.7 m/s^2, for what length of time can the plane simulate Mars gravity? Assume that the maximum change in altitude is the same for both trajectories.

   (A) 25 s
   (B) 31 s ← **CORRECT**
   (C) 41 s
   (D) 68 s
   (E) 183 s

Solution

The new acceleration of the plane should be $10\ \mathrm{m/s^2} - 3.7\ \mathrm{m/s^2} = 6.3\ \mathrm{m/s^2}$. The altitude change is supposed to be the same, so if the weightless flight took time t and the Mars flight took time τ, we have

$$\frac{1}{2}10\ \mathrm{m/s^2} \cdot t^2 = \frac{1}{2}6.3\ \mathrm{m/s^2} \cdot \tau^2.$$

Solving for τ gives 31 s.

10. A uniform solid circular disk of mass m is on a flat, frictionless horizontal table. The center of mass of the disk is at rest and the disk is spinning with angular frequency ω_0. A stone, modeled as a point object also of mass m, is placed on the edge of the disk, with zero initial velocity relative to the table. A rim built into the disk constrains the stone to slide, with friction, along the disk's edge. After the stone stops sliding with respect to the disk, what is the angular frequency of rotation of the disk and stone together?

   (A) ω_0
   (B) $2\omega_0/3$
   (C) $\omega_0/2$ ← **CORRECT**
   (D) $\omega_0/3$
   (E) $\omega_0/4$

Solution

The center of mass of the stone-disk system is initially stationary, so it remains stationary. This means the final motion of the system is purely rotation about its center of mass.

The disk and stone conserve angular momentum. If the angular momentum of the disk about its center is I_0, the initial angular momentum is

$$L_0 = I_0\omega_0.$$

We need to calculate the angular momentum of the stone-disk system about its center of mass.

The center of mass is half way between the center of the disk and the stone, so if the radius of the disk is R, and its mass m, the moment of inertia of the stone is

$$I_{\text{stone}} = m\left(\frac{R}{2}\right)^2.$$

By the parallel axis theorem, the angular momentum of the disk is

$$I_{\text{disk}} = m\left(\frac{R}{2}\right)^2 + I_0.$$

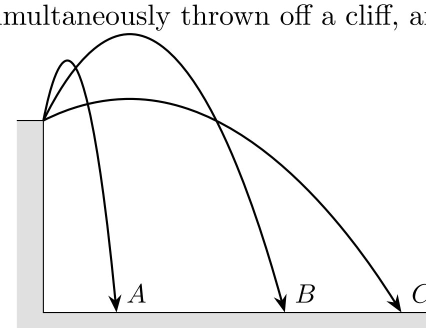
Using $I_0 = \frac{1}{2}mR^2$, the angular momentum of the stone-disk system is

$$I_{\text{total}} = mR^2 = 2I_0.$$

By angular momentum conservation, doubling the moment of inertia cuts the angular frequency in half, so the final angular frequency is

$$\omega = \frac{1}{2}\omega_0.$$

11. Projectiles A, B, and C are simultaneously thrown off a cliff, and take the trajectories shown.



Neglecting air resistance, rank the times t_A, t_B, and t_C they take to hit the ground.

(A) $t_A < t_B < t_C$

(B) $t_A < t_C < t_B$

(C) $t_C < t_B < t_A$

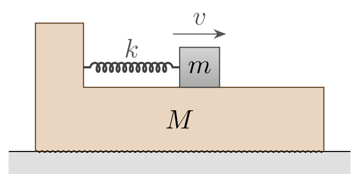
(D) $t_C < t_A < t_B$ ← **CORRECT**

(E) There is not enough information to decide.

Solution

Since the x and y directions are independent, the time to impact only depends on the initial v_y, which in turn determines the maximum height. The higher the maximum height, the longer the trajectory takes.

12. An object of mass $m = 1\,\text{kg}$ is attached to a platform of mass $M = 4\,\text{kg}$ with a spring of spring constant $k = 400\,\text{N/m}$. There is no friction between the object and the platform, and the coefficient of static friction between the platform and the ground is $\mu = 0.1$. The object is placed at its equilibrium position, and then given a horizontal velocity v.



For what v will the platform never slip on the ground?

(A) $v \leq 0.1\,\text{m/s}$
(B) $v \leq 0.2\,\text{m/s}$
(C) $v \leq 0.25\,\text{m/s}$ ← **CORRECT**
(D) $v \leq 0.4\,\text{m/s}$
(E) $v \leq 0.5\,\text{m/s}$

Solution

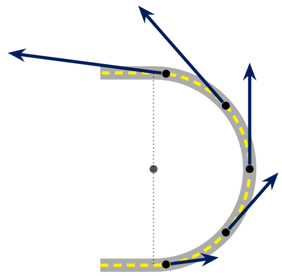
The force of the spring on the platform is $F_s = kx$, where x is the displacement of the object from equilibrium. The maximum value of x can be found by conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

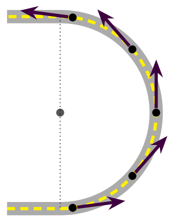
which gives a maximum spring force $F_s = v\sqrt{km}$ acting on the platform. The maximum possible friction force on the platform is $(M + m)\mu g$. Setting these two equal, the maximum velocity is

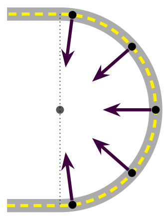
$$v = \frac{(M + m)\mu g}{\sqrt{km}} = 0.25\,\text{m/s}.$$

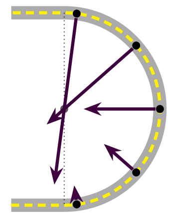
13. A car is driving on a semicircular racetrack. Its velocity at several points along the track is shown below.

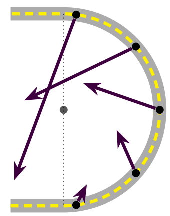


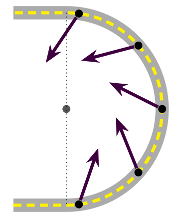
Which of the following could depict its acceleration at the corresponding points?

(A)


(B)


(C)


(D)


(E)


Solution

Because the car is speeding up, its acceleration has a tangential component, i.e. a component parallel to the racetrack. Because the car is turning, its acceleration also has a component v^2/r directed towards the center of the track. Since v increases through the turn, this inward radial acceleration increases through the turn. The only choice with all of these features is (D).

14. A bacterium cell swims by rotating its bundle of flagella to counter a viscous drag force in the medium. The drag force $F(R, v, \eta)$ only depends on the typical length scale of the cell R, its speed v, and the viscosity of the fluid η, which has units of kg/(m · s). It is observed under a microscope that a cell of length 1 μm swims at about 20 μm/s. Estimate the speed of a cell of length 0.5 μm, assuming cells of all sizes generate the same amount of force from their flagella.

(A) 5 μm/s

(B) 10 μm/s

(C) 40 μm/s ← **CORRECT**

(D) 80 μm/s

(E) There is not enough information to decide.

Solution

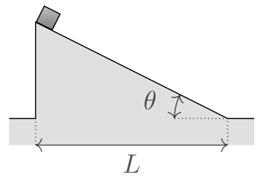
Using dimensional analysis, we can see that force scales as $F \sim Rv\eta$. To see this you can try:

$$[\mathrm{m}]^{\mathrm{a}}[\mathrm{m/s}]^{\mathrm{b}}[\mathrm{kg/(m \cdot s)}]^{\mathrm{c}} = [\mathrm{N}] = [\mathrm{kg \cdot m/s^2}].$$

The only solution is $a = b = c = 1$. Therefore, in the same medium, when the cell length reduces by 1/2, its speed doubles.

15. A block is released from rest at the top of a fixed, frictionless ramp with horizontal length L and

inclination θ.



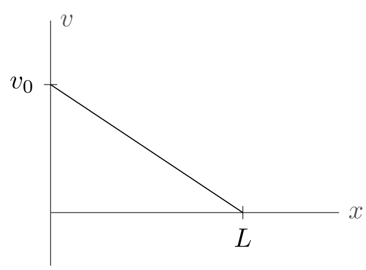
For a fixed value of L, which value of θ minimizes the time needed for the block to reach the bottom of the ramp?

(A) $30°$
(B) $45°$ ← **CORRECT**
(C) $60°$
(D) $75°$
(E) $80°$

Solution

The acceleration down the ramp is $a = g\sin\theta$, and $d = at^2/2$ where the distance $d = L/\cos\theta$. Hence $t^2 \propto d/a \propto 1/\sin\theta\cos\theta \propto 1/\sin(2\theta)$. The maximum of $\sin(2\theta)$, which minimizes t, is at $\theta = 45°$.

16. A particle begins at the point $x = 0$ and moves along the x-axis with initial rightward velocity v_0. Later, the particle reaches the point $x = L$. The velocity as a function of position during this time interval is shown below.



Consider the following three statements.

I. The particle instantaneously stops at $x = L$.
II. The particle is uniformly accelerated.
III. The particle could be performing simple harmonic motion.

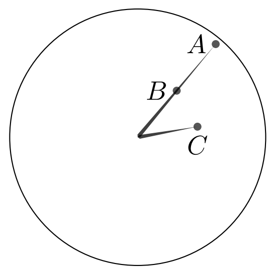
Which of these statements are true?

(A) Only I. ← **CORRECT**
(B) Only II.
(C) Both I and II.
(D) Both I and III.
(E) None of the above.

Solution

The particle instantaneously stops at $x = L$ since $v(L) = 0$. If the particle were decelerating uniformly, then by conservation of energy, $mv^2/2 + Fx$ would be constant for some constant F, which means $v(x)$ would not be a straight line. (Of course, $v(t)$ would be a straight line, but this isn't the same thing as $v(x)$.) Similarly, if the particle were performing simple harmonic motion, then $mv^2/2 + kx^2/2$ would be constant for some constant k, and the resulting $v(x)$ would also not be a straight line.

17. In a certain country, the short hand of a clock is exactly half as long as the long hand, and rotates twice for each rotation of the long hand.



The three points shown on the clock hands have accelerations of magnitude a_A, a_B, and a_C. The point B is at the midpoint of the long hand. Which of the following is true?

(A) $a_A < a_B = a_C$

(B) $a_A = a_B < a_C$

(C) $a_B < a_A < a_C$ ← **CORRECT**

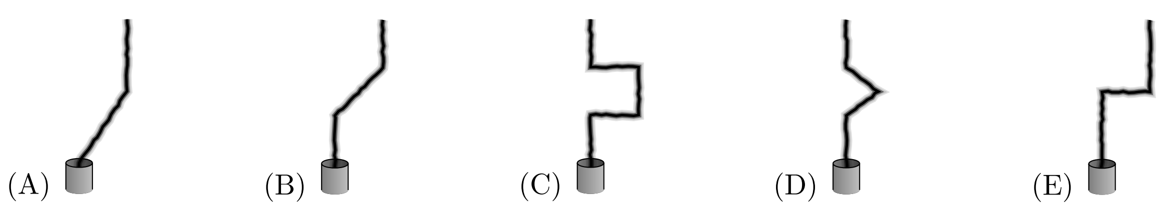
(D) $a_B < a_A = a_C$

(E) $a_B < a_C < a_A$

Solution

The centripetal acceleration is $a = \omega^2 r$. Since A and B have the same ω, $a_A = 2a_B$. Since B and C have the same r, $a_C = 4a_B$. Thus, $a_B < a_A < a_C$.

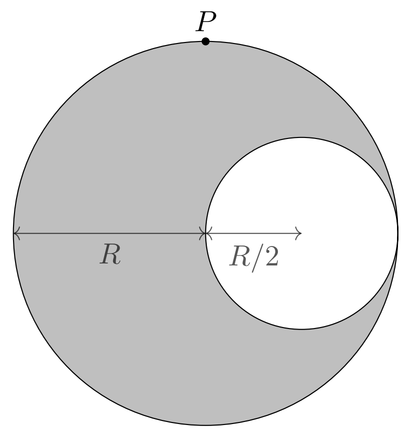
18. A factory's smokestack continually produces smoke. The smoke always rises with a constant speed relative to the air around it. Suppose that the air was initially still, then abruptly started blowing to the right, then abruptly became still again for some time. Which of the following could show the resulting shape of the smoke plume?



Solution

The smoke right next to the smokestack has just been emitted, and has only been exposed to still air, so it must rise straight upward. This rules out choice (A). The smoke high above the smokestack has experienced the full duration of the rightward wind, so it all must have been pulled to the right by the same amount, ruling out choices (C) and (D). Finally, the smoke in between has been exposed to only part of the rightward gust of wind (since some of that wind occurred before that smoke left the smokestack), so its path should smoothly connect the two vertical segments. This rules out choice (E), leaving (B) as the answer.

19. A cavity of radius $R/2$ is dug out of a spherical planet with uniform mass density of mass M and radius R. What is the magnitude of the gravitational field at point P in the diagram below?



(A) $(0.200)GM/R^2$
(B) $(0.457)GM/R^2$
(C) $(0.829)GM/R^2$
(D) $(0.900)GM/R^2$
(E) $(0.912)GM/R^2$ ← **CORRECT**

Solution

The gravitational field due to a completely filled gray sphere is $-\frac{MG}{R^2}\hat{y}$. We can treat the white circle as having negative mass. The center of the white circle is $\sqrt{5}R/2$ away from the point P. The field from the white circle has a magnitude of $\frac{MG}{8(R^2+(R/2)^2)} = \frac{MG}{10R^2}$. The field from the white circle is thus $\frac{MG}{5R^2\sqrt{5}}\hat{y} - \frac{MG}{10R^2\sqrt{5}}\hat{x}$. Adding the fields and taking the magnitude gives $(0.912)\frac{MG}{R^2}$.

20. The line between day and night on a planet or moon is called the *terminator*. How fast does the terminator of Earth's moon move across its surface at the equator? The radius of the moon is 1.74×10^6 m.

(A) 0 m/s
(B) 4.5 m/s ← **CORRECT**

(C) 83 m/s

(D) 465 m/s

(E) 2201 m/s

Solution

The same side of the moon always faces Earth. This means the terminator runs all the way around the moon once per month. The speed is

$$v = \frac{2\pi \cdot 1.74 \times 10^6 \text{ m}}{28 \text{ days}} \approx 4.5 \text{ m/s}.$$

21. A solid sphere sits at the top of a ramp of height h inclined at angle θ to the horizontal. Both the static and kinetic coefficients of friction between the sphere and incline are $\mu_k = \mu_s = 0.2$. The sphere is released from rest at the top of the incline. For which of the following values of θ is the total translational plus rotational kinetic energy of the sphere greatest when it reaches the bottom of the incline?

(A) 10° ← **CORRECT**

(B) 45°

(C) 60°

(D) 80°

(E) The mechanical energy is the same for all choices.

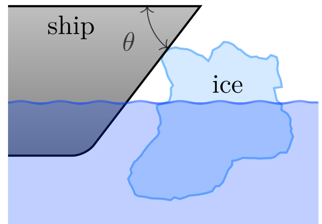
Solution

As long as the sphere rolls without slipping, it converts all its gravitational potential energy into kinetic energy, so its mechanical energy stays constant. That's what happens when the slope is very shallow. When the slope is steep, the acceleration is greater, and the torque required to roll without slipping is greater than what friction can provide. At these slopes, the sphere slips and dissipates mechanical energy, so its mechanical energy at the bottom of the slope is lower.

When rolling without slipping, a solid sphere has acceleration down an incline of $\frac{5}{7} g \sin\theta$.

The critical transition slope occurs when the friction force required to provide angular acceleration for the ball exceeds the maximum friction of $mg\cos\theta\mu$. This occurs when $\tan\theta = \frac{7}{2}\mu$, or about 35° when $\mu = 0.2$. This means that case (a) is rolling without slipping, while (b), (c), and (d) all have slipping and less kinetic energy at the bottom of the slope.

22. A ship rams into a hunk of ice floating in the sea. "Icebreakers" are ships designed so that when this happens, the ice is pushed down beneath the ship. If the coefficient of static friction between the ice and the ship is μ_s, what condition applies to the angle θ, as shown in the figure, so that the icebreaker functions as intended?



(A) $\cot\theta > \mu$ $\leftarrow$ **CORRECT**

(B) $\cos\theta > \mu$

(C) $\cot\theta < \mu$

(D) $\cos\theta < \mu$

(E) It depends on the curvature of the ice.

Solution

There is a force from the ship on the ice, which may be broken into a normal force and a friction force. The vertical component of the normal force is $-N\cos\theta$ and the vertical component of the friction force is $f\sin\theta$ with f the magnitude. The friction force can be at most $N\mu$, so the vertical force from the ship on the ice is

$$F_y \leq \mu N \sin\theta - N\cos\theta.$$

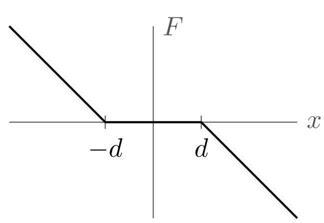
We want $F_y < 0$ in order to push the ice down, so we must have

$$N\cos\theta > \mu N \sin\theta$$

or

$$\cot\theta > \mu.$$

23. An imperfect spring has a restoring force F that depends on the displacement x from equilibrium as in the graph shown below.



The slope of the curve for $x < -d$ and $x > d$ is a constant $-k$. A mass m is attached to the spring and released from rest at the position $x = A$, where $A > d$. What is the period of the subsequent motion?

(A) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{2d}{A}\right)$

(B) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{2d}{A-d}\right)$

(C) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{4d}{A-d}\right)$ $\leftarrow$ **CORRECT**

(D) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{\pi d}{A-d}\right)$

(E) $T = \sqrt{\frac{m}{k}}\left(2\pi + \frac{2\pi d}{A-d}\right)$

Solution

The full motion is the combination of two parts: simple harmonic motion as if the spring were ideal (when the mass is at $|x| > d$), and motion with uniform velocity in the region $|x| < d$. By conservation of energy, the velocity in this region satisfies

$$\frac{1}{2}mv^2 = \frac{1}{2}k(A-d)^2$$

so the time spent in this region is

$$\frac{4d}{v} = \frac{4d}{A-d}\sqrt{\frac{m}{k}}.$$

The time spent at $|x| > d$ is $2\pi\sqrt{m/k}$, and adding these two times gives the result.

24. Two satellites are in circular orbits around a star with equal radius r, speed v, and period T. The satellites are initially diametrically opposite each other. In order to meet the second satellite in time $T/2$, the first satellite should decrease its speed to approximately

(A) $0.50v$

(B) $0.64v$ ← **CORRECT**

(C) $0.71v$

(D) $0.76v$

(E) $0.82v$

Solution

For the meeting to occur, the first satellite must switch into an orbit with period $T/2$. By Kepler's third law, the new semi-major axis should be $a = r/2^{2/3}$. Since the total energy of a satellite is $E = -GMm/2a$, the energy difference of the two satellites is

$$E_2 - E_1 = \frac{GMm}{2r}(2^{2/3} - 1) = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2$$

where v' is the speed of the first satellite right after it slows down. On the other hand, in circular motion we have $mv^2 = GMm/r$, which implies after some simplification that

$$v' = \sqrt{2 - 2^{2/3}}\,v = 0.64v.$$

25. Spaceman Fred's trusty pellet sprayer is held at rest a distance h away from the center of Planet Orb, which has radius $R \ll h$. The pellet sprayer ejects pellets radially outward, uniformly in the plane of the page. These pellets are all launched with the same speed v, so that a pellet launched

directly away from Orb by the pellet sprayer can just barely escape it. What fraction of the pellets eventually lands on Orb? (Hint: you may use the small angle approximation, $\sin\theta \approx \theta$ for $\theta \ll 1$, where θ is in radians.)

Left: the pellet sprayer relative to Orb   Right: a close-up of the pellet sprayer

(A) $\frac{1}{2\pi}\frac{R}{h}$

(B) $\frac{1}{\pi}\frac{R}{h}$

(C) $\frac{2}{\pi}\frac{R}{h}$

(D) $\frac{1}{2\pi}\sqrt{\frac{R}{h}}$

(E) $\frac{1}{\pi}\sqrt{\frac{R}{h}}$ ← **CORRECT**

Solution

Because all pellets are launched at the escape velocity, the speed of one of these pellets a distance r away from Orb is

$$v(r) = \sqrt{\frac{2GM}{r}}.$$

At the distance r_{min} of closest approach, the pellet's velocity must be perpendicular to its displacement from the planet's center. Then, by conservation of angular momentum, for a pellet of mass m launched at an angle θ from the vertical,

$$mv(h)h\sin\theta = mv(r_{\text{min}})r_{\text{min}} \implies \sqrt{h}\sin\theta = \sqrt{r_{\text{min}}}.$$

The pellet collides with the planet whenever $r_{\text{min}} < R$. Then, our equation for θ_{max} is

$$\theta_{\text{max}} \approx \sin\theta_{\text{max}} = \sqrt{\frac{R}{h}}.$$

The range of collision angles is thus $-\theta_{\text{max}} < \theta < \theta_{\text{max}}$ and the range of allowed angles is 2π, so the fraction is

$$f = \frac{1}{\pi}\sqrt{\frac{R}{h}}.$$

AAPT UNITED STATES PHYSICS TEAM
AIP 2022

2022 $F = ma$ Exam A

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
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Tengiz Bibilashvili, Abi Krishnan, Andrew Lin, Kris Lui, Kye Shi, Brian Skinner, Mike Winer and Kevin Zhou

1. A projectile is thrown upward with speed v. By the time its speed has decreased to $v/2$, it has risen a height h. Neglecting air resistance, what is the maximum height reached by the projectile?

(A) $\frac{5h}{4}$ (B) $\frac{4h}{3}$ (C) $\frac{3h}{2}$ (D) $2h$ (E) $3h$

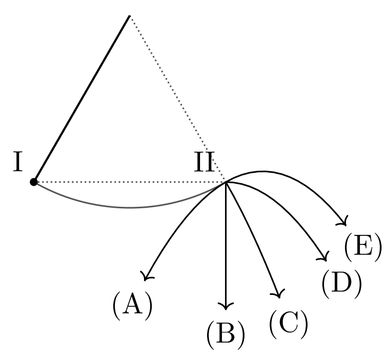
2. A car is moving at 60 miles per hour (mph), when the driver notices an obstacle ahead. Hitting the brakes, the driver decelerates at a constant rate, and manages to come to a stop just barely before hitting the obstacle. If the car had instead been moving at 70 mph, and started decelerating at the same place and at the same rate, with what speed would it have hit the obstacle?

(A) 10 mph
(B) 14 mph
(C) 28 mph
(D) 36 mph
(E) There is not enough information to decide.

3. Two blocks of mass m have an inelastic one-dimensional collision. Initially, the first block is moving with speed 5 m/s, and the second is at rest. After the collision, the first block is moving with speed 2 m/s. What percentage of the system's original kinetic energy was lost during the collision?

(A) 16% (B) 42% (C) 48% (D) 52% (E) 84%

4. A mass on an ideal pendulum is released from rest at point I. It swings over to point II, at which point the string suddenly breaks. Which of the following shows the trajectory of the mass?



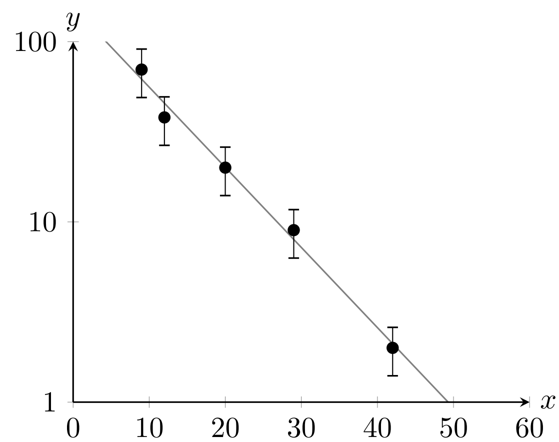
5. A uniform solid ball with mass $m = 1$ kg and radius $R = 10$ cm rolls without slipping on a horizontal plane, so that its center of mass has velocity $v = 1$ m/s. What is the ball's total kinetic energy?

(A) 0.2 J (B) 0.5 J (C) 0.7 J (D) 1 J (E) 1.4 J

6. A bob of mass m hangs from a rigid, massless rod, forming an ideal pendulum. The rod is held horizontally and released from rest. What is its maximum tension during its swing?

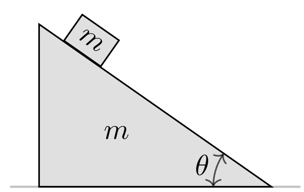
(A) mg (B) $\frac{3}{2}mg$ (C) $2mg$ (D) $3mg$ (E) $4mg$

7. The following graph shows the results of measurements of two physical quantities, y and x. What is the following best describes the functional dependence of y on x? Below, A and B are positive constants.



(A) $y = Ax + B$ (B) $y = -Ax + B$ (C) $y = A/x^B$ (D) $y = Ae^{Bx}$ (E) $y = Ae^{-Bx}$

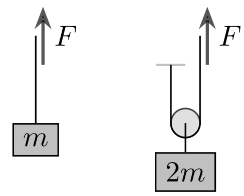
8. A block of mass m is placed on a wedge of mass m, inclined at an angle θ to the horizontal.



The coefficients of friction between the block and wedge, and the wedge and ground, are high enough for both the block and the wedge to remain static. What is the magnitude of the friction force of the ground on the wedge?

(A) $mg\sin\theta$ (B) $mg\cos\theta$ (C) $mg\sin\theta\cos\theta$ (D) $mg\tan\theta$ (E) 0

9. A person is holding a massless rope, on which hangs a mass m, as shown at left. To pull the end of the rope with constant upward velocity v, the person must exert a force F_v. To pull the end of the rope with constant upward acceleration a, the person must exert a force F_a. Now the rope is wrapped around a fixed, massless pulley, and the mass is doubled to $2m$, as shown at right.



Compared to the original setup, how do the forces F_v and F_a needed to pull the end of the rope with a given upward velocity and acceleration change? In both cases, ignore friction and air resistance.

(A) F_v stays the same, and F_a decreases.

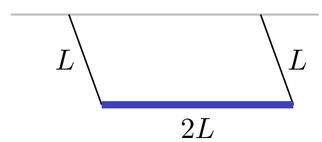
(B) Both F_v and F_a stay the same.

(C) F_v stays the same, and F_a increases.

(D) F_v increases, and F_a stays the same.

(E) Both F_v and F_a increase.

10. The two ends of a uniform rod of length $2L$ are hung on massless strings of length L.



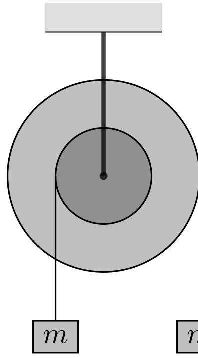
If the strings are attached to the ceiling, and the rod is pulled a small distance horizontally and released as shown, what is the period of oscillation?

(A) $2\pi\sqrt{\frac{L}{g}}$ (B) $2\pi\sqrt{\frac{7L}{6g}}$ (C) $2\pi\sqrt{\frac{4L}{3g}}$ (D) $2\pi\sqrt{\frac{2L}{g}}$ (E) $2\pi\sqrt{\frac{7L}{3g}}$

11. Two identical spherically symmetric planets, each of mass M, are somehow held at rest with respect to each other. Each planet has radius R, and the distance between the centers of the planets is $4R$. If a rocket is launched from the surface of one planet with speed v, what is the minimum speed v so that the rocket can reach the other planet?

(A) $\sqrt{\frac{2GM}{R}}$ (B) $\sqrt{\frac{GM}{R}}$ (C) $\sqrt{\frac{3GM}{4R}}$ (D) $\sqrt{\frac{2GM}{3R}}$ (E) $\sqrt{\frac{GM}{2R}}$

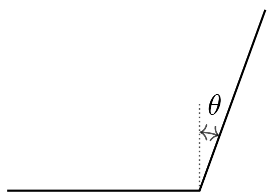
12. A pulley is constructed by attaching two concentric cylinders, with the larger cylinder having twice the radius. Ropes are wrapped around both cylinders, a mass m is hung from each rope, and the system is released from rest.



Neglect the masses of the cylinders and ropes. Each mass experiences both a gravitational and a tension force. If the *net* force experienced by the left mass is F_1, and the net force experienced by the right mass is F_2, what is the ratio F_2/F_1?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 4

13. Consider a laptop made of two identical uniform plates, each of mass $m/2$, connected by a hinge. The hinge is locked when the screen makes an angle θ to the vertical, as shown, fixing the angle between the two pieces.

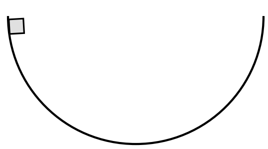


Assuming the laptop does not slip, what is the minimum force that can be exerted on the top of the laptop, in the plane of the page, to cause the bottom of the laptop to lift off the ground?

(A) $\dfrac{mg(1-\sin\theta)}{2}$ (B) $\dfrac{mg(\cos\theta+\sin\theta)}{2}$ (C) $\dfrac{mg(1-\sin\theta)}{4}$

(D) $\dfrac{mg(1+\sin\theta)}{4}$ (E) $\dfrac{mg(\cos\theta+\sin\theta)}{4}$

14. A small block is released from rest on the rim of a fixed, frictionless hemispherical bowl.



From the time the block is released, until it reaches the bottom of the bowl, which of the following is true?

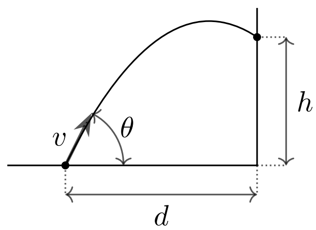
I. The speed of the block never decreases.

II. The magnitude of the horizontal component of the velocity of the block never decreases.

III. The magnitude of the vertical component of the velocity of the block never decreases.

(A) Only I. (B) Only III. (C) I and II. (D) I and III. (E) I, II, and III.

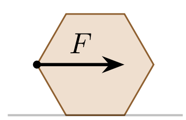
15. An egg is launched with speed v from the ground, a distance d from a vertical wall.



If v is high enough for the egg to hit the wall, which of the following could describe the angle θ that maximizes the height h at which the egg hits the wall?

(A) $\sin\theta = \dfrac{v^2}{gd}$ (B) $\tan\theta = \dfrac{v^2}{gd}$ (C) $\sin 2\theta = \dfrac{gd}{2v^2}$ (D) $\cos\theta = \dfrac{gd}{v^2}$ (E) $\sin 2\theta = \dfrac{v^2}{gd}$

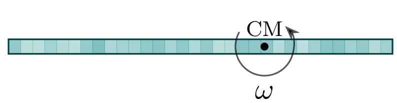
16. A hexagonal pencil of uniform density lies at rest on a horizontal table. It is pushed horizontally with a steadily increasing force halfway up its height, as shown.



What is the minimum value of the coefficient of static friction between the floor and pencil, so that the pencil will eventually begin to roll?

(A) 0 (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{\sqrt{3}}{3}$ (E) $\dfrac{\sqrt{3}}{2}$

17. A thin rod has a *nonuniform* density. It is mounted on an axle passing perpendicular to it, through its center of mass, as shown, and is then rotated about the axle.



The axle divides the rod into two parts, one on each side of it. Which of the following must be true, no matter how the mass in the rod is distributed?

(A) The two parts have the same mass.

(B) The magnitudes of the momenta of the two parts are equal.

(C) The magnitudes of the angular momenta of the two parts, about the center of mass, are equal.

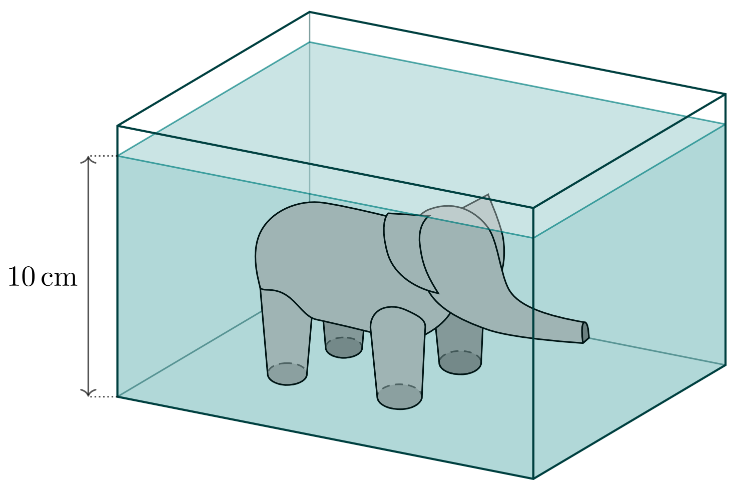
(D) The kinetic energies of the two parts are equal.

(E) At least two of the above are true.

18. A cylindrical piece of cork of density ρ_c, height h_c, and cross-sectional area A_c is in a larger empty cylindrical container of cross-sectional area A_w. Water of density $\rho_w > \rho_c$ is slowly poured into the empty container. What is the height of the water in the container when the cork starts to float?

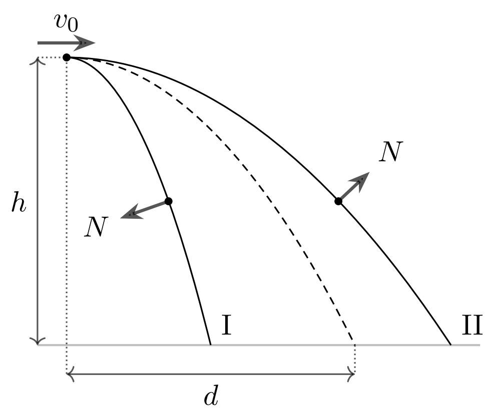
(A) $\dfrac{h_c \rho_c A_c}{\rho_w A_w}$ (B) $\dfrac{h_c \rho_c}{\rho_w}$ (C) $\dfrac{h_c \rho_w}{\rho_c}$ (D) $\dfrac{h_c \rho_c A_c}{\rho_w (A_w - A_c)}$ (E) $\dfrac{h_c \rho_c A_c^2}{\rho_w A_w^2}$

19. A toy elephant is standing on the bottom of a fish tank. The fish tank is filled with water to a depth of 10 cm, completely covering the toy. The elephant's legs are perfectly polished, so that there is no water between the bottom of the legs and the tank's floor, and the total area of contact is $0.16\,\text{cm}^2$. The water has density $\rho = 10^3\,\text{kg/m}^3$, the toy has uniform density 2ρ, the atmospheric pressure is $P_{\text{atm}} = 10^5\,\text{Pa}$, and the toy has total mass 120 g. What is the total hydrostatic force that the water exerts on the toy?



(A) 1 N, down (B) 0.6 N, down (C) 0 N (D) 0.6 N, up (E) 1 N, up

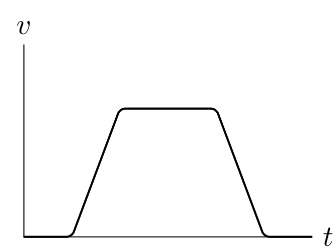
20. A bead is threaded on a frictionless wire and launched horizontally from height h with speed v_0, as shown. If the shape of the wire is steep, as in curve I, then the normal force from the wire on the bead will point inward. If it is shallow, as in curve II, then the normal force will point outward.

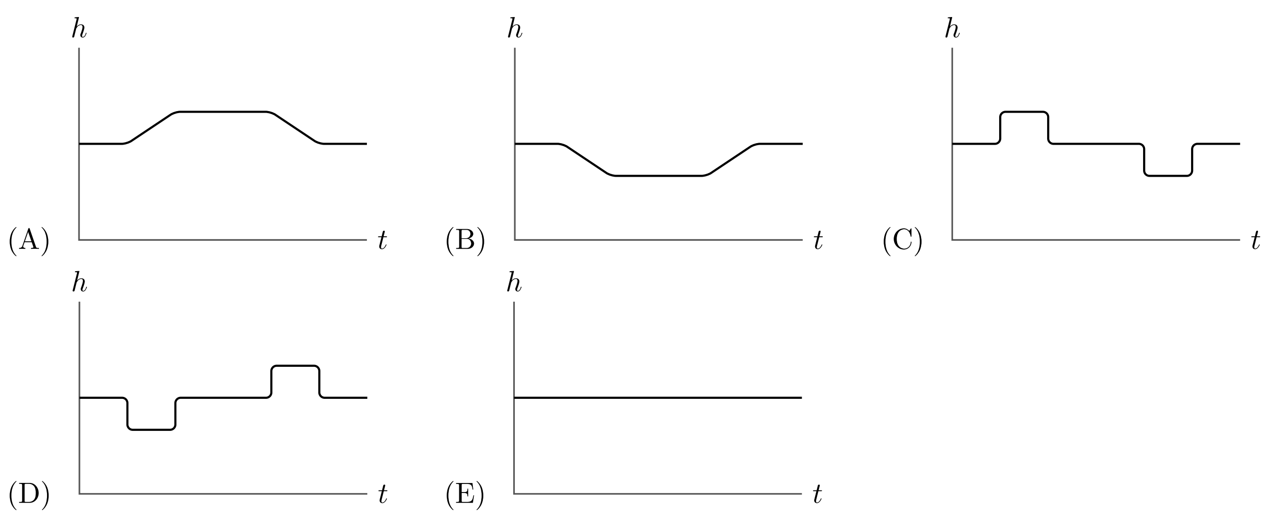


There is exactly one possible shape of wire, shown as a dotted line, for which the normal force of the wire on the bead is *always* equal to zero. What is the horizontal displacement d of the bead when it travels along this wire?

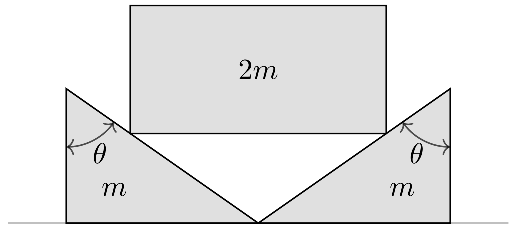
(A) $v_0\sqrt{\frac{4g}{h}}$ (B) $v_0\sqrt{\frac{2h}{g}}$ (C) $v_0\sqrt{\frac{h}{g}}$ (D) $v_0\sqrt{\frac{h}{2g}}$ (E) $v_0\sqrt{\frac{h}{4g}}$

21. A cork floating in a cup filled with a viscous fluid is placed in an elevator. Below is a plot of the velocity v of the elevator as a function of time t. Which of the following plots best describes the height h of the cork in the cup as a function of time? Assume that the fluid is viscous enough to dampen all oscillations, that the fluid does not slosh as the elevator accelerates, and that both the cork and fluid are incompressible.





22. A block of mass $2m$ is placed symmetrically on two identical wedges of mass m, as shown.



All surfaces are frictionless, and the wedges have angle θ to the vertical. If the system is released from rest, what is the downward acceleration of the block?

(A) $g\sin\theta$ (B) $g\sin(2\theta)$ (C) $g\cos\theta$ (D) $g\cos(2\theta)$ (E) $g\cos^2\theta$

23. For objects moving through air, the force of air resistance can be modeled as proportional to the speed ("linear drag") or proportional to the square of the speed ("quadratic drag"), depending on the circumstances. Two identical objects, A and B, are dropped from the same height h simultaneously, but object A is given an initial horizontal velocity v. The objects hit the ground at times t_A and t_B. Accounting for air resistance, which of the following is true?

(A) For both linear drag and quadratic drag, $t_A = t_B$.
(B) For linear drag, $t_A > t_B$, while for quadratic drag, $t_A = t_B$.
(C) For linear drag, $t_A = t_B$, while for quadratic drag, $t_A > t_B$.
(D) For both linear drag and quadratic drag, $t_A > t_B$.
(E) For both linear drag and quadratic drag, the answer depends on v and h.

24. A satellite is in orbit around a planet of mass M. Its maximum distance from the center of the planet is d, and at this point, it is traveling at a speed of $\frac{1}{2}\sqrt{\frac{GM}{d}}$. What is the area of the satellite's orbit?

(A) $\frac{8}{15}\sqrt{\frac{2}{15}}\pi d^2$ (B) $\frac{4}{7}\sqrt{\frac{1}{7}}\pi d^2$ (C) $\frac{1}{3}\sqrt{\frac{2}{3}}\pi d^2$ (D) $\frac{8}{7}\sqrt{\frac{1}{7}}\pi d^2$ (E) $\frac{2}{3}\sqrt{\frac{2}{3}}\pi d^2$

25. A cylinder is placed with its axis vertical, and a rubber band of mass m and tension T is wrapped horizontally around it. What is the minimum coefficient of static friction μ between the rubber band and the cylinder such that the band will not slide down the cylinder?

(A) $\frac{mg}{2\pi T}$ (B) $\frac{mg}{T}$ (C) $\frac{4mg}{T}$ (D) $\frac{2\pi mg}{T}$ (E) $\frac{2m^2g^2}{T^2}$

AAPT UNITED STATES PHYSICS TEAM
AIP 2022

2022 $F = ma$ Exam A

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

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1. A projectile is thrown upward with speed v. By the time its speed has decreased to $v/2$, it has risen a height h. Neglecting air resistance, what is the maximum height reached by the projectile?

(A) $\frac{5h}{4}$ $\boxed{\text{B}}$ $\frac{4h}{3}$ (C) $\frac{3h}{2}$ (D) $2h$ (E) $3h$

We use the basic kinematic equation $v^2 - v_f^2 = 2g\Delta y$. The maximum height is $v^2/2g$, while the height reached by the time the speed falls to $v/2$ is $h = (1 - 1/4)(v^2/2g)$. Therefore, the maximum height is $4h/3$.

2. A car is moving at 60 miles per hour (mph), when the driver notices an obstacle ahead. Hitting the brakes, the driver decelerates at a constant rate, and manages to come to a stop just barely before hitting the obstacle. If the car had instead been moving at 70 mph, and started decelerating at the same place and at the same rate, with what speed would it have hit the obstacle?

(A) 10 mph
(B) 14 mph
(C) 28 mph
$\boxed{\text{D}}$ 36 mph
(E) There is not enough information to decide.

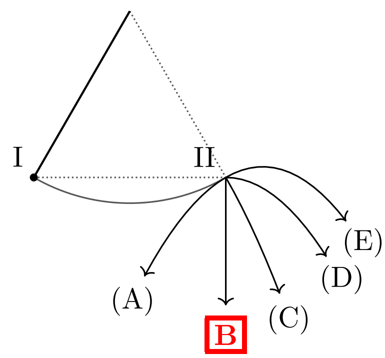
For an object decelerating at a constant rate a over a distance d, the initial and final velocities, v_f and v_i, are related by $v_f^2 = v_i^2 - 2ad$. In the case where the car was traveling with an initial speed $v_1 = 60\,\text{mph}$, this equation gives $v_1^2 = 2ad$. In the case where the car was traveling with an initial speed $v_2 = 70\,\text{mph}$, this equation gives $v_f^2 = v_2^2 - 2ad$. Thus, $v_f^2 = v_2^2 - v_1^2$, or $v_f = \sqrt{70^2 - 60^2} \approx 36\,\text{mph}$.

3. Two blocks of mass m have an inelastic one-dimensional collision. Initially, the first block is moving with speed 5 m/s, and the second is at rest. After the collision, the first block is moving with speed 2 m/s. What percentage of the system's original kinetic energy was lost during the collision?

(A) 16% (B) 42% $\boxed{\text{C}}$ 48% (D) 52% (E) 84%

We must consider two cases: either the first block is traveling in the same direction as before, or in the opposite direction. If it is traveling in the opposite direction, the other block must have speed 7 m/s by momentum conservation, so the kinetic energy is higher than before the collision, which is impossible. Therefore, the other block has speed 3 m/s, so the fraction of energy lost is $(5^2 - 2^2 - 3^2)/5^2 = 48\%$.

4. A mass on an ideal pendulum is released from rest at point I. It swings over to point II, at which point the string suddenly breaks. Which of the following shows the trajectory of the mass?



When the string breaks, the mass has zero velocity by energy conservation, so it just falls straight down. Thus, the answer is (B).

5. A uniform solid ball with mass $m = 1$ kg and radius $R = 10$ cm rolls without slipping on a horizontal plane, so that its center of mass has velocity $v = 1$ m/s. What is the ball's total kinetic energy?

(A) 0.2 J (B) 0.5 J $\boxed{\text{C}}$ 0.7 J (D) 1 J (E) 1.4 J

The translational kinetic energy is $\frac{1}{2}mv^2 = 0.5\,\text{J}$. The rotational kinetic energy is $\frac{1}{2}I\omega^2$, where $I = \frac{2}{5}mR^2$ is the moment of inertia of a uniform ball. Using $\omega = v/r$ gives a rotational kinetic energy of $\frac{1}{5}mv^2 = 0.2\,\text{J}$, for a total of $0.7\,\text{J}$.

6. A bob of mass m hangs from a rigid, massless rod, forming an ideal pendulum. The rod is held horizontally and released from rest. What is its maximum tension during its swing?

(A) mg (B) $\frac{3}{2}mg$ (C) $2mg$ $\boxed{\text{D}}$ $3mg$ (E) $4mg$

The maximum tension occurs at the bottom of the swing. If the pendulum has length ℓ, then by energy conservation,

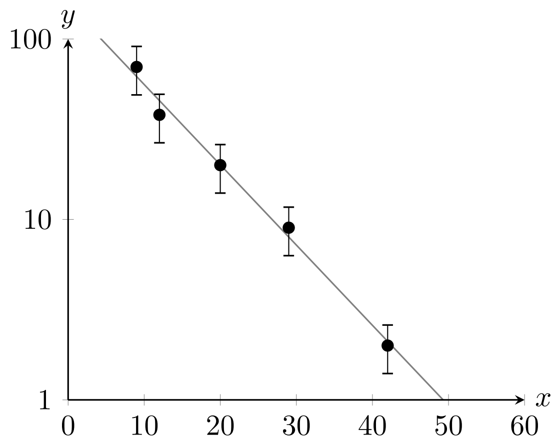
$$\frac{mv^2}{2} = mg\ell.$$

Thus, the centripetal acceleration at the bottom is

$$\frac{mv^2}{\ell} = 2mg.$$

The tension plus the weight together provide this centripetal acceleration, so the tension is $3mg$.

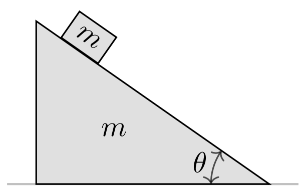
7. The following graph shows the results of measurements of two physical quantities, y and x. What is the following best describes the functional dependence of y on x? Below, A and B are positive constants.



(A) $y = Ax + B$ (B) $y = -Ax + B$ (C) $y = A/x^B$ (D) $y = Ae^{Bx}$ $\boxed{\text{E}}$ $y = Ae^{-Bx}$

The plot shows a linear relationship between x and $\log y$, which means that y is an exponential in x. Since y decreases as x increases, the asnwer must be (E).

8. A block of mass m is placed on a wedge of mass m, inclined at an angle θ to the horizontal.

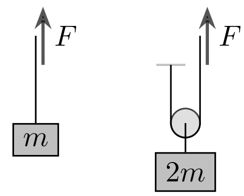


The coefficients of friction between the block and wedge, and the wedge and ground, are high enough for both the block and the wedge to remain static. What is the magnitude of the friction force of the ground on the wedge?

(A) $mg \sin\theta$ (B) $mg\cos\theta$ (C) $mg\sin\theta\cos\theta$ (D) $mg\tan\theta$ $\boxed{\text{E}}$ 0

Gravity exerts a force mg downward on the block, which means that the wedge must exert a force mg upward on the block. Thus, the block exerts a force mg downward on the wedge, and gravity also exerts a force mg downward on the wedge. Since these forces have no horizontal components, no friction with the ground is necessary to keep the wedge static.

9. A person is holding a massless rope, on which hangs a mass m, as shown at left. To pull the end of the rope with constant upward velocity v, the person must exert a force F_v. To pull the end of the rope with constant upward acceleration a, the person must exert a force F_a. Now the rope is wrapped around a fixed, massless pulley, and the mass is doubled to $2m$, as shown at right.



Compared to the original setup, how do the forces F_v and F_a needed to pull the end of the rope with a given upward velocity and acceleration change? In both cases, ignore friction and air resistance.

$\boxed{\text{A}}$ F_v stays the same, and F_a decreases.

(B) Both F_v and F_a stay the same.

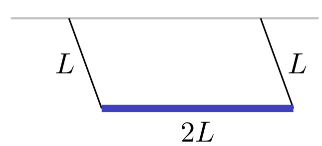
(C) F_v stays the same, and F_a increases.

(D) F_v increases, and F_a stays the same.

(E) Both F_v and F_a increase.

To pull the rope with constant velocity, the forces on the mass must be balanced, which means $F_v = mg$ originally, and $F_v = 2mg/2 = mg$ with the pulley, so F_v stays the same. In the original setup, the acceleration of the mass must be a, so we have $F_a = m(g + a)$. With the pulley, the acceleration of the mass only needs to be $a/2$, and furthermore any extra force acting on it is doubled. We have $2m(g + a/2) = 2F_a$, which means $F_a = m(g + a/2)$, which is smaller than before.

10. The two ends of a uniform rod of length $2L$ are hung on massless strings of length L.



If the strings are attached to the ceiling, and the rod is pulled a small distance horizontally and released as shown, what is the period of oscillation?

$\boxed{\text{A}}$ $2\pi\sqrt{\frac{L}{g}}$ (B) $2\pi\sqrt{\frac{7L}{6g}}$ (C) $2\pi\sqrt{\frac{4L}{3g}}$ (D) $2\pi\sqrt{\frac{2L}{g}}$ (E) $2\pi\sqrt{\frac{7L}{3g}}$

The sum of the tension forces on this pendulum act exactly like the tension force on an ordinary pendulum of length L. Thus, the period of oscillation must still be $2\pi\sqrt{L/g}$. (Unlike a physical pendulum, the moment of inertia of the rod doesn't matter, because it never rotates about its center.)

11. Two identical spherically symmetric planets, each of mass M, are somehow held at rest with respect to each other. Each planet has radius R, and the distance between the centers of the planets is $4R$. If a rocket is launched from the surface of one planet with speed v, what is the minimum speed v so that the rocket can reach the other planet?

(A) $\sqrt{\frac{2GM}{R}}$ (B) $\sqrt{\frac{GM}{R}}$ (C) $\sqrt{\frac{3GM}{4R}}$ $\boxed{\text{D}}$ $\sqrt{\frac{2GM}{3R}}$ (E) $\sqrt{\frac{GM}{2R}}$

The gravitational force vanishes at the midway point between the planets, so the rocket only needs to have enough energy to get there. The initial and final gravitational potential energies are

$$U_i = -\frac{GMm}{R} - \frac{GMm}{3R} = -\frac{4}{3}\frac{GMm}{R}, \quad U_f = -\frac{2GMm}{2R} = -\frac{GMm}{R}.$$

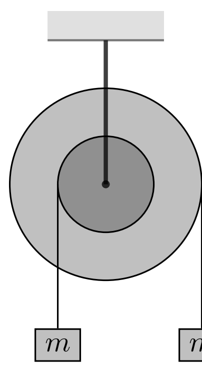
Thus, the initial kinetic energy needed is

$$\frac{1}{2}mv^2 = U_f - U_i = \frac{GMm}{3R}$$

which implies

$$v = \sqrt{\frac{2GM}{3R}}.$$

12. A pulley is constructed by attaching two concentric cylinders, with the larger cylinder having twice the radius. Ropes are wrapped around both cylinders, a mass m is hung from each rope, and the system is released from rest.

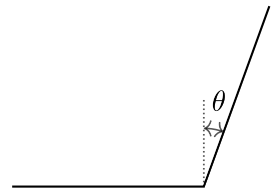


Neglect the masses of the cylinders and ropes. Each mass experiences both a gravitational and a tension force. If the *net* force experienced by the left mass is F_1, and the net force experienced by the right mass is F_2, what is the ratio F_2/F_1?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 $\boxed{\text{D}}$ 2 (E) 4

This unusual pulley is called a windlass. Since $a = R\alpha$ and the right mass is wound around a cylinder of twice the radius, the right mass has twice the acceleration. Since $F = ma$, we have $F_2/F_1 = 2$.

13. Consider a laptop made of two identical uniform plates, each of mass $m/2$, connected by a hinge. The hinge is locked when the screen makes an angle θ to the vertical, as shown, fixing the angle between the two pieces.

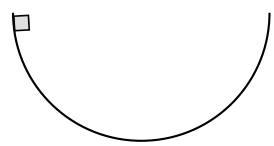


Assuming the laptop does not slip, what is the minimum force that can be exerted on the top of the laptop, in the plane of the page, to cause the bottom of the laptop to lift off the ground?

(A) $\dfrac{mg(1-\sin\theta)}{2}$ (B) $\dfrac{mg(\cos\theta+\sin\theta)}{2}$ $\boxed{\text{C}}$ $\dfrac{mg(1-\sin\theta)}{4}$
(D) $\dfrac{mg(1+\sin\theta)}{4}$ (E) $\dfrac{mg(\cos\theta+\sin\theta)}{4}$

When the bottom of the laptop is about to lift off, the normal force is concentrated at the hinge, so we take torques about that point. If the plates have length ℓ, the net torque due to gravity is $(mg/2)(\ell/2 - (\ell/2)\sin\theta)$, while the torque due to the applied force is $F\ell$. Therefore, $F = (mg/4)(1-\sin\theta)$.

14. A small block is released from rest on the rim of a fixed, frictionless hemispherical bowl.



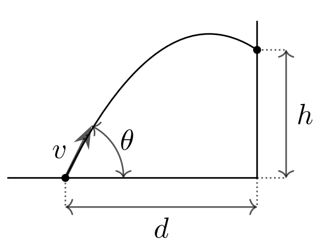
From the time the block is released, until it reaches the bottom of the bowl, which of the following is true?

I. The speed of the block never decreases.
II. The magnitude of the horizontal component of the velocity of the block never decreases.
III. The magnitude of the vertical component of the velocity of the block never decreases.

(A) Only I. (B) Only III. $\boxed{\text{C}}$ I and II. (D) I and III. (E) I, II, and III.

Choice III isn't true, because the vertical velocity starts at zero and ends at zero, so its magnitude must increase and then decrease. Choice I is true because the block is never moving upward, so its potential energy never increases. Choice II is true because the only horizontal force the block experiences is the horizontal component of the normal force, which never points to the left.

15. An egg is launched with speed v from the ground, a distance d from a vertical wall.

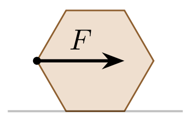


If v is high enough for the egg to hit the wall, which of the following could describe the angle θ that maximizes the height h at which the egg hits the wall?

(A) $\sin\theta = \dfrac{v^2}{gd}$ $\boxed{\text{B}}$ $\tan\theta = \dfrac{v^2}{gd}$ (C) $\sin 2\theta = \dfrac{gd}{2v^2}$ (D) $\cos\theta = \dfrac{gd}{v^2}$ (E) $\sin 2\theta = \dfrac{v^2}{gd}$

This problem is meant to be solved with limiting cases. When $v \to \infty$, we should have $\theta \to 90°$. This rules out choices (A) and (E). When $v^2 = gd$, it is just barely possible to hit the wall at all, and the projectile must be launched at $\theta = 45°$ to do this. This rules out choices (C) and (D), leaving choice (B).

16. A hexagonal pencil of uniform density lies at rest on a horizontal table. It is pushed horizontally with a steadily increasing force halfway up its height, as shown.

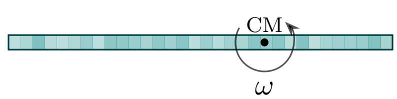


What is the minimum value of the coefficient of static friction between the floor and pencil, so that the pencil will eventually begin to roll?

(A) 0 (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ $\boxed{\text{D}}$ $\dfrac{\sqrt{3}}{3}$ (E) $\dfrac{\sqrt{3}}{2}$

If the pencil is about to start rolling, the normal force on it will be concentrated at the far end of the pencil. Taking torques about this point, the torque of the applied force must balance the torque due to gravity. If the pencil has side length r, then this implies $mg(r/2) = F(\sqrt{3}\,r/2)$, which implies $F = mg/\sqrt{3}$. For the pencil to be able to roll, the pencil must not be slipping at this point, which implies $\mu_s > 1/\sqrt{3}$.

17. A thin rod has a *nonuniform* density. It is mounted on an axle passing perpendicular to it, through its center of mass, as shown, and is then rotated about the axle.



The axle divides the rod into two parts, one on each side of it. Which of the following must be true, no matter how the mass in the rod is distributed?

(A) The two parts have the same mass.
$\boxed{\text{B}}$ The magnitudes of the momenta of the two parts are equal.
(C) The magnitudes of the angular momenta of the two parts, about the center of mass, are equal.
(D) The kinetic energies of the two parts are equal.
(E) At least two of the above are true.

For simplicity, let's suppose the rod is made of discrete masses, though the reasoning is exactly the same if the rod is continuous. By the definition of the center of mass, if r_i is the distance of mass m_i from the center of mass, then

$$\sum_{i\,\text{left}} m_i r_i = \sum_{i\,\text{right}} m_i r_i.$$

The total mass, momentum, angular momentum, and kinetic energy are

$$\sum_i m_i, \quad \sum_i m_i v_i = \omega \sum_i m_i r_i, \quad \sum_i m_i v_i r_i = \omega \sum_i m_i r_i^2, \quad \sum_i m_i v_i^2/2 = (\omega^2/2) \sum_i m_i r_i^2.$$

Only the momentum has the sum of the same form, so the magnitudes of the momenta are equal, and in general none of the other quantities are equal. (Another easy way to tell that the momenta are equal is that they must be opposite, because we know the total momentum of the rod is zero.)

18. A cylindrical piece of cork of density ρ_c, height h_c, and cross-sectional area A_c is in a larger empty cylindrical container of cross-sectional area A_w. Water of density $\rho_w > \rho_c$ is slowly poured into the empty container. What is the height of the water in the container when the cork starts to float?

(A) $\dfrac{h_c \rho_c A_c}{\rho_w A_w}$ $\boxed{\text{B}}$ $\dfrac{h_c \rho_c}{\rho_w}$ (C) $\dfrac{h_c \rho_w}{\rho_c}$ (D) $\dfrac{h_c \rho_c A_c}{\rho_w (A_w - A_c)}$ (E) $\dfrac{h_c \rho_c A_c^2}{\rho_w A_w^2}$

We will show two alternative solutions.

Archimedes' principle: The weight of the water displaced has to equal the weight of the cork for the cork to start floating. If the height of the water in the container is H, then the weight of the displaced water is

$$\rho_w g A_c H.$$

The weight of the cork is

$$\rho_c g A h.$$

Therefore, we require

$$H = \frac{\rho_c h}{\rho_w}.$$

Energy: The cork will start floating once the potential energy increase from adding water with the cork touching the bottom of the container is larger than the potential energy increase from the cork floating. Suppose we add a volume of water ΔV to the container, when the water level is already H. The former costs energy

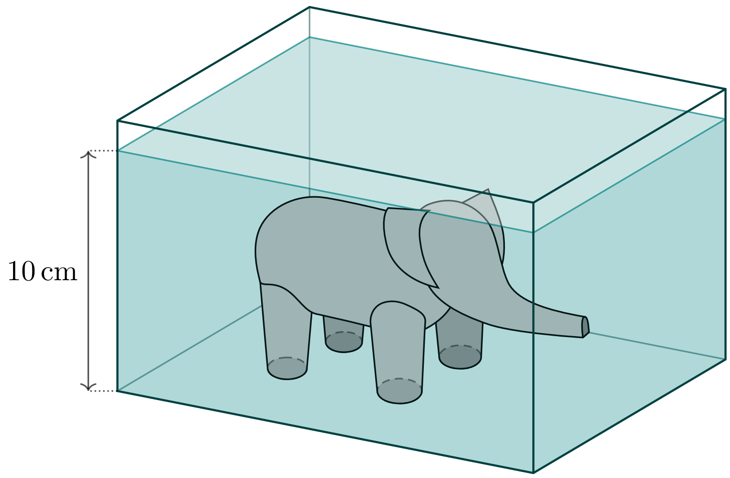
$$E_{\text{no float}} = \rho_w \Delta V g H.$$

The latter costs energy

$$E_{\text{float}} = \rho_c A_c h g \Delta s = \rho_c \Delta V g h,$$

where $\Delta s = \Delta V / A_c$ is the rise in the cork's height. Equating the two energies gives us the same result: $H = \rho_c h / \rho_w$.

19. A toy elephant is standing on the bottom of a fish tank. The fish tank is filled with water to a depth of 10 cm, completely covering the toy. The elephant's legs are perfectly polished, so that there is no water between the bottom of the legs and the tank's floor, and the total area of contact is $0.16\,\text{cm}^2$. The water has density $\rho = 10^3\,\text{kg/m}^3$, the toy has uniform density 2ρ, the atmospheric pressure is $P_{\text{atm}} = 10^5\,\text{Pa}$, and the toy has total mass 120 g. What is the total hydrostatic force that the water exerts on the toy?

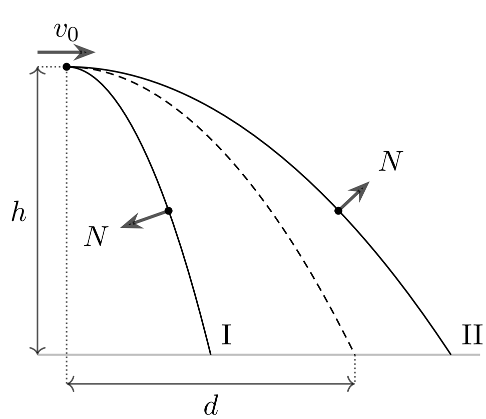


$\boxed{\text{A}}$ 1 N, down (B) 0.6 N, down (C) 0 N (D) 0.6 N, up (E) 1 N, up

If the elephant's legs weren't perfectly polished, then the buoyant force would be equal to the weight of the water displaced. Since the elephant's density is twice that as water, the force would be $mg/2 = 0.6\,\text{N}$ upward.

However, since the elephant's legs perfectly contact the floor, they experience no upward hydrostatic pressure. Therefore, we should subtract an upward force $\rho gh + P_{\text{atm}}A \approx P_{\text{atm}}A = 1.6\,\text{N}$, which means the net hydrostatic force is 1 N downward.

20. A bead is threaded on a frictionless wire and launched horizontally from height h with speed v_0, as shown. If the shape of the wire is steep, as in curve I, then the normal force from the wire on the bead will point inward. If it is shallow, as in curve II, then the normal force will point outward.

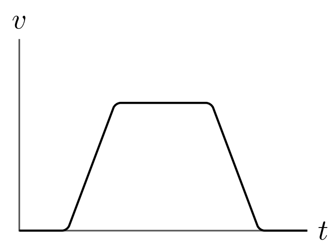


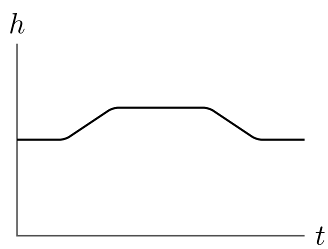
There is exactly one possible shape of wire, shown as a dotted line, for which the normal force of the wire on the bead is *always* equal to zero. What is the horizontal displacement d of the bead when it travels along this wire?

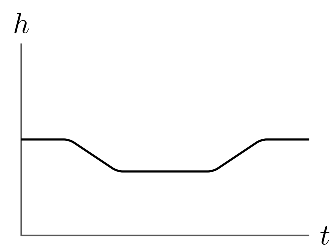
(A) $v_0\sqrt{\frac{4g}{h}}$ $\boxed{\text{B}}$ $v_0\sqrt{\frac{2h}{g}}$ (C) $v_0\sqrt{\frac{h}{g}}$ (D) $v_0\sqrt{\frac{h}{2g}}$ (E) $v_0\sqrt{\frac{h}{4g}}$

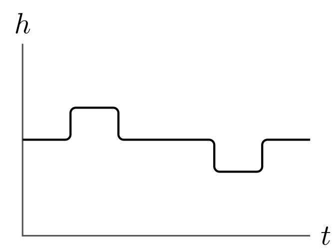
If the normal force always vanishes, then the bead is only experiencing gravity. In other words, it behaves just like a projectile, and the corresponding shape of the wire is a parabola. The bead travels for a time $t = \sqrt{2h/g}$, so the horizontal displacement is $v_0 t = v_0\sqrt{2h/g}$.

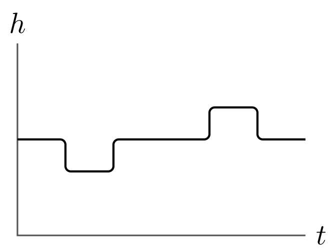
21. A cork floating in a cup filled with a viscous fluid is placed in an elevator. Below is a plot of the velocity v of the elevator as a function of time t. Which of the following plots best describes the height h of the cork in the cup as a function of time? Assume that the fluid is viscous enough to dampen all oscillations, that the fluid does not slosh as the elevator accelerates, and that both the cork and fluid are incompressible.

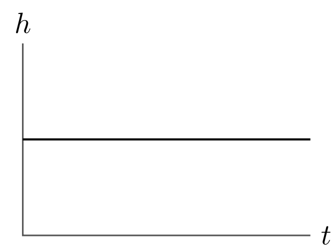


(A)


(B)


(C)


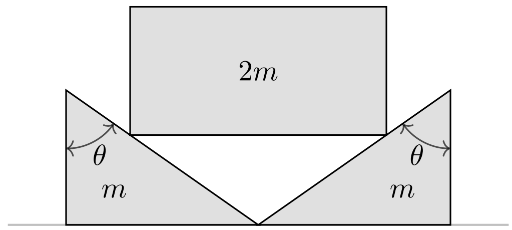
(D)


$\boxed{\text{E}}$


By Archimedes' principle, the fraction submerged only depends on the ratio of densities between the cork and the fluid, so the answer is (E).

The statement of Archimedes' principle is that the buoyant force is equal to the weight of the water displaced, and the cork floats when its weight equals the buoyant force. The elevator's acceleration affects the buoyant force and cork weight equally in the noninertial frame of the elevator (both forces are weights), so the fraction submerged does not change.

22. A block of mass $2m$ is placed symmetrically on two identical wedges of mass m, as shown.



All surfaces are frictionless, and the wedges have angle θ to the vertical. If the system is released from rest, what is the downward acceleration of the block?

(A) $g \sin\theta$ (B) $g \sin(2\theta)$ (C) $g \cos\theta$ (D) $g \cos(2\theta)$ $\boxed{\text{E}}$ $g \cos^2\theta$

If the block is moving down with speed v, then the wedges need to be moving with speed $v \tan\theta$. This means the total kinetic energy is

$$K = \frac{1}{2}(2m)v^2 + 2\,\frac{1}{2}m(v\tan\theta)^2 = \frac{1}{2}\frac{2m}{\cos^2\theta}v^2.$$

This is the same energy as an object of mass $m_{\text{eff}} = 2m/\cos^2\theta$ would have alone. But when the block moves down by an amount Δh, a gravitational potential energy $2mg\Delta h = m_{\text{eff}}(g\cos^2\theta)\Delta h$ is released. Thus, the acceleration is $g\cos^2\theta$.

23. For objects moving through air, the force of air resistance can be modeled as proportional to the speed ("linear drag") or proportional to the square of the speed ("quadratic drag"), depending on the circumstances. Two identical objects, A and B, are dropped from the same height h simultaneously, but object A is given an initial horizontal velocity v. The objects hit the ground at times t_A and t_B. Accounting for air resistance, which of the following is true?

(A) For both linear drag and quadratic drag, $t_A = t_B$.

(B) For linear drag, $t_A > t_B$, while for quadratic drag, $t_A = t_B$.

$\boxed{\text{C}}$ For linear drag, $t_A = t_B$, while for quadratic drag, $t_A > t_B$.

(D) For both linear drag and quadratic drag, $t_A > t_B$.

(E) For both linear drag and quadratic drag, the answer depends on v and h.

For linear drag, the horizontal and vertical components of the motion are completely independent,

$$a_x = -bv_x, \quad a_y = -g - bv_y$$

for some drag coefficient b. That means the time to hit the ground, which depends on the vertical motion, is independent of the initial horizontal velocity, so $t_A = t_B$. On the other hand, for quadratic drag,

$$a_y = -g - bv_y|v|$$

which means the upward drag force is larger when the horizontal velocity is larger, so $t_A > t_B$.

24. A satellite is in orbit around a planet of mass M. Its maximum distance from the center of the planet is d, and at this point, it is traveling at a speed of $\frac{1}{2}\sqrt{\frac{GM}{d}}$. What is the area of the satellite's orbit?

(A) $\frac{8}{15}\sqrt{\frac{2}{15}}\pi d^2$ $\boxed{\text{B}}$ $\frac{4}{7}\sqrt{\frac{1}{7}}\pi d^2$ (C) $\frac{1}{3}\sqrt{\frac{2}{3}}\pi d^2$ (D) $\frac{8}{7}\sqrt{\frac{1}{7}}\pi d^2$ (E) $\frac{2}{3}\sqrt{\frac{2}{3}}\pi d^2$

This problem can be solved in many ways; we will show two alternative solutions.

Geometry: By conserving angular momentum and energy, we find that the distance of closest approach to the planet is $d/7$. By the basic geometrical properties of ellipses, the semimajor axis is $a = 4d/7$ and the semiminor axis is $b = d/\sqrt{7}$. The area of an ellipse is $\pi ab = 4\pi d^2/7\sqrt{7}$.

Kepler's laws: By the vis-viva equation (or by the same steps as in the previous solution), the semimajor axis is $a = 4d/7$. By Kepler's third law, the period T obeys $T^2 = 4\pi^2 \frac{64d^3}{343GM}$. Finally, by Kepler's second law, the satellite sweeps out area at a constant rate, which is initially $dA/dt = vd/2 = \frac{1}{4}\sqrt{GMd}$. Therefore,

$$A = T\,\frac{dA}{dt} = \frac{4\pi d^2}{7\sqrt{7}}.$$

25. A cylinder is placed with its axis vertical, and a rubber band of mass m and tension T is wrapped horizontally around it. What is the minimum coefficient of static friction μ between the rubber band and the cylinder such that the band will not slide down the cylinder?

$\boxed{\text{A}}$ $\frac{mg}{2\pi T}$ (B) $\frac{mg}{T}$ (C) $\frac{4mg}{T}$ (D) $\frac{2\pi mg}{T}$ (E) $\frac{2m^2g^2}{T^2}$

Suppose the band has area A and normal force exerts a pressure P outward on the band. Imagine expanding the band slightly, so its radius increases by a small amount Δr. The volume enclosed by the band increases by $A\Delta r$, whereas its length increases by $2\pi\Delta r$. But because the band is in mechanical equilibrium, the total work done

$$W = PA\,\Delta r - T\cdot 2\pi\Delta r$$

must be zero. Thus the outward pressure is

$$P = \frac{2\pi T}{A}$$

Now, focus on any small patch of the band of area A_p. Because the band is uniform, the weight of this patch is proportional to its area:

$$F_g = mg\frac{A_p}{A}$$

The normal force on the patch is PA_p, so the force of static friction satisfies

$$F_f \le \mu\cdot 2\pi T\frac{A_p}{A}$$

Thus static friction can balance gravity ($F_f = F_g$) if

$$\mu \ge \frac{mg}{2\pi T}.$$

AAPT UNITED STATES PHYSICS TEAM
AIP 2022

2022 $F = ma$ Exam B

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 25, 2022.**

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

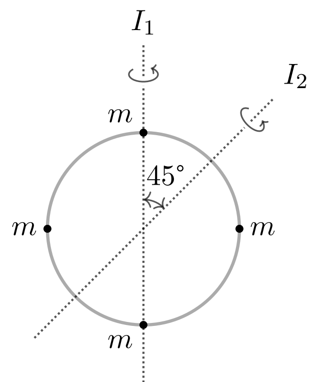
We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Abi Krishnan, Andrew Lin, Kris Lui, Kye Shi, Brian Skinner, Mike Winer and Kevin Zhou

1. A ball is held a height h above a slope, which is at an angle 45° from the horizontal. The ball is dropped from rest. Assume the ball bounces off the slope perfectly elastically. What is the distance between its first and second impact points?

(A) $2h$ (B) $2\sqrt{2}\,h$ (C) $4h$ (D) $4\sqrt{2}\,h$ (E) $8h$

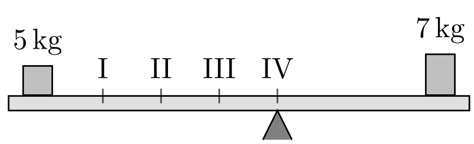
2. A massless wheel of radius R has four small masses M placed evenly along its rim. Let the moment of inertia for rotations about the center of wheel, in the plane of the wheel, be I_0.



Now consider rotations about the two axes shown, with corresponding moments of inertia I_1 and I_2. Which of the following is true?

(A) $I_1 = I_2 = I_0/2$ (B) $I_1 = I_2 = I_0/\sqrt{2}$ (C) $I_1 = I_2 = I_0$
(D) $I_1 = I_0/2, I_2 = I_0/\sqrt{2}$ (E) $I_1 = I_0, I_2 = I_0/\sqrt{2}$

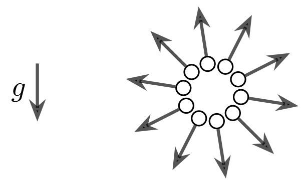
3. Four evenly spaced points are marked on a massless seesaw, as shown.



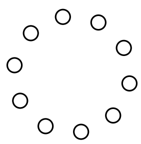
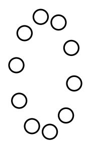
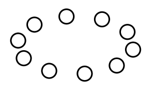
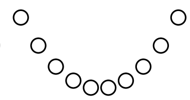
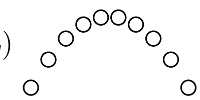
Two blocks, of masses 5 kg and 7 kg, are placed on the seesaw, so that it balances when the fulcrum is at point IV. (The diagram is not drawn to scale.) Now suppose the fulcrum is moved to point II. How much mass should be placed at point I so that the seesaw again balances?

(A) 12 kg
(B) 18 kg
(C) 24 kg
(D) 36 kg
(E) There is not enough information to decide.

4. A firework explodes, sending shells in all directions in a vertical plane, as shown.



Suppose the shells are all launched with the same speed, and ignore air resistance, but not gravity. A long time later, but before any of the shells hit the ground, what shape do the shells make?

(A)  (B)  (C)  (D)  (E) 

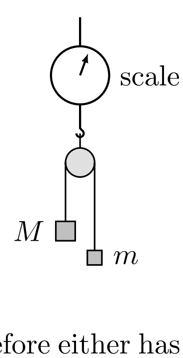
5. A swimmer swims at speed v relative to still water. A river flows from a pier to a lake, with speed u. If the swimmer swims downstream from the pier to the lake, then back upstream, what was their average speed during the trip?

(A) v (B) $\sqrt{v^2 - u^2}$ (C) $\frac{(v-u)^2}{v}$ (D) $\frac{(v+u)^2}{v}$ (E) $\frac{v^2 - u^2}{v}$

6. A block of mass m is at rest on a frictionless table. It is pushed horizontally by a constant force F, and has total momentum p when it reaches the end of the table. If a block of mass $2m$ is pushed across the table in the same way, also starting from rest, what is its momentum when it reaches the end of the table?

(A) $\frac{p}{2}$ (B) $\frac{p}{\sqrt{2}}$ (C) p (D) $\sqrt{2}\,p$ (E) $2p$

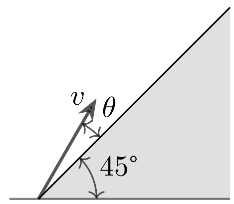
7. Two blocks of unequal mass M and m are hung from the ends of a massless, frictionless pulley, as shown. The blocks are held in place, and the entire pulley is mounted on a sensitive scale.



After the blocks are released from rest, but before either has fallen off the pulley, what is the scale reading?

(A) It is less than $(M+m)g$.
(B) It is equal to $(M+m)g$.
(C) It is more than $(M+m)g$.
(D) It is initially less than $(M+m)g$, then approaches $(M+m)g$.
(E) It is initially more than $(M+m)g$, then approaches $(M+m)g$.

8. A ball is launched with speed v at an angle θ to a fixed ramp, which itself makes a 45° angle with the horizontal.



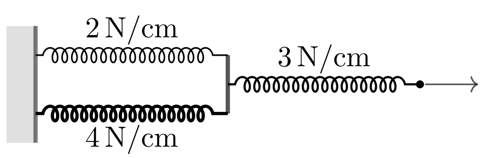
Assume that when the ball hits the ramp, it bounces elastically and frictionlessly. For what value of θ will the ball bounce off the ramp two times, and then return to its original launch point?

(A) 9.5° (B) 11.3° (C) 14.0° (D) 14.5° (E) 18.4°

9. Billy is leaning on a box of mass 30.0 kg, exerting a force 35.0° below the horizontal. If the coefficient of static friction of the box on the ground is $\mu_s = 0.400$, what is the minimum force needed for the box to slide?

(A) 120 N (B) 147 N (C) 203 N (D) 224 N (E) 342 N

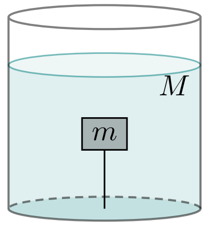
10. Two springs with spring constants 2 N/cm and 4 N/cm are connected in parallel. They are both connected in series to a spring of constant 3 N/cm, as shown.



What force must be exerted at the end to extend the system by 1 cm from its relaxed length?

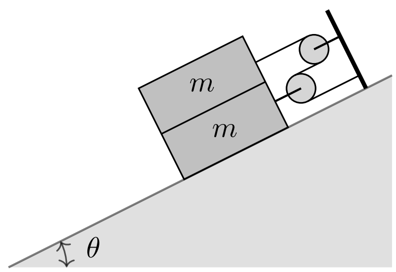
(A) 0.5 N (B) 2 N (C) 3 N (D) 6 N (E) 9 N

11. Water with total mass M is poured into a cup of cross-sectional area A. A block of mass m, whose density is half that of water, is tied to a thin string. The string is attached to the bottom of the cup, and the block floats in the water as shown. The atmospheric pressure is P_{atm}. What is the total pressure force that the water exerts on the bottom of the cup?



(A) Mg
(B) $(M + m)g$
(C) $P_{\text{atm}}A + Mg$
(D) $P_{\text{atm}}A + (M + m)g$
(E) $P_{\text{atm}}A + (M + 2m)g$

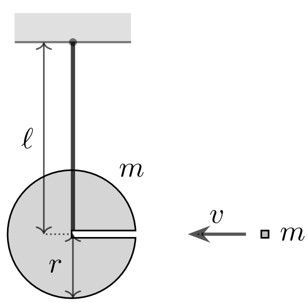
12. Two identical bricks of mass m are attached to two frictionless pulleys by a massless string, on a fixed slope of angle θ to the horizontal, as shown.



Suppose that the coefficient of static friction between the lower block and the slope is μ, and all other surfaces are frictionless. What is the minimum value of μ so that the blocks can stay static?

(A) $\frac{1}{2}\tan\theta$
(B) $\frac{2}{3}\tan\theta$
(C) $\tan\theta$
(D) $\frac{3}{2}\tan\theta$
(E) $2\tan\theta$

13. A ballistic pendulum is designed as shown below.



A uniform ball of mass m and radius r is attached to a massless rigid rod, so that its center is a distance ℓ from the ceiling. The ball has a small tunnel hollowed out. A small block of mass m and speed v goes into the tunnel and collides at the center of the ball, perfectly inelastically. Subsequently, to what maximum height does the block rise?

(A) $\dfrac{v^2}{8g}$
(B) $\dfrac{v^2}{8g(1 + r^2/5\ell^2)^2}$
(C) $\dfrac{v^2}{8g(1 + 2r^2/5\ell^2)^2}$
(D) $\dfrac{v^2}{8g(1 + r^2/5\ell^2)}$
(E) $\dfrac{v^2}{8g(1 + 2r^2/5\ell^2)}$

14. A cylinder of water is suspended in a space station. Under the influence of surface tension, the cylinder splits into droplets. After a short time, viscosity causes the droplets to settle into static shapes. Neglect the effect of evaporation. Compared to the original cylinder, the final set of droplets will have

(A) Almost exactly the same volume and surface area

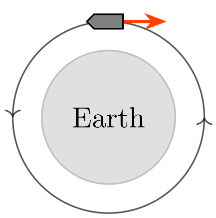
(B) More volume, but almost exactly the same surface area

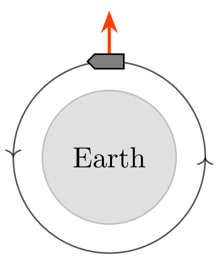
(C) Less volume, but almost exactly the same surface area

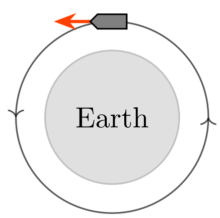
(D) More surface area, but almost exactly the same volume

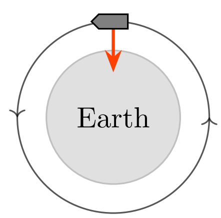
(E) Less surface area, but almost exactly the same volume

15. Five astronauts are orbiting Earth in a low circular orbit. They decided to go around the planet following the same orbit, but much faster, to set a new record for orbiting Earth. Four of them thought that they could quickly increase the velocity, then use a jet engine to maintain an orbit of the same shape; in the first four choices below, the red arrow denotes the direction that fuel will flow out of the jet nozzle. The fifth astronaut was skeptical about such a possibility. Who is right?

(A) 

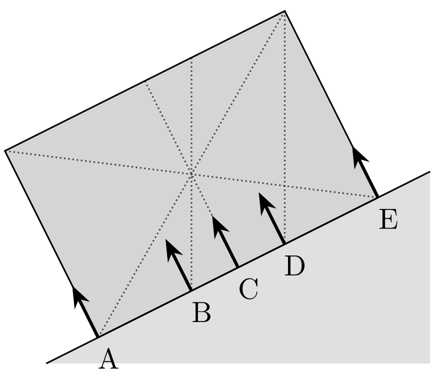
(B) 

(C) 

(D) 

(E) It is impossible to follow the same orbit faster.

16. A uniform rectangular block is in static equilibrium on an inclined plane, and experiences gravity, static friction, and the normal force from the plane. Though gravity acts on the entire volume of the block, for the purposes of torque balance, it is equivalent to a single resultant force acting at the block's center of mass. Similarly, while the normal force acts on the entire bottom surface of the block, its torque is equivalent to a resultant force acting at a single point. Which of the following best shows this point?



(A) Point A, at the lowest point of the bottom side

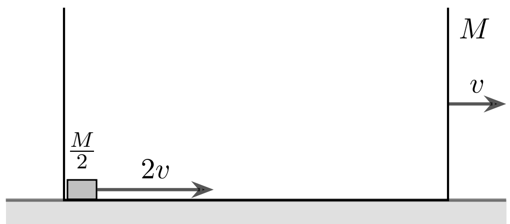
(B) Point B, directly below the center of mass

(C) Point C, at the midpoint of the bottom side

(D) Point D, directly below the top corner

(E) Point E, at the highest point of the bottom side

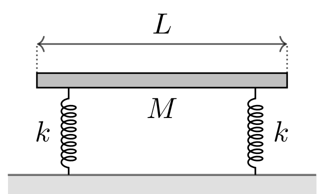
17. A box of mass M is sliding to the right with velocity v on a frictionless table. A small puck of mass $M/2$ slides frictionlessly inside the box. Initially, the puck is at the left wall of the box, with a rightward velocity of $2v$ with respect to the table. After a time T, the puck collides elastically with the right wall of the box. How much longer will it take until the puck hits the left wall of the box again?



(A) $T/2$ (B) T (C) $2T$ (D) $3T$ (E) $4T$

18. **The following information applies to problems 18 and 19.**

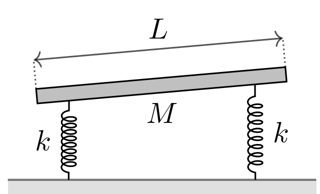
A uniform rod of length L and mass M is placed with its ends resting on two identical springs of spring constant k, as shown.



The rod is initially in equilibrium. If the rod is uniformly displaced downward and released from rest, what is the frequency f of its oscillations?

(A) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (E) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

19. Next, the rod is brought back to equilibrium. It is slightly rotated about its center of mass, then released from rest.



What is the frequency f of its oscillations?

(A) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{3k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$ (E) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

20. A uniform, hollow sphere is initially at rest on a horizontal plane. The plane begins to accelerate horizontally to the right, with acceleration a. If friction is high enough to prevent the sphere from slipping, what is its translational acceleration?

(A) $2a/3$ to the left (B) $a/3$ to the left (C) zero
(D) $2a/5$ to the right (E) $2a/3$ to the right

21. Amora and Bronko are given a long, thin rectangle of sheet metal. (It has been machined very precisely, so they can assume it is perfectly rectangular.) Using calipers, Amora measures the width of the rectangle as 1 cm with 1% uncertainty. Using a tape measure, Bronko independently measures its length as 100 cm with 0.1% uncertainty. Which of the following are closest to the relative uncertainties they should report for the area and the perimeter of the rectangle, respectively?

(A) 0.1%; 0.2% (B) 1%; 0.1% (C) 1%; 0.2% (D) 1%; 1% (E) 1%; 2%

22. A cannon just above the surface of a spherical planet with mass M and radius R launches a particle with speed v, where $\sqrt{GM/R} < v < \sqrt{2GM/R}$. The initial velocity of the particle makes an angle θ with the vertical direction. Neglect drag. For what θ will the particle never collide with the planet?

(A) All possible launch angles, $0^\circ \leq \theta \leq 90^\circ$

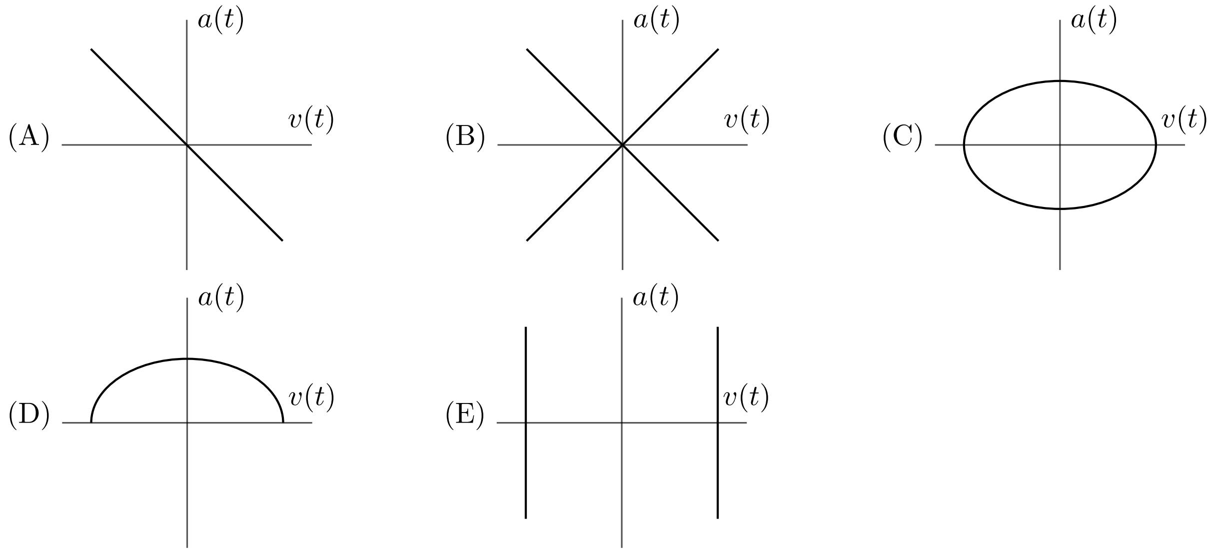
(B) $\cos^{-1}\left(\sqrt{\frac{v^2R}{GM}-1}\right) \leq \theta \leq 90^\circ$

(C) $\sin^{-1}\left(\sqrt{\frac{v^2R}{GM}-1}\right) \leq \theta \leq 90^\circ$

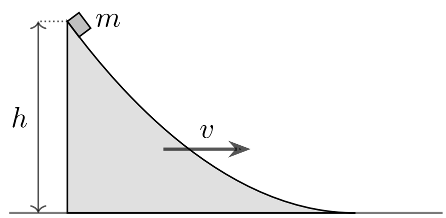
(D) Only $\theta = 90^\circ$

(E) A collision will occur for any value of θ

23. A mass attached to a spring is performing simple harmonic motion, with velocity $v(t)$ and acceleration $a(t)$. Which of the following could be a graph of the curve $(v(t), a(t))$ over a complete oscillation?



24. A ramp with height h is moving with fixed, uniform speed v to the right. A small block of mass m is placed at the top of the ramp, and is released at rest with respect to the ramp.



The block slides smoothly to the bottom of the ramp and onto the floor. How much kinetic energy does it gain in this process? Neglect friction.

(A) mgh
(B) $mgh + mv^2/2$
(C) $mgh + mv\sqrt{2gh}$
(D) $mgh + mv\sqrt{gh} + mv^2/2$
(E) $mgh + mv\sqrt{2gh} + mv^2$

25. Two masses are initially at rest, separated by a distance r, and attract each other gravitationally. If their masses are m and $2m$, then they will collide after a time T. How long would they take to collide if they both had mass $2m$?

(A) $\left(\frac{2}{3}\right)^{3/2} T$
(B) $\frac{3}{4} T$
(C) $\sqrt{\frac{2}{3}}\, T$
(D) $\sqrt{\frac{3}{4}}\, T$
(E) $\sqrt{\frac{8}{9}}\, T$

AAPT UNITED STATES PHYSICS TEAM
AIP 2022

2022 $F = ma$ Exam B

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 25, 2022.**

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Abi Krishnan, Andrew Lin, Kris Lui, Kye Shi, Brian Skinner, Mike Winer and Kevin Zhou

1. A ball is held a height h above a slope, which is at an angle 45° from the horizontal. The ball is dropped from rest. Assume the ball bounces off the slope perfectly elastically. What is the distance between its first and second impact points?

(A) $2h$ (B) $2\sqrt{2}\,h$ (C) $4h$ $\boxed{\text{D}}$ $4\sqrt{2}\,h$ (E) $8h$

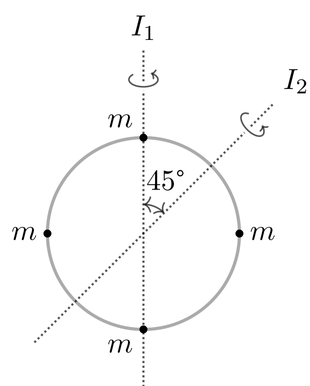
By conservation of energy, the ball has speed $v = \sqrt{2gh}$ when it hits the slope the first time. After the first impact, this velocity is redirected horizontally. Let Δx and Δy be the displacement from the first bounce to the second. Then we have

$$\Delta x = vt, \quad \Delta y = -\frac{1}{2}gt^2$$

and the second bounce obeys $\Delta x = -\Delta y$. Solving gives $t = 2v/g$ and distance

$$\sqrt{2}\,\Delta x = 2\sqrt{2}\,\frac{v^2}{g} = 4\sqrt{2}\,h.$$

2. A massless wheel of radius R has four small masses M placed evenly along its rim. Let the moment of inertia for rotations about the center of wheel, in the plane of the wheel, be I_0.

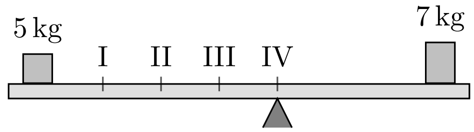


Now consider rotations about the two axes shown, with corresponding moments of inertia I_1 and I_2. Which of the following is true?

$\boxed{\text{A}}$ $I_1 = I_2 = I_0/2$ (B) $I_1 = I_2 = I_0/\sqrt{2}$ (C) $I_1 = I_2 = I_0$
(D) $I_1 = I_0/2, I_2 = I_0/\sqrt{2}$ (E) $I_1 = I_0, I_2 = I_0/\sqrt{2}$

By the definition of the moment of inertia, $I_0 = 4MR^2$, $I_1 = 2MR^2$, and $I_2 = 4M(R/\sqrt{2})^2 = 2MR^2$. Therefore, $I_1 = I_2 = I_0/2$.

3. Four evenly spaced points are marked on a massless seesaw, as shown.



Two blocks, of masses 5 kg and 7 kg, are placed on the seesaw, so that it balances when the fulcrum is at point IV. (The diagram is not drawn to scale.) Now suppose the fulcrum is moved to point II. How much mass should be placed at point I so that the seesaw again balances?

(A) 12 kg

(B) 18 kg

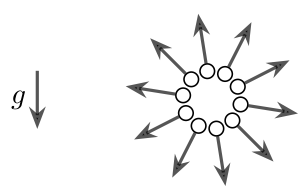
$\boxed{\text{C}}$ 24 kg

(D) 36 kg

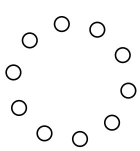
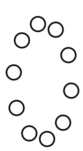
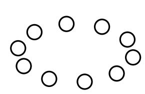
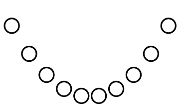
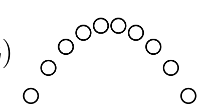
(E) There is not enough information to decide.

At first, it might seem like there isn't enough information to decide, because the locations of the masses aren't given. However, this turns out to not affect the answer. If the seesaw was initially balanced, its center of mass must have been directly over the fulcrum. Therefore, the total weight force of 120 N can be regarded as acting at point IV. After the fulcrum is moved, balancing torques about point II implies that the new mass should have a weight of 240 N, and therefore a mass 24 kg.

4. A firework explodes, sending shells in all directions in a vertical plane, as shown.



Suppose the shells are all launched with the same speed, and ignore air resistance, but not gravity. A long time later, but before any of the shells hit the ground, what shape do the shells make?

$\boxed{\text{A}}$  (B)  (C)  (D)  (E) 

In the absence of gravity, the shells would always form a circle. Adding gravity simply shifts all of their locations downward by $gt^2/2$, so the shape is still always a circle.

5. A swimmer swims at speed v relative to still water. A river flows from a pier to a lake, with speed u. If the swimmer swims downstream from the pier to the lake, then back upstream, what was their average speed during the trip?

(A) v (B) $\sqrt{v^2 - u^2}$ (C) $\frac{(v-u)^2}{v}$ (D) $\frac{(v+u)^2}{v}$ $\boxed{\text{E}}$ $\frac{v^2 - u^2}{v}$

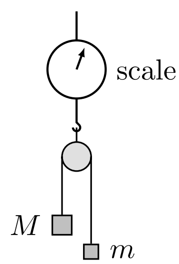
Assuming the distance from pier to lake is d, it takes time $d/(v+u)$ to get to the lake and $d/(v-u)$ to get back. This works out to distance $2d$ traveled in time $2dv/(v^2 - u^2)$, or an average speed of $(v^2 - u^2)/v$.

6. A block of mass m is at rest on a frictionless table. It is pushed horizontally by a constant force F, and has total momentum p when it reaches the end of the table. If a block of mass $2m$ is pushed across the table in the same way, also starting from rest, what is its momentum when it reaches the end of the table?

(A) $\frac{p}{2}$ (B) $\frac{p}{\sqrt{2}}$ (C) p $\boxed{\text{D}}$ $\sqrt{2}\,p$ (E) $2p$

Since the forces and distances are the same, the total work done is the same. The kinetic energy is related to momentum by $K = p^2/2m$. Thus, $p \propto \sqrt{m}$, so the momentum of the second block is $\sqrt{2}p$.

7. Two blocks of unequal mass M and m are hung from the ends of a massless, frictionless pulley, as shown. The blocks are held in place, and the entire pulley is mounted on a sensitive scale.

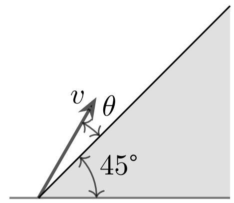


After the blocks are released from rest, but before either has fallen off the pulley, what is the scale reading?

$\boxed{\text{A}}$ It is less than $(M + m)g$.
(B) It is equal to $(M + m)g$.
(C) It is more than $(M + m)g$.
(D) It is initially less than $(M + m)g$, then approaches $(M + m)g$.
(E) It is initially more than $(M + m)g$, then approaches $(M + m)g$.

When the blocks on a pulley are released, both blocks accelerate, with the heavier block accelerating downward. This means the center of mass is uniformly accelerating downward, which means the force acting on the pulley is less than the weight $(M + m)g$.

8. A ball is launched with speed v at an angle θ to a fixed ramp, which itself makes a 45° angle with the horizontal.



Assume that when the ball hits the ramp, it bounces elastically and frictionlessly. For what value of θ will the ball bounce off the ramp two times, and then return to its original launch point?

(A) 9.5° (B) 11.3° (C) 14.0° (D) 14.5° $\boxed{\text{E}}$ 18.4°

Work with axes aligned with the ramp. In these coordinates, gravity has a component $g/\sqrt{2}$ towards the ramp, and $g/\sqrt{2}$ along the ramp. Meanwhile, the initial velocity has a component $v \sin\theta$ away from the ramp, and $v \cos\theta$ along the ramp.

A collision happens once the component of velocity away from the ramp goes from $v \sin\theta$ to $-v \sin\theta$, at which point the collision resets the velocity to $v \sin\theta$. Since three collisions happen (with the final one being at the original launch point itself), the total time must be

$$t = \frac{6v \sin\theta}{g/\sqrt{2}}.$$

In order to have arrived back the launch point at this point, the velocity along the ramp must have gone from $v \cos\theta$ to $-v \cos\theta$, which means

$$gt/\sqrt{2} = 2v \cos\theta.$$

Combining these results gives $\tan\theta = 1/3$.

9. Billy is leaning on a box of mass 30.0 kg, exerting a force 35.0° below the horizontal. If the coefficient of static friction of the box on the ground is $\mu_s = 0.400$, what is the minimum force needed for the box to slide?

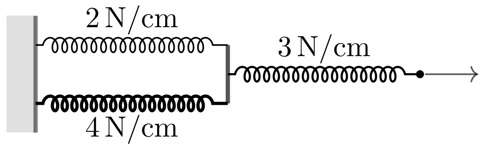
(A) 120 N (B) 147 N $\boxed{\text{C}}$ 203 N (D) 224 N (E) 342 N

Suppose Billy exerts force F. Then the maximum friction force is

$$f = \mu_s N = (0.400)(300\,\text{N} + F\sin(35^\circ)).$$

The horizontal applied force is $F_x = F\cos(35^\circ)$. For this to move the box, we need $F_x > f$, and solving yields $F \geq 203\,\text{N}$.

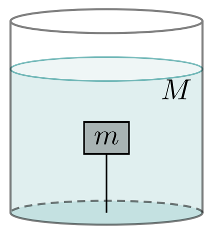
10. Two springs with spring constants $2\,\text{N/cm}$ and $4\,\text{N/cm}$ are connected in parallel. They are both connected in series to a spring of constant $3\,\text{N/cm}$, as shown.



What force must be exerted at the end to extend the system by 1 cm from its relaxed length?

(A) 0.5 N $\boxed{\text{B}}$ 2 N (C) 3 N (D) 6 N (E) 9 N

11. Water with total mass M is poured into a cup of cross-sectional area A. A block of mass m, whose density is half that of water, is tied to a thin string. The string is attached to the bottom of the cup, and the block floats in the water as shown. The atmospheric pressure is P_{atm}. What is the total pressure force that the water exerts on the bottom of the cup?

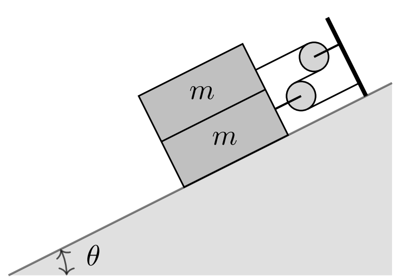


(A) Mg (B) $(M + m)g$ (C) $P_{\text{atm}}A + Mg$
(D) $P_{\text{atm}}A + (M + m)g$ $\boxed{\text{E}}$ $P_{\text{atm}}A + (M + 2m)g$

Consider force balance on the system of the block and water. The atmospheric pressure applies a downward force $P_{\text{atm}}A$, and gravity applies a downward force $(M + m)g$. Now, the buoyant force on the block is $2mg$ because water is twice as dense, which means the string applies a downward tension force mg to keep the block in place. To balance all these forces, the base of the cup must exert an upward force $P_{\text{atm}}A + (M + 2m)g$.

There is also a shorter but trickier solution. The pressure force depends only on the height of the water, so it is unchanged if the block is replaced with water. Then the total mass of water is $M + 2m$, so the upward force must be $P_{\text{atm}}A + (M + 2m)g$.

12. Two identical bricks of mass m are attached to two frictionless pulleys by a massless string, on a fixed slope of angle θ to the horizontal, as shown.



Suppose that the coefficient of static friction between the lower block and the slope is μ, and all other surfaces are frictionless. What is the minimum value of μ so that the blocks can stay static?

$\boxed{\text{A}}$ $\frac{1}{2}\tan\theta$ (B) $\frac{2}{3}\tan\theta$ (C) $\tan\theta$ (D) $\frac{3}{2}\tan\theta$ (E) $2\tan\theta$

Let the tension in the string that wraps around both pulleys be T. Applying force balance to the upper brick, we must have $T = mg\sin\theta$. The pulley applies a force $2T$ to the lower brick, so applying force balance gives

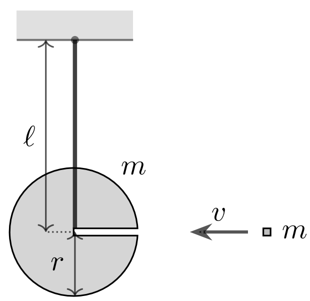
$$2T = mg\sin\theta + f_{\text{fric}}.$$

When the friction is just barely sufficient, we have

$$f_{\text{fric}} = \mu N = 2\mu mg\cos\theta$$

and combining these two equations gives $\mu = (\tan\theta)/2$.

13. A ballistic pendulum is designed as shown below.



A uniform ball of mass m and radius r is attached to a massless rigid rod, so that its center is a distance ℓ from the ceiling. The ball has a small tunnel hollowed out. A small block of mass m and speed v goes into the tunnel and collides at the center of the ball, perfectly inelastically. Subsequently, to what maximum height does the block rise?

(A) $\dfrac{v^2}{8g}$ (B) $\dfrac{v^2}{8g(1+r^2/5\ell^2)^2}$ (C) $\dfrac{v^2}{8g(1+2r^2/5\ell^2)^2}$

$\boxed{\text{D}}$ $\dfrac{v^2}{8g(1+r^2/5\ell^2)}$ (E) $\dfrac{v^2}{8g(1+2r^2/5\ell^2)}$

We treat the impact by conserving angular momentum about the pivot, which is equal to $mv\ell$. The final moment of inertia about the pivot is

$$I = \frac{2}{5}mr^2 + 2m\ell^2.$$

To find the height of the highest point, we conserve energy,

$$2mgh = \frac{L^2}{2I} = \frac{m^2v^2\ell^2}{4m\ell^2 + \frac{4}{5}mr^2}.$$

Solving this for h gives

$$h = \frac{v^2}{8g} \frac{1}{1 + r^2/5\ell^2}.$$

Note that conserving linear momentum about the pivot would not give the correct answer, because the rigid rod provides a horizontal impulse during the collision.

14. A cylinder of water is suspended in a space station. Under the influence of surface tension, the cylinder splits into droplets. After a short time, viscosity causes the droplets to settle into static shapes. Neglect the effect of evaporation. Compared to the original cylinder, the final set of droplets will have

(A) Almost exactly the same volume and surface area

(B) More volume, but almost exactly the same surface area

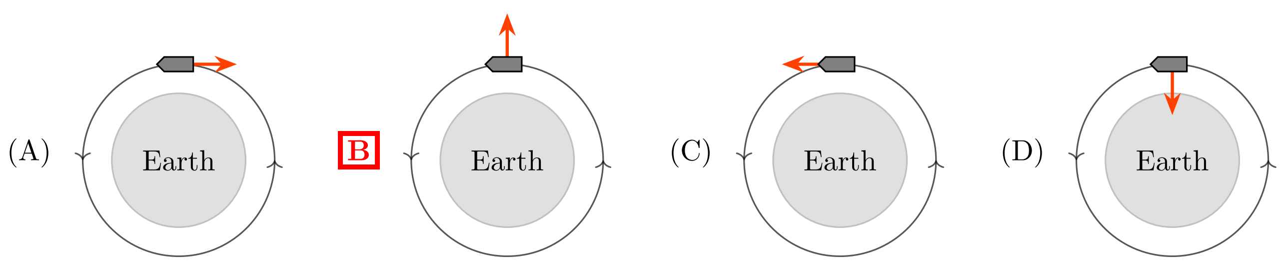
(C) Less volume, but almost exactly the same surface area

(D) More surface area, but almost exactly the same volume

$\boxed{\text{E}}$ Less surface area, but almost exactly the same volume

Because water is almost incompressible, the volume stays almost exactly the same. During the process, energy is dissipated due to viscosity, and the energy due to surface tension is proportional to the total surface area. Thus, the surface area must decrease.

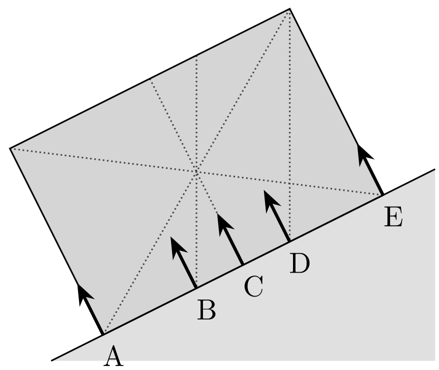
15. Five astronauts are orbiting Earth in a low circular orbit. They decided to go around the planet following the same orbit, but much faster, to set a new record for orbiting Earth. Four of them thought that they could quickly increase the velocity, then use a jet engine to maintain an orbit of the same shape; in the first four choices below, the red arrow denotes the direction that fuel will flow out of the jet nozzle. The fifth astronaut was skeptical about such a possibility. Who is right?



(E) It is impossible to follow the same orbit faster.

The centripetal acceleration required to follow a circular orbit is v^2/r. If the speed is increased, the inward acceleration must be increased, so the engine should be pointed outward to provide an inward force.

16. A uniform rectangular block is in static equilibrium on an inclined plane, and experiences gravity, static friction, and the normal force from the plane. Though gravity acts on the entire volume of the block, for the purposes of torque balance, it is equivalent to a single resultant force acting at the block's center of mass. Similarly, while the normal force acts on the entire bottom surface of the block, its torque is equivalent to a resultant force acting at a single point. Which of the following best shows this point?



(A) Point A, at the lowest point of the bottom side

$\boxed{B}$ Point B, directly below the center of mass

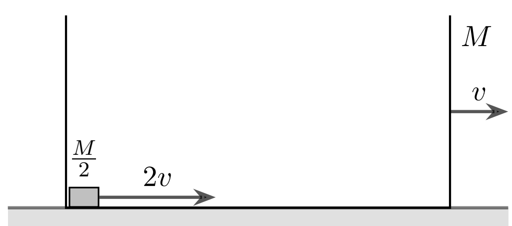
(C) Point C, at the midpoint of the bottom side

(D) Point D, directly below the top corner

(E) Point E, at the highest point of the bottom side

Consider the point of contact between the block and plane, directly below the center of mass. Gravity effectively acts at the center of mass, and thus exerts no torque about this point. The friction force points along the plane, and thus also exerts no torque about this point. Therefore, since the block is in static equilibrium, the net normal force must effectively be applied at this point, yielding choice (B).

17. A box of mass M is sliding to the right with velocity v on a frictionless table. A small puck of mass $M/2$ slides frictionlessly inside the box. Initially, the puck is at the left wall of the box, with a rightward velocity of $2v$ with respect to the table. After a time T, the puck collides elastically with the right wall of the box. How much longer will it take until the puck hits the left wall of the box again?

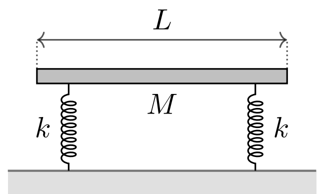


(A) $T/2$ $\boxed{B}$ T (C) $2T$ (D) $3T$ (E) $4T$

As can be shown by working in the center of mass frame, during an elastic collision, the relative velocity of the two objects only changes in sign. Therefore, the relative speed remains the same, so the time until the next collision is also T.

18. **The following information applies to problems 18 and 19.**

A uniform rod of length L and mass M is placed with its ends resting on two identical springs of spring constant k, as shown.

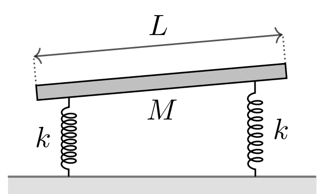


The rod is initially in equilibrium. If the rod is uniformly displaced downward and released from rest, what is the frequency f of its oscillations?

(A) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ $\boxed{\text{D}}$ $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (E) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

If the rod is displaced down by $\Delta\ell$, then the vertical net force it experiences is $2k\Delta\ell$. This is just like a spring-mass system where the spring constant is effectively $2k$, so the angular frequency is $\sqrt{2k/M}$.

19. Next, the rod is brought back to equilibrium. It is slightly rotated about its center of mass, then released from rest.



What is the frequency f of its oscillations?

(A) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{3k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$ $\boxed{\text{E}}$ $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

If the rod is rotated by a small angle θ, the net torque on the rod is $\tau = 2(L/2)(kL\theta/2) = kL^2\theta/2$. The rotational form of Newton's second law, $\tau = I\alpha$, gives

$$\frac{1}{12}ML^2\alpha = -\frac{kL^2}{2}\theta$$

or equivalently $\alpha = -(6k/M)\theta$, which has the form of a simple harmonic motion equation with $\omega = \sqrt{6k/M}$.

20. A uniform, hollow sphere is initially at rest on a horizontal plane. The plane begins to accelerate horizontally to the right, with acceleration a. If friction is high enough to prevent the sphere from slipping, what is its translational acceleration?

(A) $2a/3$ to the left (B) $a/3$ to the left (C) zero

$\boxed{\text{D}}$ $2a/5$ to the right (E) $2a/3$ to the right

It's easiest to begin by working in the noninertial frame of the plane. In this frame, there is a fictitious force Ma on the sphere, directed to the left, effectively acting at its center. Taking torques about the contact point with the ground, the angular acceleration of the sphere is $\alpha = 3a/5R$, so the acceleration of the center of mass is $\alpha R = 3a/5$ to the left. Transforming back to the original frame, the acceleration of the sphere is $2a/5$ to the right.

21. Amora and Bronko are given a long, thin rectangle of sheet metal. (It has been machined very precisely, so they can assume it is perfectly rectangular.) Using calipers, Amora measures the width of the rectangle as 1 cm with 1% uncertainty. Using a tape measure, Bronko independently measures its length as 100 cm with 0.1% uncertainty. Which of the following are closest to the relative uncertainties they should report for the area and the perimeter of the rectangle, respectively?

(A) 0.1%; 0.2% $\boxed{\text{B}}$ 1%; 0.1% (C) 1%; 0.2% (D) 1%; 1% (E) 1%; 2%

Calculating the area involves multiplying the two measurements, and so the uncertainty in the area depends on the relative uncertainties of the inputs. Amora's 1% relative uncertainty dominates Bronko's 0.1%, so the result carries her 1% uncertainty. (It remains 1% because the area depends on each input to the first power.)

Calculating the perimeter involves adding the two measurements, and so the uncertainty in the perimeter depends on the absolute uncertainties of the inputs. Bronko's 0.1 cm absolute uncertainty dominates Amora's 0.01 cm, so the result carries his 0.1% uncertainty. (Note that while the perimeter is approximately twice Bronko's measurement, the *relative* uncertainty is unaffected by this observation, so the answer is not 0.2%.)

22. A cannon just above the surface of a spherical planet with mass M and radius R launches a particle with speed v, where $\sqrt{GM/R} < v < \sqrt{2GM/R}$. The initial velocity of the particle makes an angle θ with the vertical direction. Neglect drag. For what θ will the particle never collide with the planet?

(A) All possible launch angles, $0^\circ \leq \theta \leq 90^\circ$

(B) $\cos^{-1}\left(\sqrt{\dfrac{v^2R}{GM}} - 1\right) \leq \theta \leq 90^\circ$

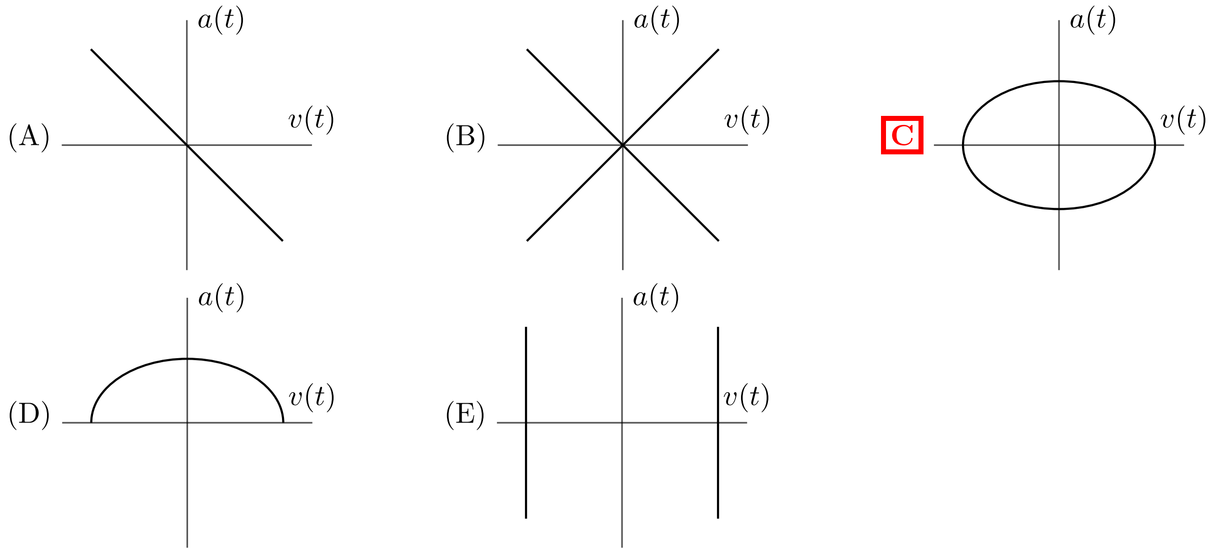
(C) $\sin^{-1}\left(\sqrt{\dfrac{v^2R}{GM}} - 1\right) \leq \theta \leq 90^\circ$

$\boxed{\text{D}}$ Only $\theta = 90^\circ$

(E) A collision will occur for any value of θ

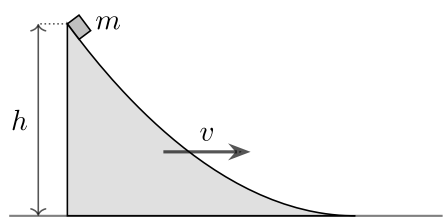
The launch speed is above the circular orbit speed $\sqrt{GM/R}$, so a launch angle of exactly 90° will result in an orbit that doesn't collide with the planet; the orbit is an ellipse tangent to the planet's surface at the launch point. On the other hand, the launch speed is below the escape speed $\sqrt{2GM/R}$, which implies, by Kepler's first law, that the orbit is a closed ellipse. Therefore, if the launch angle was anything besides 90°, the orbit would have to intersect the planet. Therefore, the answer is (D).

23. A mass attached to a spring is performing simple harmonic motion, with velocity $v(t)$ and acceleration $a(t)$. Which of the following could be a graph of the curve $(v(t), a(t))$ over a complete oscillation?



By conservation of energy, $E = kx^2/2 + mv^2/2$. In addition, the acceleration is $a = -kx/m$, which implies that $E = k(ma/k)^2/2 + mv^2/2$. Therefore, when a is plotted against v, the result is an ellipse. All combinations of signs of a and v are possible, so the ellipse occupies all four quadrants, giving choice (B).

24. A ramp with height h is moving with fixed, uniform speed v to the right. A small block of mass m is placed at the top of the ramp, and is released at rest with respect to the ramp.



The block slides smoothly to the bottom of the ramp and onto the floor. How much kinetic energy does it gain in this process? Neglect friction.

(A) mgh (B) $mgh + mv^2/2$ $\boxed{\text{C}}$ $mgh + mv\sqrt{2gh}$

(D) $mgh + mv\sqrt{gh} + mv^2/2$ (E) $mgh + mv\sqrt{2gh} + mv^2$

In the frame of reference of the ramp, the block starts at rest, and ends up with horizontal velocity $\sqrt{2gh}$ by energy conservation. Thus, in the original frame, it has velocity $v + \sqrt{2gh}$, which means

$$\Delta K = \frac{1}{2}m((v + \sqrt{2gh})^2 - v^2) = mgh + mv\sqrt{2gh}.$$

In particular, mgh is not the correct answer because the horizontal motion of the ramp also does work on the block as it slides.

25. Two masses are initially at rest, separated by a distance r, and attract each other gravitationally. If their masses are m and $2m$, then they will collide after a time T. How long would they take to collide if they both had mass $2m$?

(A) $\left(\frac{2}{3}\right)^{3/2} T$ (B) $\frac{3}{4}T$ (C) $\sqrt{\frac{2}{3}}T$ $\boxed{\text{D}}$ $\sqrt{\frac{3}{4}}T$ (E) $\sqrt{\frac{8}{9}}T$

Compare the relative speed of the masses when they are a distance $r' < r$ apart. In the first case, if the mass $2m$ has speed v_1, then the mass m has speed $2v_1$ by momentum conservation. By energy conservation,

$$\frac{1}{2}m(2v_1)^2 + \frac{1}{2}(2m)v_1^2 = 2Gm^2\left(\frac{1}{r} - \frac{1}{r'}\right).$$

Therefore, the relative speed is

$$3v_1 = \sqrt{6Gm(1/r - 1/r')}.$$

In the second case, both the masses have the same speed v_2, where

$$\frac{1}{2}(2m)v_2^2 + \frac{1}{2}(2m)v_2^2 = 4Gm^2\left(\frac{1}{r} - \frac{1}{r'}\right).$$

Therefore, the relative speed is

$$2v_2 = \sqrt{8Gm(1/r - 1/r')}.$$

In the second case, the relative speed is always $\sqrt{4/3}$ times higher than in the first case, when the masses have the same separation r'. Therefore, the time to collision is $T\sqrt{3/4}$.

AAPT UNITED STATES PHYSICS TEAM
AIP 2023

2023 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

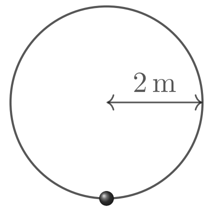
- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 25, 2023.**

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Kellan Colburn, Samuel Gebretsadkan, Abi Krishnan, Natalie LeBaron, Kye Shi, Mike Winer, and Kevin Zhou

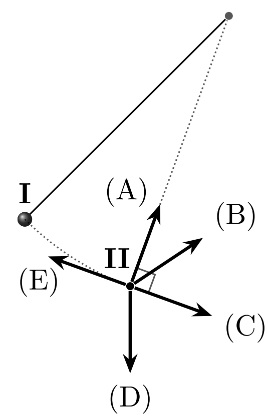
1. A bead on a circular hoop with radius 2 m travels counterclockwise for 10 s and completes 2.25 rotations, at which point it reaches the position shown.



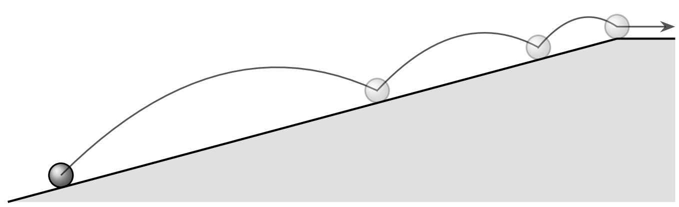
In the past 10 s, what were its average speed and the direction of its average velocity?

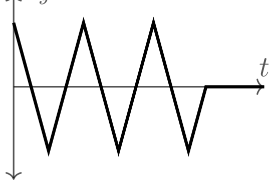
(A) $\frac{\sqrt{2}}{5}\,\frac{\text{m}}{\text{s}}, \swarrow$ (B) $\frac{2\pi}{5}\,\frac{\text{m}}{\text{s}}, \swarrow$ (C) $\frac{9\pi}{10}\,\frac{\text{m}}{\text{s}}, \swarrow$ (D) $\frac{2\pi}{5}\,\frac{\text{m}}{\text{s}}, \searrow$ (E) $\frac{9\pi}{10}\,\frac{\text{m}}{\text{s}}, \searrow$

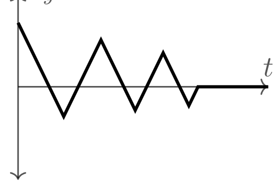
2. A mass on an ideal pendulum is released from rest at point I. When it reaches point II, which of the following shows the direction of its acceleration?

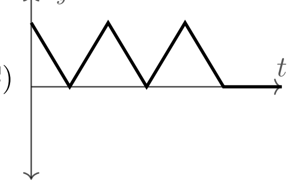


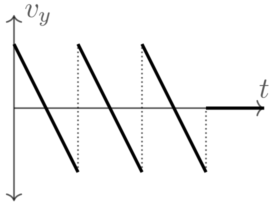
3. A soccer ball is kicked up a hill with a flat top, as shown. The ball bounces twice on the hill, at the points shown, then lands on the top and begins rolling horizontally. Which of the following shows the vertical component of its velocity as a function of time?

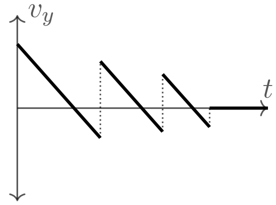


(A)


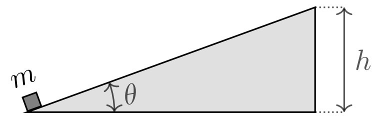
(B)


(C)


(D)


(E)


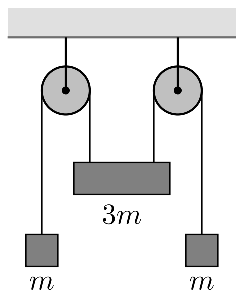
4. A box of mass m is at the bottom of an inclined plane with angle θ to the horizontal, and height h.



A person drags the box very slowly up the plane, by applying a force parallel to the plane. The coefficient of kinetic friction between the box and plane is μ_k. When the box reaches the top of the plane, how much work has the person done?

(A) $mgh\,(1 + \mu_k \sin\theta)$ (B) $mgh\,(1 + \mu_k \cos\theta)$ (C) $mgh\,(1 + \mu_k \tan\theta)$
(D) $mgh\,(1 + \mu_k \csc\theta)$ (E) $mgh\,(1 + \mu_k \cot\theta)$

5. Two blocks of mass m and a block of mass $3m$ are attached to a system of massless fixed pulleys and massless string, as shown.



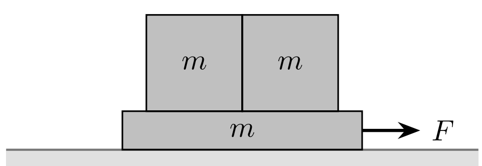
Assume all surfaces are frictionless. What is the acceleration of each mass m?

(A) $\dfrac{g}{8}$ (B) $\dfrac{g}{5}$ (C) $\dfrac{g}{4}$ (D) $\dfrac{g}{3}$ (E) $\dfrac{2g}{3}$

6. A ball at the end of a rope of length 0.5 m is swung in a horizontal circle, with a speed of 15 m/s. The other end of the rope is fixed in place. What is the height difference between the ends of the rope?

(A) 1.1 cm (B) 2.2 cm (C) 3.8 cm (D) 4.9 cm (E) 7.5 cm

7. Two boxes are stacked side-by-side on top of a larger box as shown.



All three boxes have mass m, the coefficient of static friction between the left box and the bottom box is μ_s, and all other surfaces are frictionless. A rightward force F is applied to the bottom box. What is the minimum value of μ_s so that the upper boxes don't slide?

(A) $\dfrac{2F}{mg}$ (B) $\dfrac{3F}{mg}$ (C) $\dfrac{F}{2mg}$ (D) $\dfrac{2F}{3mg}$ (E) $\dfrac{F}{3mg}$

8. Two stars α and β, with masses satisfying $m_\alpha/m_\beta = 10$, are in circular orbits around each other. In the rest frame of this system, find the ratio of the speeds v_α/v_β.

(A) $\frac{1}{11}$ (B) $\frac{1}{10}$ (C) $\frac{1}{9}$ (D) 9 (E) 10

9. A helium balloon is released from the floor in a room at rest, then slowly rises and comes to rest touching the ceiling. During this process, the gravitational potential energy of the balloon has increased. Since energy is conserved, the energy of something else must have decreased during this process. Which of the following is the main contribution to this decrease?

(A) The kinetic energy of the balloon decreased.
(B) The elastic potential energy of the balloon decreased.
(C) The thermal energy of the air in the balloon decreased.
(D) The thermal energy of the air in the room decreased.
(E) The gravitational potential energy of the air in the room decreased.

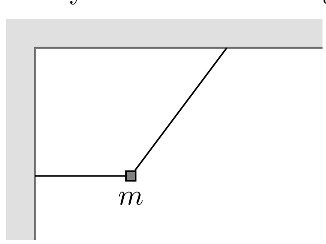
10. An archer takes aim at a target that is 100 m away. Assuming she holds the bow at the same height as the center of the target and shoots an arrow with velocity $v = 100\,\mathrm{m/s}$, at what angle above the horizontal should she aim the bow so that the arrow hits the center of the target?

(A) $\frac{\arccos(1/5)}{2}$ (B) $\frac{\arcsin(1/5)}{2}$ (C) $\frac{\arccos(1/10)}{2}$ (D) $\frac{\arcsin(1/10)}{2}$ (E) $\frac{\arctan(2/5)}{2}$

11. A projectile is thrown from a horizontal surface, and reaches a maximum height h and also lands a distance h from the launch point. Neglecting air resistance, what is the maximum height for a projectile thrown directly upward with the same initial speed?

(A) $\frac{17h}{16}$ (B) $\frac{13h}{12}$ (C) $\frac{9h}{8}$ (D) $\frac{5h}{4}$ (E) $2h$

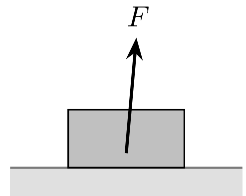
12. A block of mass m is initially held in place by two massless strings, as shown.



The tension in the diagonal string is T_1. Next, the horizontal string is cut, and immediately afterward the tension in the diagonal string is T_2. Which of the following is true?

(A) $T_1 < mg < T_2$ (B) $T_2 < mg < T_1$ (C) $T_1 < T_2 < mg$ (D) $mg < T_2 < T_1$ (E) $T_1 = T_2 < mg$

13. A uniform box with mass m is at rest on a horizontal surface, and the coefficient of static friction between them is μ_s. A force directed at an angle of 85° above the horizontal is applied to the center of the box, with a linearly increasing magnitude $F = \beta t$.



The box will eventually slide or lift off the ground. Which of the following is correct?

(A) If $\mu_s < \tan 85°$, the box will lift off the ground first.

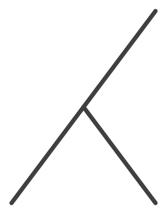
(B) If $\mu_s < \tan 85°$, the box will slide first.

(C) For any value of μ_s, the box will lift off the ground first.

(D) For any value of μ_s, the box will slide first.

(E) The answer depends on the values of β, g, and m.

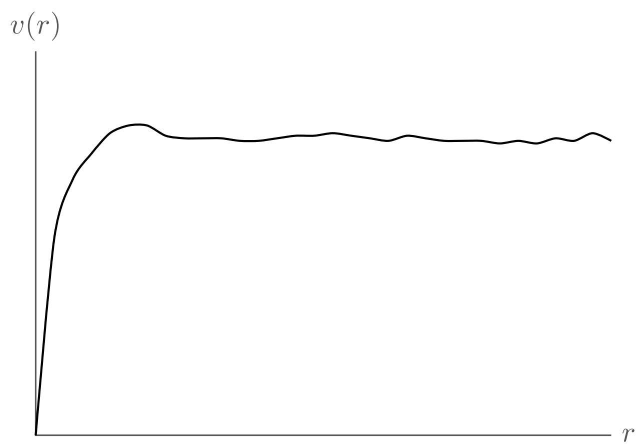
14. The object shown below is made of three rigidly connected, identical rods with uniform density.



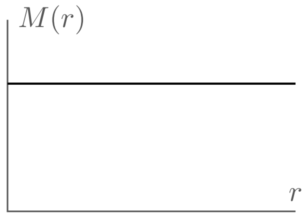
When it stands upright on a horizontal table, what fraction of its weight rests on the left leg?

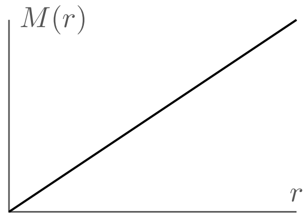
(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{5}{12}$

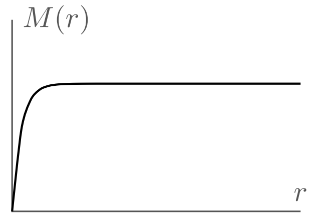
15. The following plot shows the speed $v(r)$ at which stars orbit about the center of a galaxy, as a function of their distance r from the center.

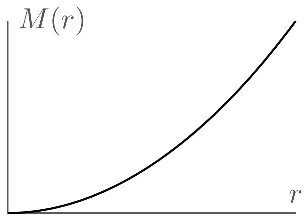


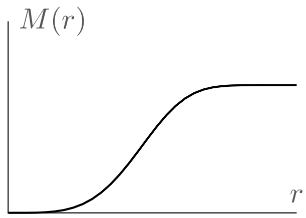
Assuming the galaxy has a spherically symmetric mass distribution, which plot best shows the mass of the galaxy $M(r)$ enclosed within radius r?

(A)


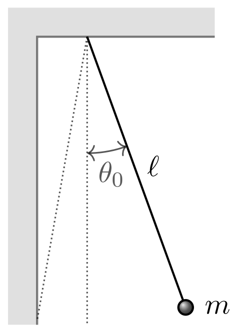
(B)


(C)


(D)


(E)


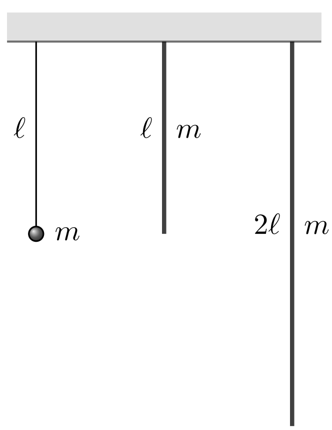
16. A bead attached to a string of length $\ell = 10\,\mathrm{m}$ is released from a very small angle θ_0 to the vertical. A wall is placed in the path of the bead such that the bead collides elastically with the wall when the string is at an angle $\theta_0/2$ to the vertical, as shown.



What is the time interval between the bead's collisions with the wall?

(A) $\dfrac{2\pi}{3}$ s (B) $\dfrac{3\pi}{4}$ s (C) $\dfrac{4\pi}{3}$ s (D) $\dfrac{3\pi}{2}$ s (E) 2π s

17. Three physical pendulums are built as shown. The first is a typical pendulum with a massless rope, and the second and third are made of uniform rods.



What is the correct ranking of the moments of inertia I_1, I_2, and I_3 about the pivot points?

(A) $I_1 > I_2 > I_3$ (B) $I_3 > I_2 > I_1$ (C) $I_1 = I_3 > I_2$ (D) $I_2 > I_3 > I_1$ (E) $I_3 > I_1 > I_2$

18. Alice, Bob, and Carol are each given identical airtight bags containing identical rocks, and a large tub of water with a scale sitting on the bottom. Each of them measures the weight of their bag and rock by putting the bag on the scale, using three slightly different procedures.

- Alice closes the bag carefully, so that there is no air inside.
- Bob fills the rest of the bag with water before closing it.
- Carol closes the bag loosely, so that it contains some air.

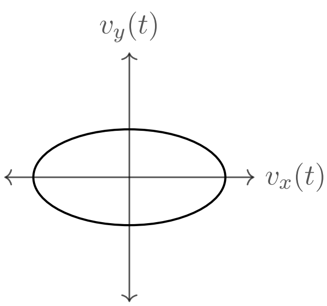
What is the correct ranking of their measured weights W_A, W_B, and W_C?

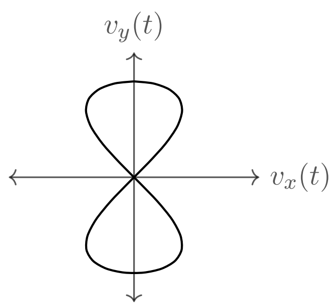
(A) $W_C < W_A < W_B$ (B) $W_A < W_C < W_B$ (C) $W_A < W_C = W_B$
(D) $W_A = W_C < W_B$ (E) $W_C < W_A = W_B$

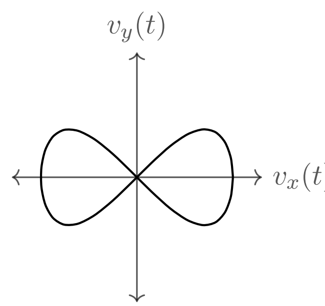
19. A student sets up a simple pendulum, measures its length to be (0.50 ± 0.01) m, and observes a period of oscillation of (1.4 ± 0.1) s. Using this data, the student computes $g = 10.1\,\mathrm{m/s^2}$. What is the uncertainty of this measurement?

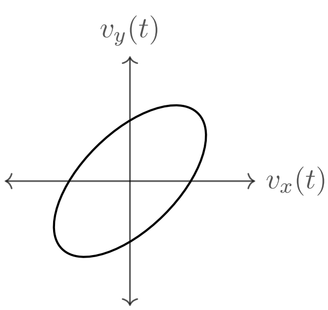
(A) $0.7\,\mathrm{m/s^2}$ (B) $1.2\,\mathrm{m/s^2}$ (C) $1.4\,\mathrm{m/s^2}$ (D) $1.9\,\mathrm{m/s^2}$ (E) $2.7\,\mathrm{m/s^2}$

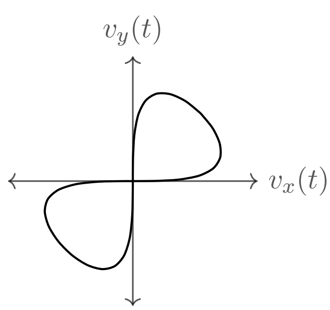
20. A mass attached to the end of a string oscillates like a pendulum with small amplitude. The mass has horizontal velocity $v_x(t)$ and vertical velocity $v_y(t)$. Which of the following could be a graph of the curve $(v_x(t), v_y(t))$ over a complete oscillation?

(A) 

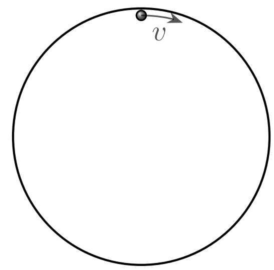
(B) 

(C) 

(D) 

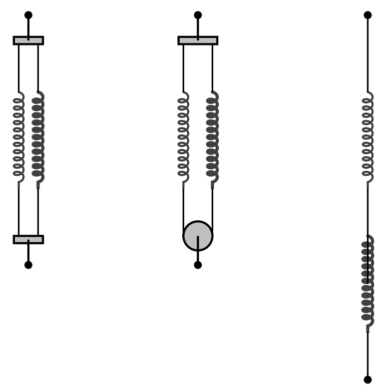
(E) 

21. A smooth ring of radius R and mass m lies on a frictionless surface. A point mass, also of mass m, is placed just inside the ring and given a speed v tangent to the inner surface of the ring. How long does it take for the point mass to return to its initial position relative to the ring?



(A) $\frac{\pi R}{v}$ (B) $\frac{\sqrt{2}\pi R}{v}$ (C) $\frac{2\pi R}{v}$ (D) $\frac{2\sqrt{2}\pi R}{v}$ (E) $\frac{4\pi R}{v}$

22. Two springs with different spring constants are connected in three ways, as shown.

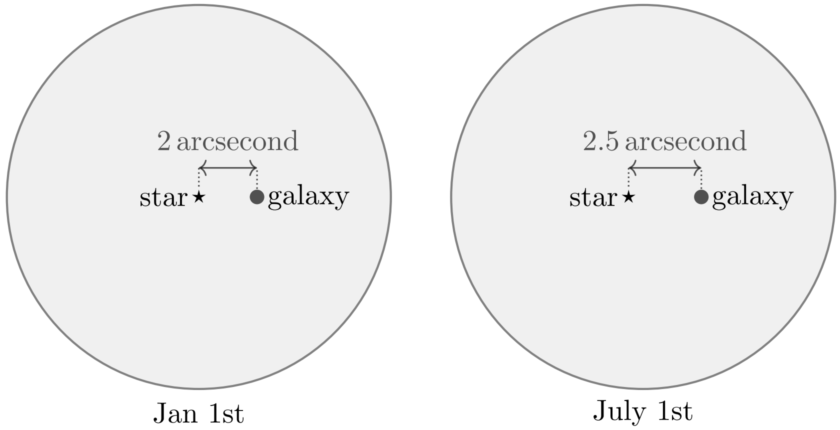


In the second case, the springs are connected to opposite ends of a string, which runs under a massless frictionless pulley. In each case, the two springs act like a single spring with an effective spring constant k_A, k_B, or k_C. Which of the following is correct?

(A) $k_A > k_B > k_C$ (B) $k_A > k_C > k_B$ (C) $k_C > k_B > k_A$ (D) $k_C > k_A > k_B$ (E) $k_B > k_A > k_C$

23. An astronomer on Earth, which is a distance $L_\odot$ from the Sun, observes a star and galaxy. The star is a distance $L_s \gg L_\odot$ away, and the galaxy is much further away than the star. Throughout the year, the angular distance between the star and galaxy appears to vary, reaching a minimum of 2 arcseconds on January 1st and a maximum of 2.5 arcseconds on July 1st. (One degree is equal to 3600 arcseconds.) Assume the Sun, star, and galaxy do not move relative to each other, and that the Earth's orbit lies

within their plane. What is the ratio $L_s/L_\odot$?



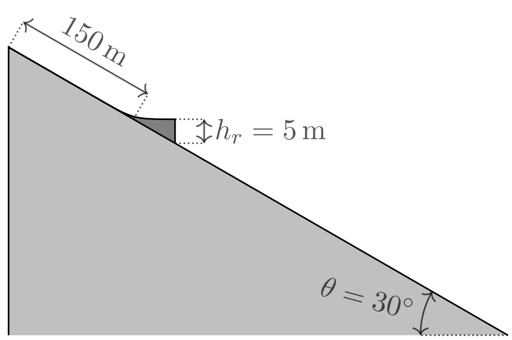
(astronomer's view through the telescope)

(A) 1.4×10^4 (B) 7×10^4 (C) 4×10^5 (D) 8×10^5 (E) 4×10^6

24. A very heavy plate continuously moves up and down with a small speed v, switching directions after each time t. At a random time, a ball is dropped from rest far above the plate, then bounces elastically off of it. How does the ball's speed change during this collision?

(A) The ball always slows down.
(B) The ball is more likely to slow down than to speed up.
(C) The ball is equally likely to slow down or speed up.
(D) The ball is more likely to speed up than to slow down.
(E) The ball always speeds up.

25. A skier slides from rest along a frictionless slope with incline angle $\theta = 30°$ for 150 m, then smoothly transitions into a horizontal jumping ramp of height $h_r = 5$ m. Neglecting air resistance, after the skier leaves the ramp, about how far will they travel *horizontally* before landing?



(A) 164 m (B) 173 m (C) 181 m (D) 200 m (E) 210 m

AAPT UNITED STATES PHYSICS TEAM
AIP 2023

2023 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

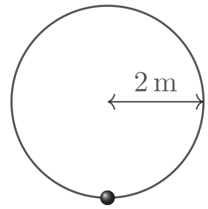
- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 25, 2023.**

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Kellan Colburn, Samuel Gebretsadkan, Abi Krishnan, Natalie LeBaron, Kye Shi, Mike Winer, and Kevin Zhou

1. A bead on a circular hoop with radius 2 m travels counterclockwise for 10 s and completes 2.25 rotations, at which point it reaches the position shown.

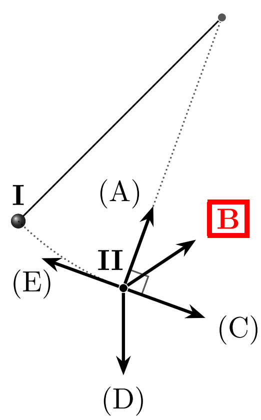


In the past 10 s, what were its average speed and the direction of its average velocity?

(A) $\frac{\sqrt{2}}{5}\,\frac{\text{m}}{\text{s}}, \swarrow$ (B) $\frac{2\pi}{5}\,\frac{\text{m}}{\text{s}}, \swarrow$ (C) $\frac{9\pi}{10}\,\frac{\text{m}}{\text{s}}, \swarrow$ (D) $\frac{2\pi}{5}\,\frac{\text{m}}{\text{s}}, \searrow$ $\boxed{\text{E}}$ $\frac{9\pi}{10}\,\frac{\text{m}}{\text{s}}, \searrow$

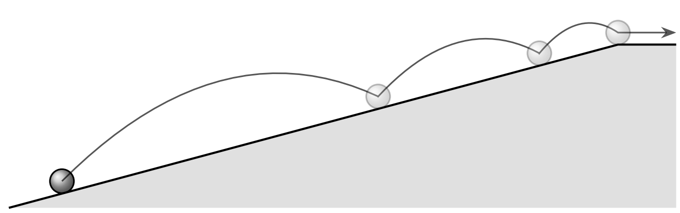
The total distance traveled is $(2.25)(2\pi \cdot 2\,\text{m}) = 9\pi\,\text{m}$, and the average speed is the distance per time, $(9\pi/10)\,\text{m/s}$. This rules out all choices but (C) and (E). To find the direction we note that 10 seconds ago, the bead was initially at the left side of the hoop, so the direction of the average velocity is $\searrow$.

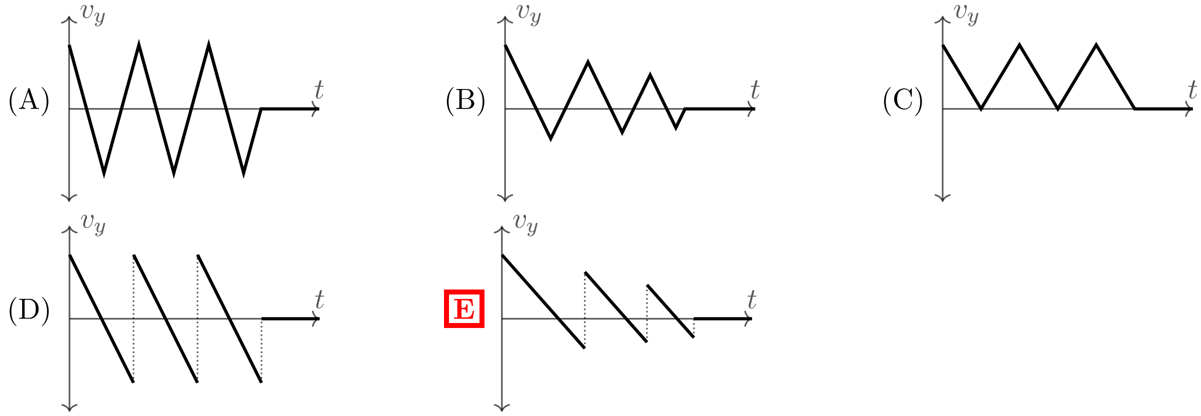
2. A mass on an ideal pendulum is released from rest at point I. When it reaches point II, which of the following shows the direction of its acceleration?



At this point, this is both an inward centripetal force because the mass is moving in a circle, and a tangential force because the mass is speeding up. Thus, the answer has to be a positive sum of vectors directed along (A) and (C), so it must be (B).

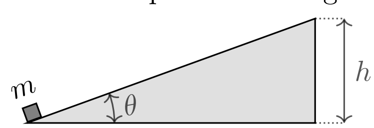
3. A soccer ball is kicked up a hill with a flat top, as shown. The ball bounces twice on the hill, at the points shown, then lands on the top and begins rolling horizontally. Which of the following shows the vertical component of its velocity as a function of time?





Since gravity provides a uniform acceleration downward, the graph of v_y should be decreasing with constant slope everywhere except at the collisions, ruling out all but choices (D) and (E). The collisions provide the discontinuities. To decide between the two, we note that the typical vertical velocities shown in the figure get smaller as the ball nears the top, so the correct choice is (E).

4. A box of mass m is at the bottom of an inclined plane with angle θ to the horizontal, and height h.



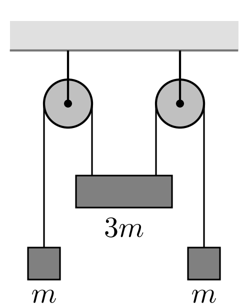
A person drags the box very slowly up the plane, by applying a force parallel to the plane. The coefficient of kinetic friction between the box and plane is μ_k. When the box reaches the top of the plane, how much work has the person done?

(A) $mgh\,(1 + \mu_k \sin\theta)$ (B) $mgh\,(1 + \mu_k \cos\theta)$ (C) $mgh\,(1 + \mu_k \tan\theta)$

(D) $mgh\,(1 + \mu_k \csc\theta)$ $\boxed{\text{E}}$ $mgh\,(1 + \mu_k \cot\theta)$

When the box is moving up the ramp, there is a gravitational force $mg\sin\theta$ down the ramp, and a friction force $\mu_k mg\cos\theta$ down the ramp. The box must be dragged a total distance $h/\sin\theta$ along the ramp, so the work done is $mgh(\sin\theta + \mu_k\cos\theta)/\sin\theta = mgh(1 + \mu_k\cot\theta)$.

5. Two blocks of mass m and a block of mass $3m$ are attached to a system of massless fixed pulleys and massless string, as shown.



Assume all surfaces are frictionless. What is the acceleration of each mass m?

(A) $\frac{g}{8}$ $\boxed{\text{B}}$ $\frac{g}{5}$ (C) $\frac{g}{4}$ (D) $\frac{g}{3}$ (E) $\frac{2g}{3}$

The system is equivalent to two disconnected Atwood's machines with masses m and $3m/2$, by splitting the large block down the middle. Hence the acceleration is $(3m/2 - m)g/(3m/2 + m) = g/5$.

6. A ball at the end of a rope of length 0.5 m is swung in a horizontal circle, with a speed of 15 m/s. The other end of the rope is fixed in place. What is the height difference between the ends of the rope?

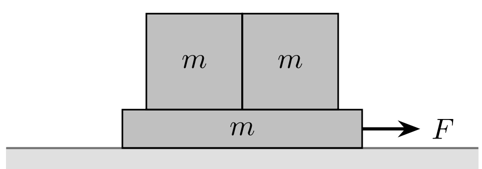
$\boxed{\text{A}}$ 1.1 cm (B) 2.2 cm (C) 3.8 cm (D) 4.9 cm (E) 7.5 cm

For all of the answer choices, the rope is close to horizontal, so we can use the small angle approximation. The angle θ the rope makes with the horizontal must obey

$$\theta \approx \tan\theta = \frac{mg}{mv^2/r}$$

where the radius of the horizontal circle is $r = \ell\cos\theta \approx \ell$. Thus, the height difference is $\ell\sin\theta \approx \ell\theta = g\ell^2/v^2$, and plugging in the numbers gives the answer.

7. Two boxes are stacked side-by-side on top of a larger box as shown.



All three boxes have mass m, the coefficient of static friction between the left box and the bottom box is μ_s, and all other surfaces are frictionless. A rightward force F is applied to the bottom box. What is the minimum value of μ_s so that the upper boxes don't slide?

(A) $\frac{2F}{mg}$ (B) $\frac{3F}{mg}$ (C) $\frac{F}{2mg}$ $\boxed{\text{D}}$ $\frac{2F}{3mg}$ (E) $\frac{F}{3mg}$

The acceleration of the entire system is $F/3m$, so the total horizontal force needed to accelerate the top two blocks is $2F/3$. This must be completely supplied by the friction force μmg on the left box, so we conclude $\mu \geq 2F/3mg$.

8. Two stars α and β, with masses satisfying $m_\alpha/m_\beta = 10$, are in circular orbits around each other. In the rest frame of this system, find the ratio of the speeds v_α/v_β.

(A) $\frac{1}{11}$ $\boxed{\text{B}}$ $\frac{1}{10}$ (C) $\frac{1}{9}$ (D) 9 (E) 10

Because the total momentum is zero, $m_\alpha v_\alpha = m_\beta v_\beta$, so $v_\alpha / v_\beta = m_\beta / m_\alpha = 1/10$.

9. A helium balloon is released from the floor in a room at rest, then slowly rises and comes to rest touching the ceiling. During this process, the gravitational potential energy of the balloon has increased. Since energy is conserved, the energy of something else must have decreased during this process. Which of the following is the main contribution to this decrease?

(A) The kinetic energy of the balloon decreased.

(B) The elastic potential energy of the balloon decreased.

(C) The thermal energy of the air in the balloon decreased.

(D) The thermal energy of the air in the room decreased.

$\boxed{\text{E}}$ The gravitational potential energy of the air in the room decreased.

The main contribution is choice (E). In terms of energy, objects that are lighter than air float upward because the air can move downward to where the object originally was, thereby reducing its gravitational potential energy by more than the gravitational potential energy of the object went up.

10. An archer takes aim at a target that is 100 m away. Assuming she holds the bow at the same height as the center of the target and shoots an arrow with velocity $v = 100\,\text{m/s}$, at what angle above the horizontal should she aim the bow so that the arrow hits the center of the target?

(A) $\dfrac{\arccos(1/5)}{2}$ (B) $\dfrac{\arcsin(1/5)}{2}$ (C) $\dfrac{\arccos(1/10)}{2}$ $\boxed{\text{D}}$ $\dfrac{\arcsin(1/10)}{2}$ (E) $\dfrac{\arctan(2/5)}{2}$

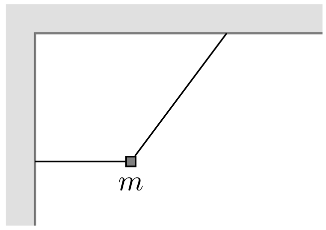
Since the initial and final heights are the same, we can apply the range equation $R = v^2 \sin(2\theta)/g$. Setting $R = 100\,\text{m}$ and $v = 100\,\text{m/s}$ and solving for θ gives the answer. Of course, an analysis using the usual kinematic equations would also give the same answer, after some more effort.

11. A projectile is thrown from a horizontal surface, and reaches a maximum height h and also lands a distance h from the launch point. Neglecting air resistance, what is the maximum height for a projectile thrown directly upward with the same initial speed?

$\boxed{\text{A}}$ $\dfrac{17h}{16}$ (B) $\dfrac{13h}{12}$ (C) $\dfrac{9h}{8}$ (D) $\dfrac{5h}{4}$ (E) $2h$

From the maximum height, we know that $h = v_y^2/2g$, and from the range, we know that $h = v_x \Delta t = 2v_x v_y/g$. Combining these equations gives $v_y = 4v_x$. The maximum height if the projectile is thrown upward is $v^2/2g = (v_x^2 + v_y^2)/2g = (17/16)(v_x^2/2g) = 17h/16$.

12. A block of mass m is initially held in place by two massless strings, as shown.

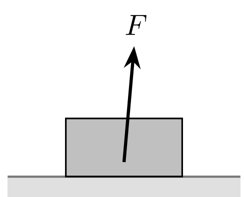


The tension in the diagonal string is T_1. Next, the horizontal string is cut, and immediately afterward the tension in the diagonal string is T_2. Which of the following is true?

(A) $T_1 < mg < T_2$ $\boxed{\text{B}}$ $T_2 < mg < T_1$ (C) $T_1 < T_2 < mg$ (D) $mg < T_2 < T_1$ (E) $T_1 = T_2 < mg$

Let the angle from the vertical be θ. Before the string is cut, balancing forces gives $T_1 = mg/\cos\theta$. Right after the string is cut, balancing forces along the direction of the string gives $T_2 = mg\cos\theta$. Thus, $T_2 < mg < T_1$.

13. A uniform box with mass m is at rest on a horizontal surface, and the coefficient of static friction between them is μ_s. A force directed at an angle of 85° above the horizontal is applied to the center of the box, with a linearly increasing magnitude $F = \beta t$.



The box will eventually slide or lift off the ground. Which of the following is correct?

(A) If $\mu_s < \tan 85°$, the box will lift off the ground first.

(B) If $\mu_s < \tan 85°$, the box will slide first.

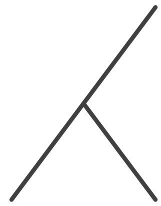
(C) For any value of μ_s, the box will lift off the ground first.

$\boxed{\text{D}}$ For any value of μ_s, the box will slide first.

(E) The answer depends on the values of β, g, and m.

The box will slide first, no matter what the other parameters are. The box will only lift off the ground once the normal force goes to zero, which occurs when the vertical component of the applied force exceeds the weight of the box. Right before that moment, the normal force will be very small, so the maximal possible friction force is also very small, and cannot balance the horizontal component of the applied force.

14. The object shown below is made of three rigidly connected, identical rods with uniform density.



When it stands upright on a horizontal table, what fraction of its weight rests on the left leg?

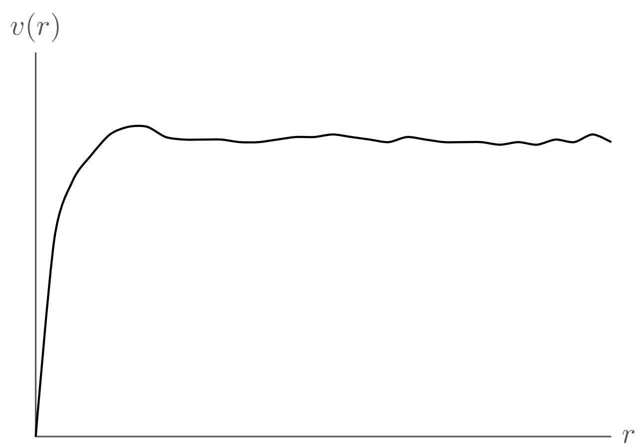
(A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ $\boxed{\text{E}}$ $\frac{5}{12}$

Let each rod have weight $W/3$, let the normal force on the left leg be N, and let the distance between the legs be ℓ. If we balance torques about bottom of the right leg, then the only contributions are from the weights of the rods and the normal force N, giving

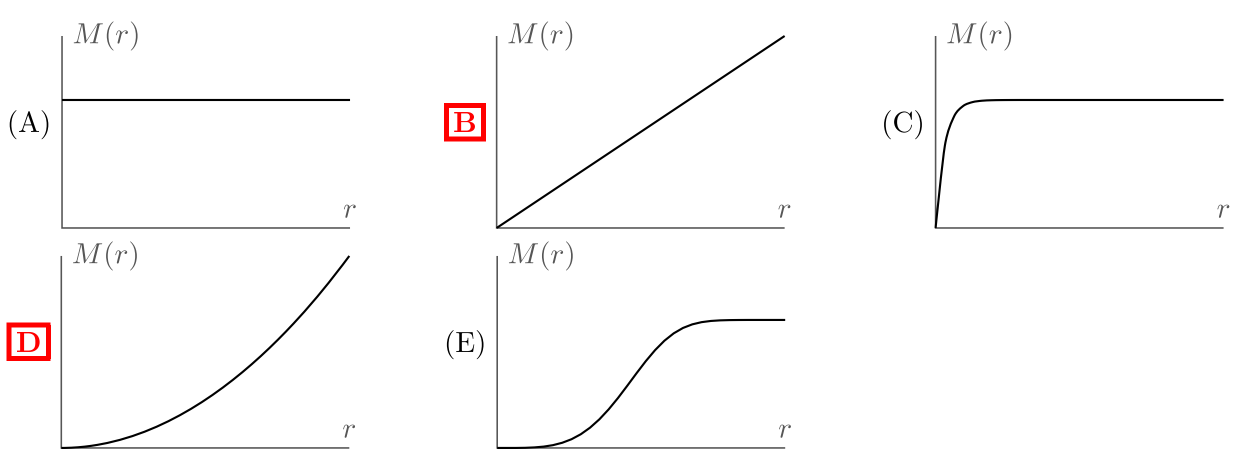
$$N\ell = \frac{W}{3}\left(\frac{\ell}{4} + \frac{\ell}{4} + \frac{3\ell}{4}\right)$$

which gives $N = 5W/12$.

15. The following plot shows the speed $v(r)$ at which stars orbit about the center of a galaxy, as a function of their distance r from the center.



Assuming the galaxy has a spherically symmetric mass distribution, which plot best shows the mass of the galaxy $M(r)$ enclosed within radius r?



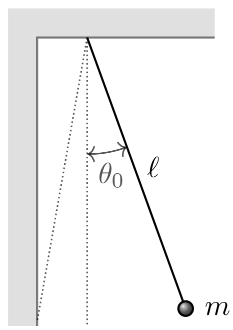
The graph shows that rotational speed becomes roughly constant once a certain distance from the center of the galaxy, but the rotational speed of a mass m is governed by

$$\frac{GM(r)m}{r^2} = \frac{mv(r)^2}{r}.$$

Then we must have a linear $M(r) \propto r$ to keep $v(r)$ constant, which correspond to choice (B).

Choice (D) does not match this behavior because it is parabolic, $M(r) \propto r^2$. However, at the level of resolution given in the figures, it is somewhat difficult to see that its curve indeed remains parabolic at large r, rather than approaching a linear $M(r)$. Furthermore, it is below linear at small r, which is consistent with the small r behavior of $v(r)$. Thus, we also accepted (D) as correct.

16. A bead attached to a string of length $\ell = 10\,\text{m}$ is released from a very small angle θ_0 to the vertical. A wall is placed in the path of the bead such that the bead collides elastically with the wall when the string is at an angle $\theta_0/2$ to the vertical, as shown.

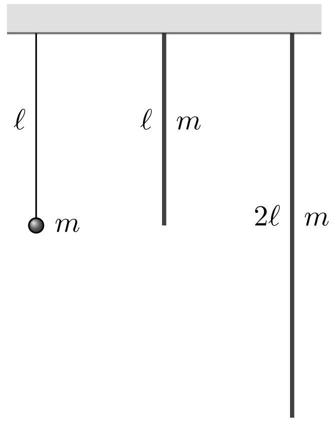


What is the time interval between the bead's collisions with the wall?

(A) $\frac{2\pi}{3}$ s (B) $\frac{3\pi}{4}$ s $\boxed{\text{C}}$ $\frac{4\pi}{3}$ s (D) $\frac{3\pi}{2}$ s (E) 2π s

If the wall was not there, then the angle of the pendulum would be $\theta(t) = \theta_0 \cos(t\sqrt{g/L})$, which corresponds to period 2π seconds. When the bead collides with the wall, it effectively skips from a phase angle 120° in its oscillation to a phase angle 240°, which removes 1/3 of the oscillation cycle. Thus, the period is $(2/3)(2\pi) = 4\pi/3$ seconds.

17. Three physical pendulums are built as shown. The first is a typical pendulum with a massless rope, and the second and third are made of uniform rods.



What is the correct ranking of the moments of inertia I_1, I_2, and I_3 about the pivot points?

(A) $I_1 > I_2 > I_3$ (B) $I_3 > I_2 > I_1$ (C) $I_1 = I_3 > I_2$ (D) $I_2 > I_3 > I_1$ $\boxed{\text{E}}$ $I_3 > I_1 > I_2$

This is a straightforward application of the standard moment of inertia formulas,

$$I_1 = m\ell^2, \qquad I_2 = \frac{1}{3}m\ell^2, \qquad I_3 = \frac{4}{3}m\ell^2$$

which gives $I_3 > I_1 > I_2$.

However, if you don't remember these formulas, you can get to the same result by noting that moment of inertia grows with distance from the pivot, so we must have $I_1 > I_2$ since the point mass is at least as far from the pivot as every point on the rod. We also have $I_3 > I_1$ by the parallel axis theorem, as the third pendulum has a nonzero moment of inertia about its center of mass, while the first pendulum doesn't.

18. Alice, Bob, and Carol are each given identical airtight bags containing identical rocks, and a large tub of water with a scale sitting on the bottom. Each of them measures the weight of their bag and rock by putting the bag on the scale, using three slightly different procedures.

- Alice closes the bag carefully, so that there is no air inside.
- Bob fills the rest of the bag with water before closing it.
- Carol closes the bag loosely, so that it contains some air.

What is the correct ranking of their measured weights W_A, W_B, and W_C?

(A) $W_C < W_A < W_B$ (B) $W_A < W_C < W_B$ (C) $W_A < W_C = W_B$

(D) $W_A = W_C < W_B$ $\boxed{\text{E}}$ $W_C < W_A = W_B$

The scale's measurement is the weight of the object not supported by the buoyant force. In other words, the scale measures $mg - \rho V g$, where m is the mass of the bag, ρ is the density of water, and V is the volume of water. For Alice,

$$W_A = m_A g - \rho V_A g.$$

If Bob adds volume ΔV of water, he adds mass $\rho \Delta V$ as well, so

$$W_B = (m_A + \rho \Delta V)g - \rho(V_A + \Delta V)g = W_A.$$

Carol adds a volume ΔV while adding a negligible mass from the air, so

$$W_C = W_A - \rho \Delta V g.$$

Therefore, the answer is (E).

19. A student sets up a simple pendulum, measures its length to be (0.50 ± 0.01) m, and observes a period of oscillation of (1.4 ± 0.1) s. Using this data, the student computes $g = 10.1\,\mathrm{m/s^2}$. What is the uncertainty of this measurement?

(A) $0.7\,\mathrm{m/s^2}$ (B) $1.2\,\mathrm{m/s^2}$ $\boxed{\text{C}}$ $1.4\,\mathrm{m/s^2}$ (D) $1.9\,\mathrm{m/s^2}$ (E) $2.7\,\mathrm{m/s^2}$

We have $g = 4\pi^2 L/T^2$, and we can use the standard rules for propagation of relative uncertainties: the relative uncertainty of a product (or ratio) of two quantities adds in quadrature, and the relative uncertainty of a squared quantity is doubled. Thus,

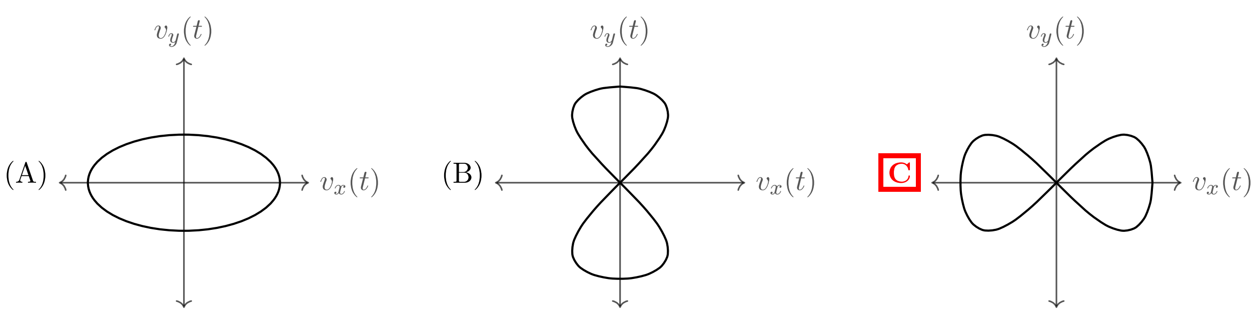
$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta T}{T}\right)^2}$$

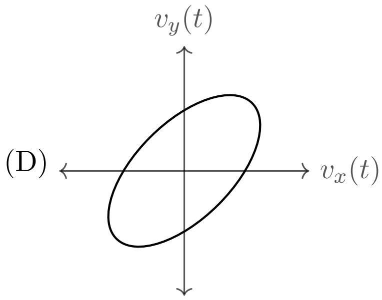
which gives $\Delta g = 1.4\,\mathrm{m/s^2}$. (At the level of precision needed here, you could also note that $\Delta L/L$ is negligible, so we just have $\Delta g = g(2\Delta T/T)$, which would give the same answer.) If one instead used

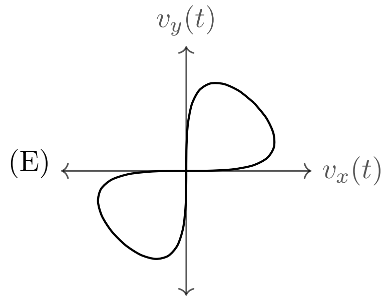
$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

then the answer would instead be $\Delta g = 1.6\,\mathrm{m/s^2}$, which is not an answer choice. However, this is incorrect because independent relative uncertainties add in quadrature, rather than directly. For further details, see the solution to problem 12 of the 2018 $F = ma$ exam A.

20. A mass attached to the end of a string oscillates like a pendulum with small amplitude. The mass has horizontal velocity $v_x(t)$ and vertical velocity $v_y(t)$. Which of the following could be a graph of the curve $(v_x(t), v_y(t))$ over a complete oscillation?

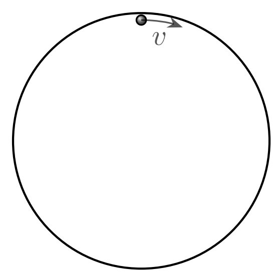






The answer is choice (C), as one can show by plotting several points. For example, one simple way to rule out the other answers is to note that on a pendulum, $v_y(t)$ hits zero four times per cycle, which only choice (C) satisfies.

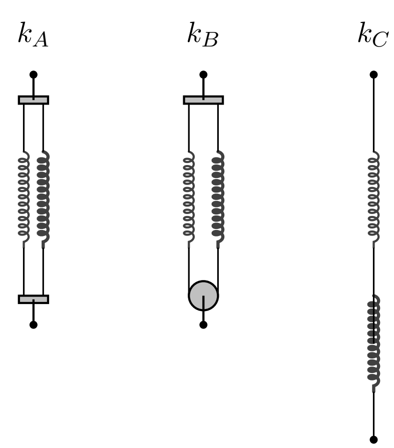
21. A smooth ring of radius R and mass m lies on a frictionless surface. A point mass, also of mass m, is placed just inside the ring and given a speed v tangent to the inner surface of the ring. How long does it take for the point mass to return to its initial position relative to the ring?



(A) $\dfrac{\pi R}{v}$ (B) $\dfrac{\sqrt{2}\pi R}{v}$ $\boxed{\text{C}}$ $\dfrac{2\pi R}{v}$ (D) $\dfrac{2\sqrt{2}\pi R}{v}$ (E) $\dfrac{4\pi R}{v}$

The masses are undergoing small elastic collisions. During an elastic collision, the magnitude of the relative velocities between the masses are conserved, so the tangential velocity of the mass m, relative to the ring, is always v. Thus, the time is $2\pi R/v$.

22. Two springs with different spring constants are connected in three ways, as shown.



In the second case, the springs are connected to opposite ends of a string, which runs under a massless frictionless pulley. In each case, the two springs act like a single spring with an effective spring constant k_A, k_B, or k_C. Which of the following is correct?

$\boxed{\text{A}}$ $k_A > k_B > k_C$ (B) $k_A > k_C > k_B$ (C) $k_C > k_B > k_A$ (D) $k_C > k_A > k_B$ (E) $k_B > k_A > k_C$

Let the springs have spring constants k_1 and k_2. In the first case, the forces of the springs simply add, so $k_A = k_1 + k_2$. In the third case, the springs adjust so they exert an equal force. If the total displacement is ℓ, then $\ell = \ell_1 + \ell_2$ and $F = k_1\ell_1 = k_2\ell_2$. Solving for the force gives an effective spring constant

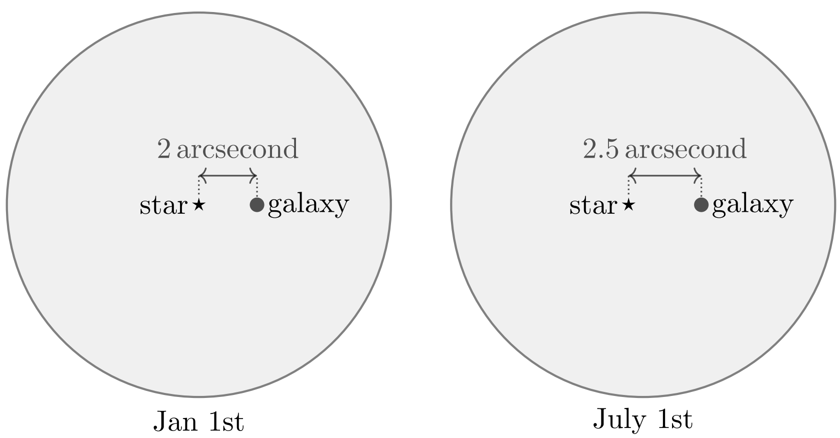
$$k_C = \frac{F}{\ell} = \frac{k_1 k_2}{k_1 + k_2}.$$

The second case is a bit less familiar, but it is actually similar to the third case. Again, we have the constraint that the forces exerted by the springs are equal, but the difference is that (1) pulling a distance ℓ now increases the length of the spring system by 2ℓ, and (2) the forces from each spring add together. Each doubles the resulting force, so

$$k_B = \frac{4k_1 k_2}{k_1 + k_2}$$

and so clearly $k_B > k_C$. We also have $k_A > k_B$ since the arithmetic mean is always greater than the geometric mean.

23. An astronomer on Earth, which is a distance $L_\odot$ from the Sun, observes a star and galaxy. The star is a distance $L_s \gg L_\odot$ away, and the galaxy is much further away than the star. Throughout the year, the angular distance between the star and galaxy appears to vary, reaching a minimum of 2 arcseconds on January 1st and a maximum of 2.5 arcseconds on July 1st. (One degree is equal to 3600 arcseconds.) Assume the Sun, star, and galaxy do not move relative to each other, and that the Earth's orbit lies within their plane. What is the ratio $L_s/L_\odot$?



(astronomer's view through the telescope)

(A) 1.4×10^4 (B) 7×10^4 (C) 4×10^5 $\boxed{\text{D}}$ 8×10^5 (E) 4×10^6

The change in apparent position of the star is due to parallax. Using the small angle approximation,

$$\Delta\theta = \frac{2L_\odot}{L_s}$$

where $\Delta\theta = 0.5\,\text{arcseconds} = 1°/7200 = 2.4 \times 10^{-6}\,\text{rad}$, and there is a factor of 2 since the change in position of the Earth across half a orbit is $2L_\odot$. Solving for the ratio gives $L_s/L_\odot = 8 \times 10^5$.

24. A very heavy plate continuously moves up and down with a small speed v, switching directions after each time t. At a random time, a ball is dropped from rest far above the plate, then bounces elastically off of it. How does the ball's speed change during this collision?

(A) The ball always slows down.

(B) The ball is more likely to slow down than to speed up.

(C) The ball is equally likely to slow down or speed up.

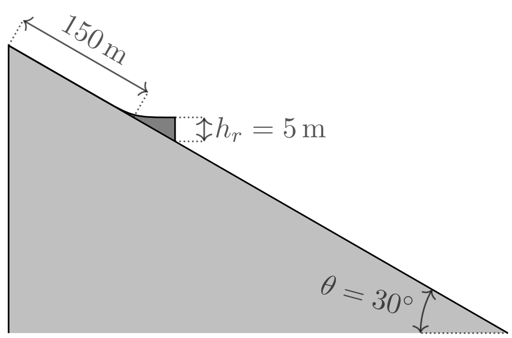
$\boxed{\text{D}}$ The ball is more likely to speed up than to slow down.

(E) The ball always speeds up.

When the lowest point of the ball enters the collision region, the plates move up or down with equal probability. if the plate moves up, then the speed of the ball will increase. If the plate moves down, it still has time to turn around and collide with the ball as it will move up.

This is a simple model for how a gas heats up in a container with hot walls, or more exotically, the Fermi acceleration mechanism for cosmic rays.

25. A skier slides from rest along a frictionless slope with incline angle $\theta = 30°$ for 150 m, then smoothly transitions into a horizontal jumping ramp of height $h_r = 5\,\text{m}$. Neglecting air resistance, after the skier leaves the ramp, about how far will they travel *horizontally* before landing?



(A) 164 m (B) 173 m $\boxed{\text{C}}$ 181 m (D) 200 m (E) 210 m

Upon leaving the ramp, the skier has speed $v = \sqrt{2gd\sin\theta} = \sqrt{gd}$. From there the skier travels a horizontal distance $x = vt$, where t is the time of flight. To find t, shift into the rotated coordinate frame (x' along the slope, y' perpendicular/normal to the slope); in this frame, the skier is launched with initial "vertical" velocity $v'_y = v\sin\theta = v/2 = \sqrt{gd}/2$ from height $h'_r = h_r\cos\theta = h_r\sqrt{3}/2$ and accelerates downward at a constant rate $g' = g\cos\theta = g\sqrt{3}/2$. Then the time of flight is

$$t = \frac{v'_y + \sqrt{(v'_y)^2 + 2g'h'}}{g'} = \frac{\sqrt{gd}/2 + \sqrt{gd/4 + 3gh_r/2}}{g\sqrt{3}/2} = \frac{\sqrt{d} + \sqrt{d + 6h_r}}{\sqrt{3g}},$$

and so the horizontal distance traveled is

$$x = vt = \sqrt{gd}t = \frac{d + \sqrt{d(d + 6h_r)}}{\sqrt{3}} = 181\,\mathrm{m}.$$

AAPT UNITED STATES PHYSICS TEAM
AIP 2024

2024 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout, unless otherwise specified.
- You may write in this question booklet and the scratch paper provided by the proctor.
- This test has 25 multiple choice questions. Select the best response to each question, and use a No.2 pencil to completely fill the box corresponding to your choice. If you change an answer, completely erase the previous mark. Only use the boxes numbered 1 through 25 on the answer sheet.
- All questions are equally weighted, but are not necessarily equally difficult.
- You will receive one point for each correct answer, and zero points for each incorrect or blank answer. There is no additional penalty for incorrect answers.
- You may use a hand-held calculator. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any external references, such as books or formula sheets.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **To maintain exam security, do not communicate any information about the questions or their solutions until after February 24, 2024.**

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Orhun Ciftcioglu, Kellan Colburn, Natalie LeBaron, Brian Skinner, Elena Yudovina and Kevin Zhou

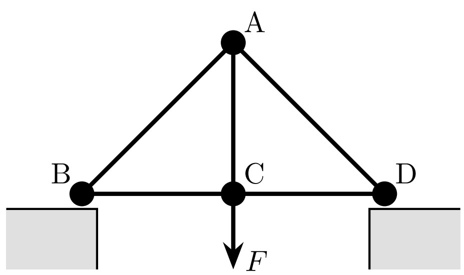
1. An archer fires an arrow from the ground so that it passes through two hoops, which are both a height h above the ground. The arrow passes through the first hoop one second after the arrow is launched, and through the second hoop another second later. What is the value of h?

   (A) 5 m
   (B) 10 m
   (C) 12 m
   (D) 15 m
   (E) There is not enough information to decide.

2. An amusement park ride consists of a circular, horizontal room. A rider leans against its frictionless outer walls, which are angled back at 30° with respect to the vertical, so that the rider's center of mass is 5.0 m from the center of the room. When the room begins to spin about its center, at what angular velocity will the rider's feet first lift off the floor?

   (A) 1.9 rad/s (B) 2.3 rad/s (C) 3.5 rad/s (D) 4.0 rad/s (E) 5.6 rad/s

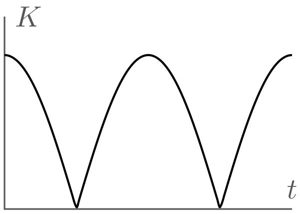
3. A simple bridge is made of five thin rods rigidly connected at four vertices.

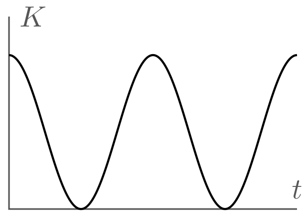


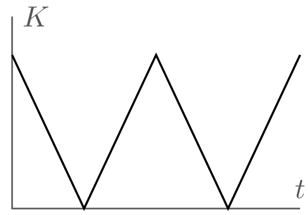
The ground is frictionless, so that it can only exert vertical normal forces at B and D. The weight of the bridge is negligible, but a person stands at its middle, exerting a downward force F at vertex C. In static equilibrium, each rod can be experiencing either tension or compression. Which of the following is true?

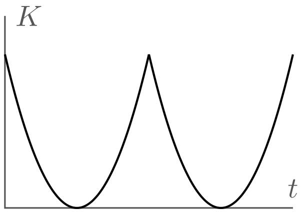
   (A) Only the vertical rod is in tension.
   (B) Only the horizontal rods are in tension.
   (C) Both the vertical rod and the diagonal rods are in tension.
   (D) Both the vertical rod and the horizontal rods are in tension.
   (E) All of the rods are in tension.

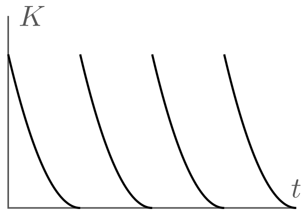
4. A bouncy ball is thrown vertically upward from the ground. Air resistance is negligible, and the ball's collisions with the ground are perfectly elastic. Which of the following shows the kinetic energy of the ball as a function of time? Assume the collisions are too quick for their duration to be seen in the plot.

(A) 

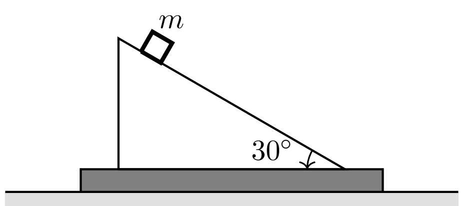
(B) 

(C) 

(D) 

(E) 

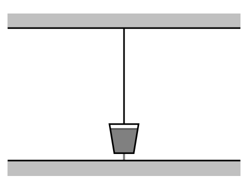
5. A massless inclined plane with angle 30° to the horizontal is fixed to a scale. A block of mass m is released from the top of the plane, which is frictionless.



As the block slides down the plane, what is the reading on the scale?

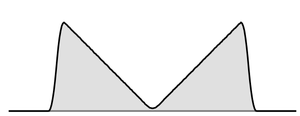
(A) $\sqrt{3}\,mg/4$ (B) $mg/2$ (C) $3mg/4$ (D) $\sqrt{3}\,mg/2$ (E) mg

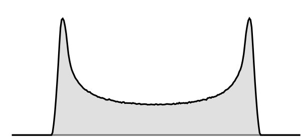
6. A pendulum is made with a string and a bucket full of water. When the string is vertical, the bottom of the bucket is near the ground.

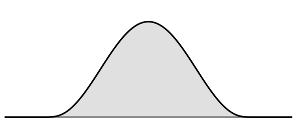


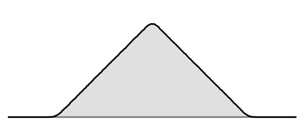
Then, the pendulum is set swinging with a small amplitude, and a very small hole is opened at the bottom of the bucket, which leaks water at a constant rate. After a few full swings, which of the following best shows the amount of water that has landed on the ground as a function of position?

(A) 

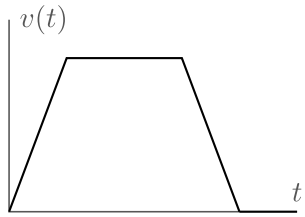
(B) 

(C) 

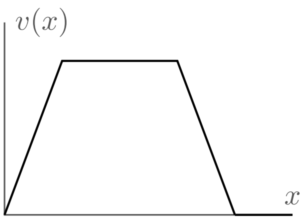
(D) 

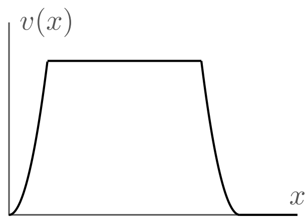
(E) 

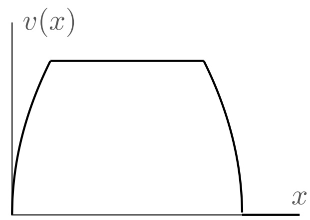
7. A particle travels in a straight line. Its velocity as a function of time is shown below.

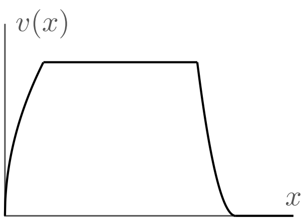


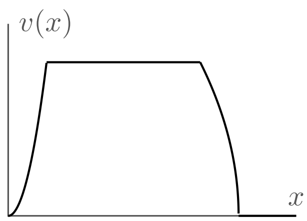
Which of the following shows the velocity as a function of distance x from its initial position?

(A) 

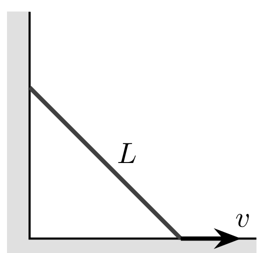
(B) 

(C) 

(D) 

(E) 

8. A rod of length L is sliding down a frictionless wall.



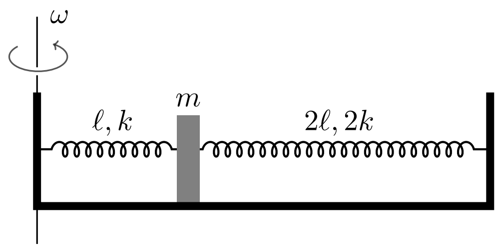
When the rod makes an angle of 45° to the horizontal, the bottom of the rod has speed v. At this moment, what is the speed of the middle of the rod?

(A) $v/2$ (B) $v/\sqrt{2}$ (C) v (D) $\sqrt{2}\,v$ (E) $2v$

9. When a car's brakes are fully engaged, it takes 100 m to stop on a dry road, which has coefficient of kinetic friction $\mu_k = 0.8$ with the tires. Now suppose only the first 50 m of the road is dry, and the rest is covered with ice, with $\mu_k = 0.2$. What total distance does the car need to stop?

(A) 150 m (B) 200 m (C) 250 m (D) 400 m (E) 850 m

10. A block of mass m is connected to the walls of a frictionless box by two massless springs with relaxed lengths ℓ and 2ℓ, and spring constants k and $2k$ respectively. The length of the box is 3ℓ. The system rotates with a constant angular velocity ω about one of its walls.



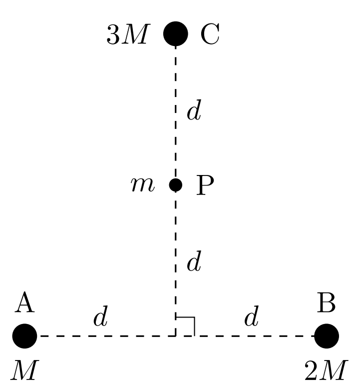
Suppose the block stays at a constant distance r from the axis of rotation, without touching either of the walls. What is the value of r?

(A) $\dfrac{2k\ell}{2k - m\omega^2}$ (B) $\dfrac{2k\ell}{2k + m\omega^2}$ (C) $\dfrac{2k\ell}{3k + m\omega^2}$ (D) $\dfrac{3k\ell}{3k - m\omega^2}$ (E) $\dfrac{3k\ell}{3k + m\omega^2}$

11. Two hemispherical shells can be pressed together to form a airtight sphere of radius 40 cm. Suppose the shells are pressed together at a high altitude, where the air pressure is half its value at sea level. The sphere is then returned to sea level, where the air pressure is 10^5 Pa. What force F, applied directly outward to each hemisphere, is required to pull them apart?

(A) 25,000 N (B) 50,000 N (C) 100,000 N (D) 200,000 N (E) 400,000 N

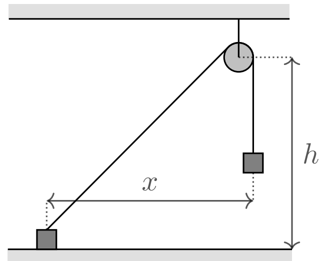
12. A space probe with mass m at point P traverses through a cluster of three asteroids, at points A, B, and C. The masses and locations of the asteroids are shown below.



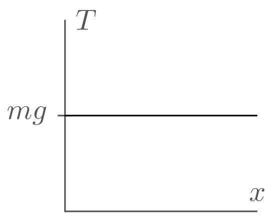
What is the torque on the probe about point C?

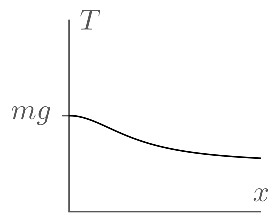
(A) $\dfrac{1}{2\sqrt{2}}\dfrac{GMm}{d}$ (B) $\dfrac{1}{2}\dfrac{GMm}{d}$ (C) $\dfrac{1}{\sqrt{2}}\dfrac{GMm}{d}$ (D) $\dfrac{GMm}{d}$ (E) $\dfrac{\sqrt{2}\,GMm}{d}$

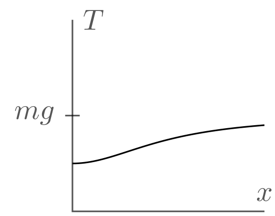
13. Two frictionless blocks of mass m are connected by a massless string which passes through a fixed massless pulley, which is a height h above the ground. Suppose the blocks are initially held with horizontal separation x, and the length of the string is chosen so that the right block hangs in the air as shown.

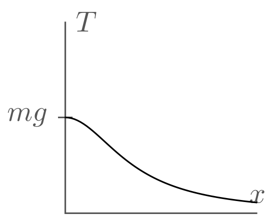


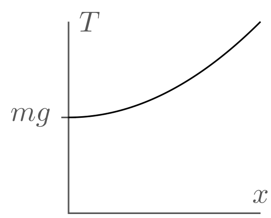
If the blocks are relased, the tension in the string immediately afterward will be T. Which of the following shows a plot of T versus x?

(A) 

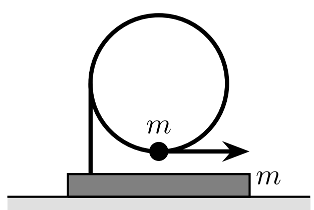
(B) 

(C) 

(D) 

(E) 

14. A bead of mass m can slide frictionlessly on a vertical circular wire hoop of radius 20 cm.



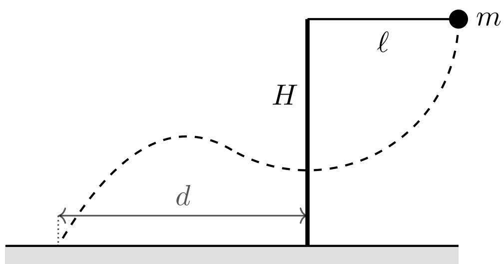
The hoop is attached to a stand of mass m, which can slide frictionlessly on the ground. Initially, the bead is at the bottom of the hoop, the stand is at rest, and the bead has velocity 2 m/s to the right. At some point, the bead will stop moving with respect to the hoop. At that moment, through what angle along the hoop has the bead traveled?

(A) 30° (B) 45° (C) 60° (D) 90° (E) 120°

15. The viscous force between two plates of area A, with relative speed v and separation d, is $F = \eta Av/d$, where η is the viscosity. In fluid mechanics, the Ohnesorge number is a dimensionless number proportional to η which characterizes the importance of viscous forces, in a drop of fluid of density ρ, surface tension γ, and length scale ℓ. Which of the following could be the definition of the Ohnesorge number?

(A) $\dfrac{\eta\ell}{\sqrt{\rho\gamma}}$ (B) $\eta\ell\sqrt{\dfrac{\rho}{\gamma}}$ (C) $\eta\sqrt{\dfrac{\rho}{\gamma\ell}}$ (D) $\eta\sqrt{\dfrac{\rho\ell}{\gamma}}$ (E) $\dfrac{\eta}{\sqrt{\rho\gamma\ell}}$

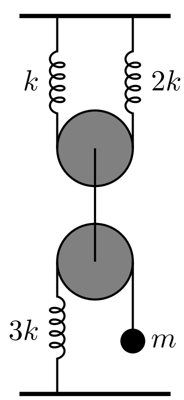
16. A child of mass m holds onto the end of a massless rope of length ℓ, which is attached to a pivot a height H above the ground. The child is released from rest when the rope is straight and horizontal.



At some point, the child lets go of the rope, flies through the air, and lands on the ground a horizontal distance d from the pivot. On Earth, the maximum possible value of d is d_E. If the setup is moved to the Moon, which has 1/6 the gravitational acceleration, what is the new maximum possible value of d?

(A) $d_E/6$ (B) $d_E/\sqrt{6}$ (C) d_E (D) $\sqrt{6}\,d_E$ (E) $6\,d_E$

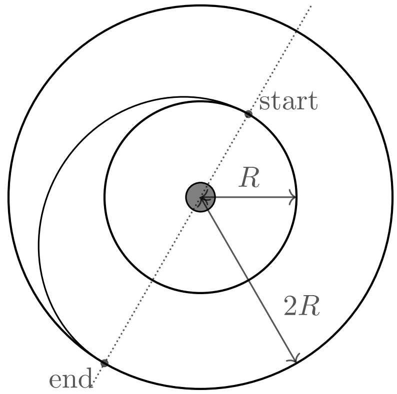
17. Consider the following system of massless and frictionless pulleys, ropes, and springs.



Initially, a block of mass m is attached to the end of a rope, and the system is in equilibrium. Next the block is doubled in mass, and the system is allowed to come to equilibrium again. During the transition between these equilibria, how far does the end of the rope (where the block is suspended) move?

(A) $\dfrac{7}{12}\dfrac{mg}{k}$ (B) $\dfrac{11}{12}\dfrac{mg}{k}$ (C) $\dfrac{13}{12}\dfrac{mg}{k}$ (D) $\dfrac{7}{6}\dfrac{mg}{k}$ (E) $\dfrac{11}{6}\dfrac{mg}{k}$

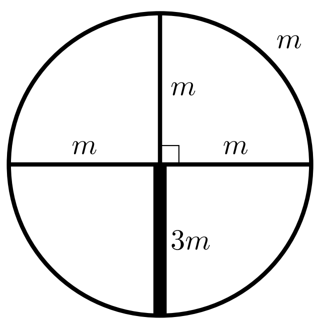
18. A satellite is initially in a circular orbit of radius R around a planet of mass M. It fires its rockets to instantaneously increase its speed by Δv, keeping the direction of its velocity the same, so that it enters an elliptical orbit whose maximum distance from the planet is $2R$.



What is the value of Δv? (Hint: when the satellite is in an elliptical orbit with semimajor axis a, its total energy per unit mass is $-GM/2a$.)

(A) $0.08\sqrt{\dfrac{GM}{R}}$ (B) $0.15\sqrt{\dfrac{GM}{R}}$ (C) $0.22\sqrt{\dfrac{GM}{R}}$ (D) $0.29\sqrt{\dfrac{GM}{R}}$ (E) $0.41\sqrt{\dfrac{GM}{R}}$

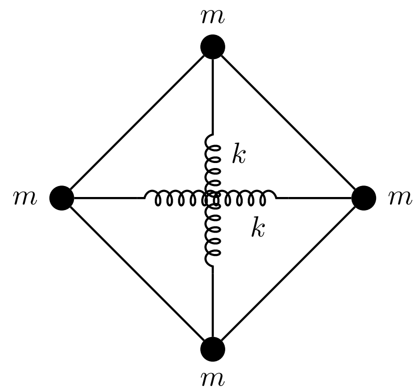
19. A wheel of radius R has a thin rim and four spokes, each of which have uniform density.



The entire rim has mass m, three of the spokes each have mass m, and the fourth spoke has mass $3m$. The wheel is suspended on a horizontal frictionless axle passing through its center. If the wheel is slightly rotated from its equilibrium position, what is the angular frequency of small oscillations?

(A) $\sqrt{\dfrac{g}{3R}}$ (B) $\sqrt{\dfrac{g}{2R}}$ (C) $\sqrt{\dfrac{2g}{3R}}$ (D) $\sqrt{\dfrac{g}{R}}$ (E) $\sqrt{\dfrac{7g}{6R}}$

20. Four massless rigid rods are connected into a quadrilateral by four hinges. The hinges have mass m, and allow the rods to freely rotate. A spring of spring constant k is connected across each of the diagonals, so that the springs are at their relaxed length when the rods form a square.



Assume the springs do not interfere with each other. If the square is slightly compressed along one of its diagonals, its shape will oscillate over time. What is the period of these oscillations?

(A) $2\pi\sqrt{\frac{m}{4k}}$ (B) $2\pi\sqrt{\frac{m}{2k}}$ (C) $2\pi\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{2m}{k}}$ (E) $2\pi\sqrt{\frac{4m}{k}}$

21. A syringe is filled with water of density ρ and negligible viscosity. Its body is a cylinder of cross-sectional area A_1, which gradually tapers into a needle with cross-sectional area $A_2 \ll A_1$. The syringe is held in place and its end is slowly pushed inward by a force F, so that it moves with constant speed v. Water shoots straight out of the needle's tip. What is the approximate value of F?

(A) $\rho v^2 A_1$ (B) $\frac{\rho v^2 A_1^2}{2A_2}$ (C) $\frac{\rho v^2 A_1^2}{A_2}$ (D) $\frac{\rho v^2 A_1^3}{2A_2^2}$ (E) $\frac{\rho v^2 A_1^3}{A_2^2}$

22. A spherical shell is made from a thin sheet of material with a mass per area of σ. Consider two points, P_1 and P_2, which are close to each other, but just inside and outside the sphere, respectively. If the accelerations due to gravity at these points are $\mathbf{g}_1$ and $\mathbf{g}_2$, respectively, what is the value of $|\mathbf{g}_1 - \mathbf{g}_2|$?

(A) $\pi G\sigma$ (B) $4\pi G\sigma/3$ (C) $2\pi G\sigma$ (D) $4\pi G\sigma$ (E) $8\pi G\sigma$

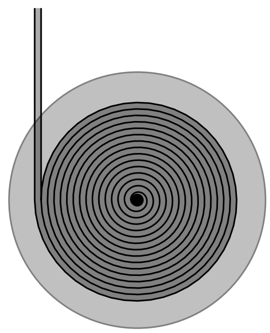
23. Collisions between ping pong balls and paddles are not perfectly elastic. Suppose that if a player holds a paddle still and drops a ball on top of it from any height h, it will bounce back up to height $h/2$. To keep the ball bouncing steadily, the player moves the paddle up and down, so that it is moving upward with speed $1.0\,\mathrm{m/s}$ whenever the ball hits it. What is the height to which the ball is bouncing?

(A) $0.21\,\mathrm{m}$
(B) $0.45\,\mathrm{m}$
(C) $1.0\,\mathrm{m}$
(D) $1.7\,\mathrm{m}$
(E) There is not enough information to determine the height.

24. When a projectile falls through a fluid, it experiences a drag force proportional to the product of its cross-sectional area, the fluid density ρ_f, and the square of its speed. Suppose a sphere of density $\rho_s \gg \rho_f$ of radius R is dropped in the fluid from rest. When the projectile has reached half of its terminal velocity, which of the following is its displacement proportional to?

(A) $R\sqrt{\rho_s/\rho_f}$ (B) $R\,\rho_s/\rho_f$ (C) $R\,(\rho_s/\rho_f)^{3/2}$ (D) $R\,(\rho_s/\rho_f)^2$ (E) $R\,(\rho_s/\rho_f)^3$

25. A yo-yo consists of two massive uniform disks of radius R connected by a thin axle. A thick string is wrapped many times around the axle, so that the end of the string is initially a distance R from the axle. Then, the end of the string is held in place and the yo-yo is dropped from rest. Assume that energy losses are negligible, and that the string has negligible mass and always remains vertical. Below, we show a cross-section of the yo-yo partway through its descent.



Between the moment the yo-yo is released and the moment the string completely unwinds, which of the following is true regarding the yo-yo's acceleration?

(A) It is always zero.
(B) It points downward, but decreases in magnitude over time.
(C) It points downward and has constant magnitude.
(D) It points downward, but increases in magnitude over time.
(E) None of the above.

AAPT UNITED STATES PHYSICS TEAM
AIP 2024

2024 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout, unless otherwise specified.
- You may write in this question booklet and the scratch paper provided by the proctor.
- This test has 25 multiple choice questions. Select the best response to each question, and use a No.2 pencil to completely fill the box corresponding to your choice. If you change an answer, completely erase the previous mark. Only use the boxes numbered 1 through 25 on the answer sheet.
- All questions are equally weighted, but are not necessarily equally difficult.
- You will receive one point for each correct answer, and zero points for each incorrect or blank answer. There is no additional penalty for incorrect answers.
- You may use a hand-held calculator. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any external references, such as books or formula sheets.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **To maintain exam security, do not communicate any information about the questions or their solutions until after February 24, 2024.**

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Orhun Ciftcioglu, Kellan Colburn, Natalie LeBaron, Brian Skinner, Elena Yudovina and Kevin Zhou

1. An archer fires an arrow from the ground so that it passes through two hoops, which are both a height h above the ground. The arrow passes through the first hoop one second after the arrow is launched, and through the second hoop another second later. What is the value of h?

(A) 5 m
$\boxed{\text{B}}$ 10 m
(C) 12 m
(D) 15 m
(E) There is not enough information to decide.

The times t_1 and t_2 satisfy the equation $gt^2/2 - v_0 t + h = 0$, where v_0 is the initial upward velocity. Therefore, $t^2 - 2v_0 t/g + 2h/g = (t - t_1)(t - t_2)$, from which we read off $t_1 t_2 = 2h/g$. Plugging in the numbers gives gives $h = 10\,\text{m}$.

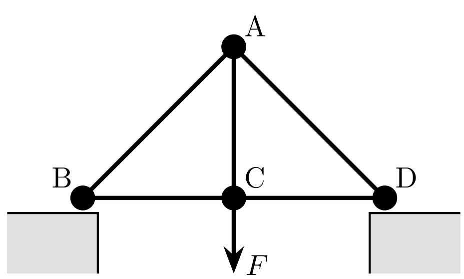
Alternatively, notice that if the arrow is at the same height at time $t_1 = 1\,\text{s}$ and $t_2 = 2\,\text{s}$, then it must have reached the top of its trajectory at time $t = 1.5\,\text{s}$, so it was launched with vertical velocity $15\,\text{m/s}$. In the first second, the average vertical velocity is therefore $(15 + 5)/2 = 10\,\text{m/s}$, giving $h = 10\,\text{m}$.

2. An amusement park ride consists of a circular, horizontal room. A rider leans against its frictionless outer walls, which are angled back at 30° with respect to the vertical, so that the rider's center of mass is 5.0 m from the center of the room. When the room begins to spin about its center, at what angular velocity will the rider's feet first lift off the floor?

$\boxed{\text{A}}$ 1.9 rad/s (B) 2.3 rad/s (C) 3.5 rad/s (D) 4.0 rad/s (E) 5.6 rad/s

When the rider's feet just lift off, the normal force from the floor vanishes, so the riders only experience a normal force from the walls. That normal force must both cancel the downward gravitational force mg and provide the inward centripetal force $mv^2/R = m\omega^2 R$. Thus, $\tan\theta = g/\omega^2 R$. Solving for ω using $\tan 30° = 1/\sqrt{3}$ gives $\omega = 1.86\,\text{rad/s}$.

3. A simple bridge is made of five thin rods rigidly connected at four vertices.



The ground is frictionless, so that it can only exert vertical normal forces at B and D. The weight of the bridge is negligible, but a person stands at its middle, exerting a downward force F at vertex C. In static equilibrium, each rod can be experiencing either tension or compression. Which of the following is true?

(A) Only the vertical rod is in tension.

(B) Only the horizontal rods are in tension.

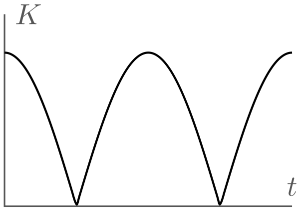
(C) Both the vertical rod and the diagonal rods are in tension.

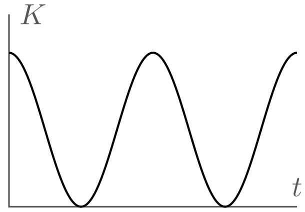
$\boxed{\text{D}}$ Both the vertical rod and the horizontal rods are in tension.

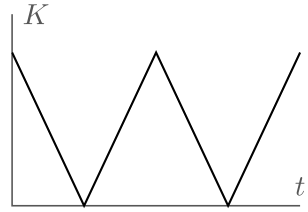
(E) All of the rods are in tension.

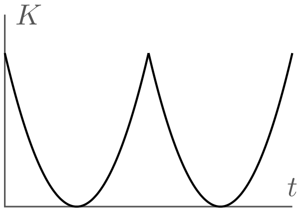
To balance vertical forces at C, the vertical rod has to be in tension. To balance vertical forces at A, the diagonal rods have to be in compression. And to balance horizontal forces at B and D, the horizontal rods have to be in tension.

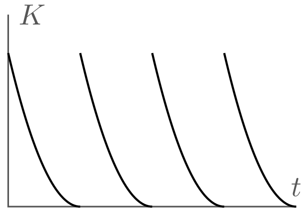
4. A bouncy ball is thrown vertically upward from the ground. Air resistance is negligible, and the ball's collisions with the ground are perfectly elastic. Which of the following shows the kinetic energy of the ball as a function of time? Assume the collisions are too quick for their duration to be seen in the plot.

(A)


(B)


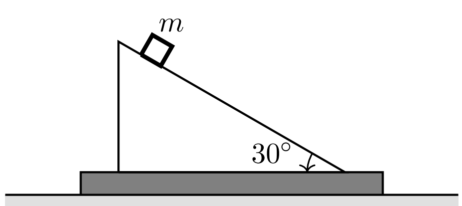
(C)


$\boxed{\text{D}}$


(E)


When the ball is in the air, its velocity is $v(t) = v_0 - gt$, so its kinetic energy is proportional to $v^2 = (v_0 - gt)^2$, which is a concave up parabola. When the ball bounces, its speed stays the same, so the kinetic energy stays the same and there's no discontinuity.

5. A massless inclined plane with angle 30° to the horizontal is fixed to a scale. A block of mass m is released from the top of the plane, which is frictionless.

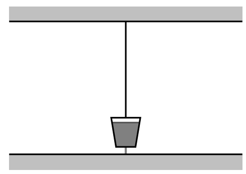


As the block slides down the plane, what is the reading on the scale?

(A) $\sqrt{3}\,mg/4$ (B) $mg/2$ $\boxed{\text{C}}$ $3mg/4$ (D) $\sqrt{3}\,mg/2$ (E) mg

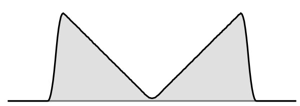
The scale reading is equal to the vertical component of the normal force of the block on the plane. The magnitude of the normal force is $mg\cos\theta$, and the vertical component is $\cos\theta$ times this, so the reading is $mg\cos^2(30°) = 3mg/4$.

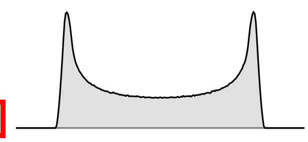
6. A pendulum is made with a string and a bucket full of water. When the string is vertical, the bottom of the bucket is near the ground.

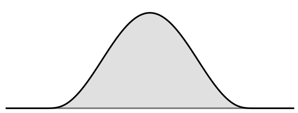


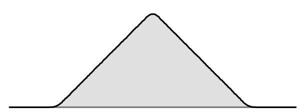
Then, the pendulum is set swinging with a small amplitude, and a very small hole is opened at the bottom of the bucket, which leaks water at a constant rate. After a few full swings, which of the following best shows the amount of water that has landed on the ground as a function of position?

(A) 

(B) 

$\boxed{\text{C}}$ 

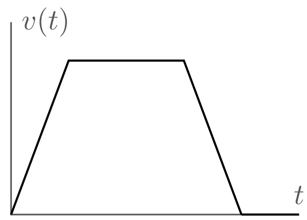
(D) 

(E) 

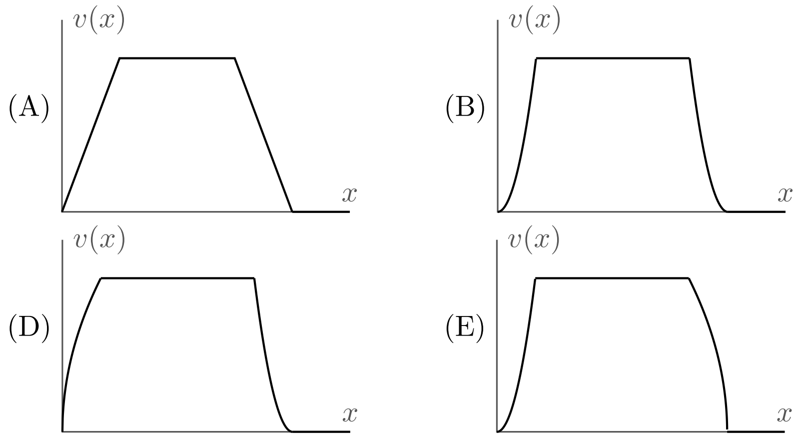
The amount of water at each location is proportional to how long the bucket spends there, so it is inversely proportional to the velocity. Therefore, it is the lowest (but nonzero) at the middle, where the bucket has maximum velocity, and highest at the endpoints, where the bucket is turning around.

Technically speaking, there are corrections to this due to the bucket's motion and the continuous draining of the water. For example, the horizontal velocity of the bucket will be imparted to the water, spreading out the water a bit more near the center. In addition, the water level in the bucket will decrease over time, changing the oscillation amplitude and the flow rate. However, such effects are negligible here because we explicitly assumed that the bucket moved with a small amplitude, and had a very small hole.

7. A particle travels in a straight line. Its velocity as a function of time is shown below.

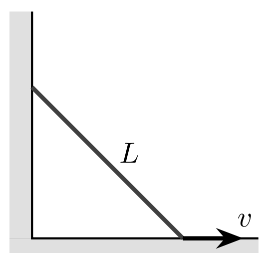


Which of the following shows the velocity as a function of distance x from its initial position?



The particle accelerates uniformly, then travels at a constant velocity for some time, then decelerates uniformly down to resting. As a function of distance, then, the velocity must also increase, then stay constant for some time, then decrease down to 0. Since velocity vs. time is symmetric, velocity vs. distance must be symmetric as well; if the movie were played backwards, it should look the same. When the particle is accelerating uniformly, its velocity is proportional to t, while the distance is proportional to t^2 (e.g. as in free fall): consequently, velocity is proportional to the square root of distance.

8. A rod of length L is sliding down a frictionless wall.



When the rod makes an angle of 45° to the horizontal, the bottom of the rod has speed v. At this moment, what is the speed of the middle of the rod?

(A) $v/2$ $\boxed{\text{B}}$ $v/\sqrt{2}$ (C) v (D) $\sqrt{2}\,v$ (E) $2v$

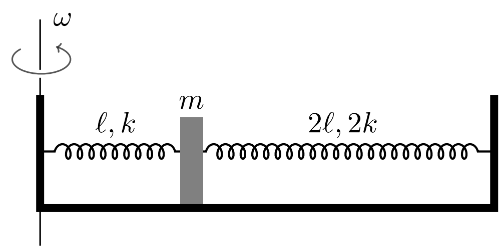
The bottom of the rod has rightward velocity v, so by symmetry, the top of the rod has downward velocity v. The velocity of the middle of the rod is the average of these velocities, and thus has magnitude $v/\sqrt{2}$.

9. When a car's brakes are fully engaged, it takes 100 m to stop on a dry road, which has coefficient of kinetic friction $\mu_k = 0.8$ with the tires. Now suppose only the first 50 m of the road is dry, and the rest is covered with ice, with $\mu_k = 0.2$. What total distance does the car need to stop?

(A) 150 m (B) 200 m $\boxed{\text{C}}$ 250 m (D) 400 m (E) 850 m

When the road is dry, the work done on the car by friction is equal to $\mu_d mgx$, where μ_d is the coefficient of friction with the dry road and $x = 100$ m is the braking distance. This work should be equal to the car's initial kinetic energy, $mv^2/2$. Equating them gives $v^2/2 = \mu_d gx$. When the car partially skids on ice, this same energy balance is $mv^2/2 = \mu_d gx/2 + \mu_i gy$, where μ_i is the coefficient of friction on ice and y is the portion of the braking distance that takes place on ice. Inserting the result for $v^2/2$ from the previous equation and solving for y gives $y = (x/2) \times (\mu_d/\mu_i) = 200$ m. So the total braking distance is $50 + 200 = 250$ m.

10. A block of mass m is connected to the walls of a frictionless box by two massless springs with relaxed lengths ℓ and 2ℓ, and spring constants k and $2k$ respectively. The length of the box is 3ℓ. The system rotates with a constant angular velocity ω about one of its walls.



Suppose the block stays at a constant distance r from the axis of rotation, without touching either of the walls. What is the value of r?

(A) $\dfrac{2k\ell}{2k - m\omega^2}$ (B) $\dfrac{2k\ell}{2k + m\omega^2}$ (C) $\dfrac{2k\ell}{3k + m\omega^2}$ $\boxed{\text{D}}$ $\dfrac{3k\ell}{3k - m\omega^2}$ (E) $\dfrac{3k\ell}{3k + m\omega^2}$

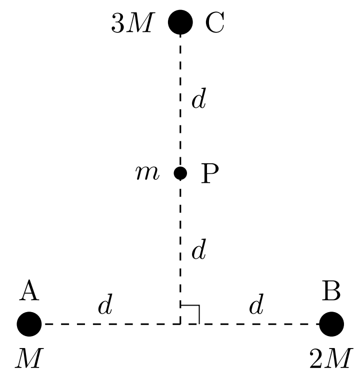
We balance forces on the block in the reference frame rotating with the box, to give $m\omega^2 r = k(r - \ell) + 2k(r - \ell)$. Solving for r gives the answer.

11. Two hemispherical shells can be pressed together to form a airtight sphere of radius 40 cm. Suppose the shells are pressed together at a high altitude, where the air pressure is half its value at sea level. The sphere is then returned to sea level, where the air pressure is 10^5 Pa. What force F, applied directly outward to each hemisphere, is required to pull them apart?

$\boxed{\text{A}}$ 25,000 N (B) 50,000 N (C) 100,000 N (D) 200,000 N (E) 400,000 N

The inward pressure force on each hemisphere is $\pi r^2 \Delta P$, where $r = 40$ cm and $\Delta P = (10^5 \text{ Pa})/2$. Plugging in the numbers gives a force of $25,000$ N.

12. A space probe with mass m at point P traverses through a cluster of three asteroids, at points A, B, and C. The masses and locations of the asteroids are shown below.



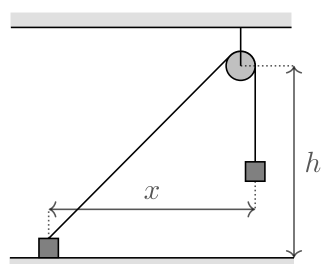
What is the torque on the probe about point C?

$\boxed{\text{A}}$ $\frac{1}{2\sqrt{2}}\frac{GMm}{d}$ (B) $\frac{1}{2}\frac{GMm}{d}$ (C) $\frac{1}{\sqrt{2}}\frac{GMm}{d}$ (D) $\frac{GMm}{d}$ (E) $\frac{\sqrt{2}\,GMm}{d}$

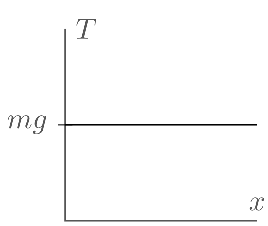
Only asteroids A and B contribute to the torque. Using the standard formula for torque, we have

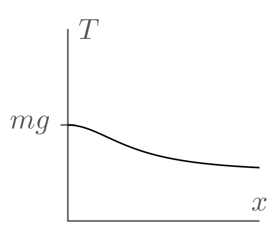
$$\tau = \frac{GMm}{(\sqrt{2}\,d)^2}\left(2\frac{d}{\sqrt{2}} - \frac{d}{\sqrt{2}}\right) = \frac{1}{2\sqrt{2}}\frac{GMm}{d}.$$

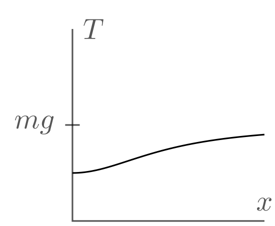
13. Two frictionless blocks of mass m are connected by a massless string which passes through a fixed massless pulley, which is a height h above the ground. Suppose the blocks are initially held with horizontal separation x, and the length of the string is chosen so that the right block hangs in the air as shown.

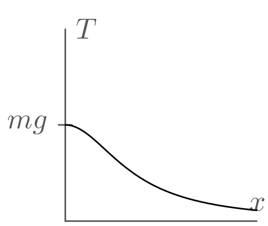


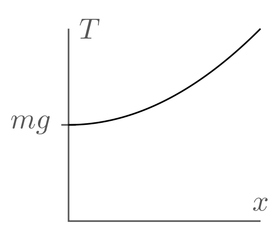
If the blocks are released, the tension in the string immediately afterward will be T. Which of the following shows a plot of T versus x?

(A)


$\boxed{\text{B}}$


(C)


(D)


(E)


This problem can be solved with limiting cases. When $x = 0$, this reduces to an Atwood's machine with equal masses. Both masses are static, so the tension $T = mg$. When $x \to \infty$, the string becomes horizontal, so equating the accelerations of the two masses gives

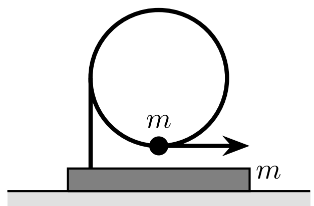
$$a = g - \frac{T}{m} = \frac{T}{m}$$

which yields $T = mg/2$. The only choice satisfying these limits is (B). Alternatively, with a little trigonometry, you can show that the tension in general is

$$T = \frac{h^2 + x^2}{h^2 + 2x^2} mg.$$

Note that in general, the tension also depends on the initial speeds of the blocks. In this problem, we have made the simplifying assumption that the initial speeds vanish.

14. A bead of mass m can slide frictionlessly on a vertical circular wire hoop of radius 20 cm.



The hoop is attached to a stand of mass m, which can slide frictionlessly on the ground. Initially, the bead is at the bottom of the hoop, the stand is at rest, and the bead has velocity 2 m/s to the right. At some point, the bead will stop moving with respect to the hoop. At that moment, through what angle along the hoop has the bead traveled?

(A) 30° (B) 45° $\boxed{\text{C}}$ 60° (D) 90° (E) 120°

When the bead stops moving with respect to the hoop, they must have the same velocity. By conservation of horizontal momentum, that velocity must be half the initial velocity. Thus, by conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}(2m)(v/2)^2 + mg\Delta h$$

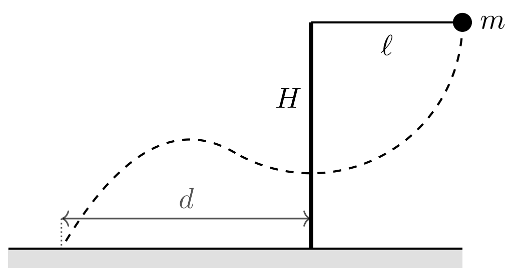
where Δh is the change in height of the bead. Solving gives $\Delta h = 10$ cm, which implies the bead slides by 60° along the hoop.

15. The viscous force between two plates of area A, with relative speed v and separation d, is $F = \eta Av/d$, where η is the viscosity. In fluid mechanics, the Ohnesorge number is a dimensionless number proportional to η which characterizes the importance of viscous forces, in a drop of fluid of density ρ, surface tension γ, and length scale ℓ. Which of the following could be the definition of the Ohnesorge number?

(A) $\dfrac{\eta\ell}{\sqrt{\rho\gamma}}$ (B) $\eta\ell\sqrt{\dfrac{\rho}{\gamma}}$ (C) $\eta\sqrt{\dfrac{\rho}{\gamma\ell}}$ (D) $\eta\sqrt{\dfrac{\rho\ell}{\gamma}}$ $\boxed{\text{E}}$ $\dfrac{\eta}{\sqrt{\rho\gamma\ell}}$

From the definition of viscosity, the units of η are kg/(m s). In addition, the units of ρ are kg/m^3 and the units of ℓ are m. As for the surface tension, it is a force per length, so its units are kg/s^2. We thus conclude that $\sqrt{\rho\gamma\ell}$ has the same units as η, so that choice (E) is the only possibility proportional to η that is properly dimensionless.

16. A child of mass m holds onto the end of a massless rope of length ℓ, which is attached to a pivot a height H above the ground. The child is released from rest when the rope is straight and horizontal.

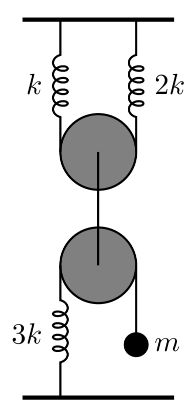


At some point, the child lets go of the rope, flies through the air, and lands on the ground a horizontal distance d from the pivot. On Earth, the maximum possible value of d is d_E. If the setup is moved to the Moon, which has 1/6 the gravitational acceleration, what is the new maximum possible value of d?

(A) $d_E/6$ (B) $d_E/\sqrt{6}$ $\boxed{\text{C}}$ d_E (D) $\sqrt{6}\,d_E$ (E) $6\,d_E$

This is most easily solved by dimensional analysis: d has to be some function of l, H, m, and g. Of these, g is the only quantity with seconds in it; since d doesn't have any seconds in it, the distance cannot depend on the value of g.

17. Consider the following system of massless and frictionless pulleys, ropes, and springs.



Initially, a block of mass m is attached to the end of a rope, and the system is in equilibrium. Next the block is doubled in mass, and the system is allowed to come to equilibrium again. During the transition between these equilibria, how far does the end of the rope (where the block is suspended) move?

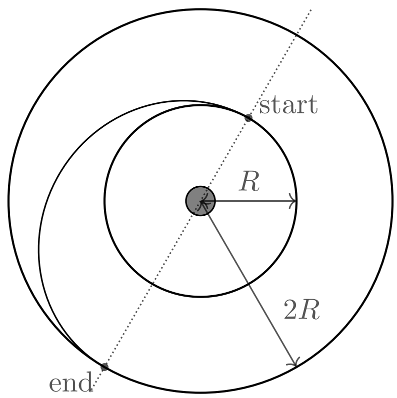
(A) $\frac{7}{12}\frac{mg}{k}$ (B) $\frac{11}{12}\frac{mg}{k}$ (C) $\frac{13}{12}\frac{mg}{k}$ (D) $\frac{7}{6}\frac{mg}{k}$ $\boxed{\text{E}}$ $\frac{11}{6}\frac{mg}{k}$

When the block is doubled in mass, it increases the tension in the bottom string by mg, so the third spring's length increases by $\Delta x_3 = mg/3k$. Since the bottom pulley has two ropes pulling down on it, the downward force on it increases by $2mg$, which means the upward force on the top pulley also has to increase by $2mg$, so the tension in the top string also increases by mg. Then the first and second springs increases in length by $\Delta x_1 = mg/k$ and $\Delta x_2 = mg/2k$.

Finally, by considering how changes in length of each spring affect the position of the mass, we conclude

$$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 = \frac{11mg}{6k}.$$

18. A satellite is initially in a circular orbit of radius R around a planet of mass M. It fires its rockets to instantaneously increase its speed by Δv, keeping the direction of its velocity the same, so that it enters an elliptical orbit whose maximum distance from the planet is $2R$.



What is the value of Δv? (Hint: when the satellite is in an elliptical orbit with semimajor axis a, its total energy per unit mass is $-GM/2a$.)

(A) $0.08\sqrt{\frac{GM}{R}}$ $\boxed{\text{B}}$ $0.15\sqrt{\frac{GM}{R}}$ (C) $0.22\sqrt{\frac{GM}{R}}$ (D) $0.29\sqrt{\frac{GM}{R}}$ (E) $0.41\sqrt{\frac{GM}{R}}$

The initial speed in the circular orbit is $v_i = \sqrt{GM/R}$. In the elliptical orbit, the semimajor axis is $a = 3R/2$, so the total energy per satellite mass is

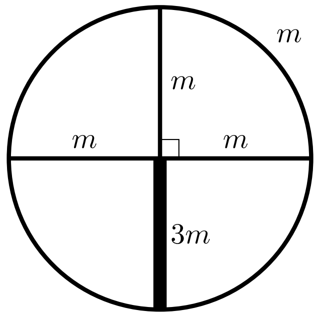
$$-\frac{GM}{3R} = -\frac{GM}{R} + \frac{1}{2}v_f^2$$

which implies

$$v_f^2 = \frac{4}{3}\frac{GM}{R}.$$

Computing $v_f - v_i$ gives the answer.

19. A wheel of radius R has a thin rim and four spokes, each of which have uniform density.



The entire rim has mass m, three of the spokes each have mass m, and the fourth spoke has mass $3m$. The wheel is suspended on a horizontal frictionless axle passing through its center. If the wheel is slightly rotated from its equilibrium position, what is the angular frequency of small oscillations?

$\boxed{\text{A}}$ $\sqrt{\frac{g}{3R}}$ (B) $\sqrt{\frac{g}{2R}}$ (C) $\sqrt{\frac{2g}{3R}}$ (D) $\sqrt{\frac{g}{R}}$ (E) $\sqrt{\frac{7g}{6R}}$

This system is a physical pendulum. The moment of inertia about the center is

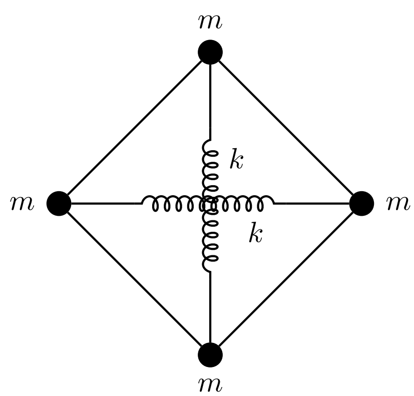
$$I = mR^2 + \frac{1}{3}(m + m + m + 3m)R^2 = 3mR^2.$$

When the system is rotated a small angle θ from equilibrium, the restoring torque comes from the difference in masses of the top and bottom spokes, so

$$\tau = -(3m - m)g\frac{R}{2}\sin\theta \approx -mgR\theta.$$

As a result, we have $\alpha = -(g/3R)\theta$ from which we read off $\omega = \sqrt{g/3R}$.

20. Four massless rigid rods are connected into a quadrilateral by four hinges. The hinges have mass m, and allow the rods to freely rotate. A spring of spring constant k is connected across each of the diagonals, so that the springs are at their relaxed length when the rods form a square.



Assume the springs do not interfere with each other. If the square is slightly compressed along one of its diagonals, its shape will oscillate over time. What is the period of these oscillations?

(A) $2\pi\sqrt{\frac{m}{4k}}$ $\boxed{\text{B}}$ $2\pi\sqrt{\frac{m}{2k}}$ (C) $2\pi\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{2m}{k}}$ (E) $2\pi\sqrt{\frac{4m}{k}}$

Suppose that one of the masses has moved a distance x from its original position, and it currently has speed v. By symmetry, all of the other masses have also moved a distance x, and also have a speed v, so the total kinetic energy is $K = (4m)v^2/2$. Each spring is either stretched or compressed by a distance $2x$, so the total elastic potential energy is $U = (8k)x^2/2$. Thus, the kinetic and potential energy are the same as that of a system of mass $m_{\text{eff}} = 4m$ on a spring of spring constant $k_{\text{eff}} = 8k$, so the angular frequency is $\sqrt{k_{\text{eff}}/m_{\text{eff}}} = \sqrt{2k/m}$.

21. A syringe is filled with water of density ρ and negligible viscosity. Its body is a cylinder of cross-sectional area A_1, which gradually tapers into a needle with cross-sectional area $A_2 \ll A_1$. The syringe is held in place and its end is slowly pushed inward by a force F, so that it moves with constant speed v. Water shoots straight out of the needle's tip. What is the approximate value of F?

(A) $\rho v^2 A_1$ (B) $\dfrac{\rho v^2 A_1^2}{2A_2}$ (C) $\dfrac{\rho v^2 A_1^2}{A_2}$ $\boxed{\text{D}}$ $\dfrac{\rho v^2 A_1^3}{2A_2^2}$ (E) $\dfrac{\rho v^2 A_1^3}{A_2^2}$

If the water has speed v' when it leaves the needle tip, then $A_1 v = A_2 v'$. The water outside the syringe has atmospheric pressure, so by Bernoulli's principle, the pressure P of the water inside the syringe obeys

$$P - P_{\text{atm}} = \frac{1}{2}\rho(v'^2 - v^2) \approx \frac{1}{2}\rho v'^2 = \frac{\rho v^2 A_1^2}{2A_2^2}.$$

The required force is the area times the pressure difference.

Incidentally, if you don't know Bernoulli's principle, you can also solve the problem by equating the power Fv with the rate of change of kinetic energy of the water. However, if you try to equate the force F to the rate of change of momentum of the water, you'll get $F = \rho v^2 A_1^2/A_2$ which is incorrect. The reason this method is wrong is that it doesn't account for the other forces needed to hold the syringe in place.

22. A spherical shell is made from a thin sheet of material with a mass per area of σ. Consider two points, P_1 and P_2, which are close to each other, but just inside and outside the sphere, respectively. If the accelerations due to gravity at these points are $\mathbf{g}_1$ and $\mathbf{g}_2$, respectively, what is the value of $|\mathbf{g}_1 - \mathbf{g}_2|$?

(A) $\pi G\sigma$ (B) $4\pi G\sigma/3$ (C) $2\pi G\sigma$ $\boxed{\text{D}}$ $4\pi G\sigma$ (E) $8\pi G\sigma$

Suppose the shell has radius R. By the shell theorem, the gravitational field vanishes inside, so $\mathbf{g}_2 = 0$. The gravitational field just outside has magnitude $g_1 = GM/R^2$ where $M = 4\pi\sigma R^2$ is the total mass of the shell. Thus, the unknown variable R cancels out, and the answer is $4\pi G\sigma$.

Note that this is actually a universal statement. Above, we found that it didn't depend on the sphere's radius, but more generally, it is true regardless of the shell's shape. The reason is that $|\mathbf{g}_1 - \mathbf{g}_2|$ is solely determined by the mass near P_1 and P_2, and if one zooms in enough to a patch of any surface, then it always looks like an infinite plane, so the answer doesn't depend on the shape of the surface away from that patch.

23. Collisions between ping pong balls and paddles are not perfectly elastic. Suppose that if a player holds a paddle still and drops a ball on top of it from any height h, it will bounce back up to height $h/2$. To keep the ball bouncing steadily, the player moves the paddle up and down, so that it is moving upward with speed 1.0 m/s whenever the ball hits it. What is the height to which the ball is bouncing?

(A) 0.21 m

(B) 0.45 m

(C) 1.0 m

$\boxed{\text{D}}$ 1.7 m

(E) There is not enough information to determine the height.

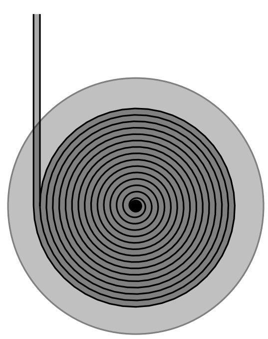
In the reference frame of the paddle, the ball loses half its energy during the collision, so the speed of the ball immediately after the collision is $1/\sqrt{2}$ of its speed immediately before the collision. Let v be the speed of the ball relative to the ground immediately before the collision, then we have $(v - 1\,\mathrm{m/s}) = (v + 1\,\mathrm{m/s})/\sqrt{2}$, or $v = 5.83\,\mathrm{m/s}$. The height of the bouncing is $v^2/(2g) = 1.7\,\mathrm{m}$.

24. When a projectile falls through a fluid, it experiences a drag force proportional to the product of its cross-sectional area, the fluid density ρ_f, and the square of its speed. Suppose a sphere of density $\rho_s \gg \rho_f$ of radius R is dropped in the fluid from rest. When the projectile has reached half of its terminal velocity, which of the following is its displacement proportional to?

(A) $R\sqrt{\rho_s/\rho_f}$ $\boxed{\text{B}}$ $R\,\rho_s/\rho_f$ (C) $R\,(\rho_s/\rho_f)^{3/2}$ (D) $R\,(\rho_s/\rho_f)^2$ (E) $R\,(\rho_s/\rho_f)^3$

If the ball falls a distance d, then its speed in the absence of drag would be $v \sim \sqrt{gd}$. Thus, if the drag force has not had a large effect yet, then its magnitude would be $F_d \sim \rho_f A v^2 \sim \rho_f R^2 g d$, while the gravitational force has magnitude $F_g \sim \rho_s R^3 g$. The object begins to approach terminal velocity when these are comparable, $\rho_f R^2 g d \sim \rho_s R^3 g$, which implies $d \sim (\rho_s/\rho_f)R$.

25. A yo-yo consists of two massive uniform disks of radius R connected by a thin axle. A thick string is wrapped many times around the axle, so that the end of the string is initially a distance R from the axle. Then, the end of the string is held in place and the yo-yo is dropped from rest. Assume that energy losses are negligible, and that the string has negligible mass and always remains vertical. Below, we show a cross-section of the yo-yo partway through its descent.



Between the moment the yo-yo is released and the moment the string completely unwinds, which of the following is true regarding the yo-yo's acceleration?

(A) It is always zero.

(B) It points downward, but decreases in magnitude over time.

(C) It points downward and has constant magnitude.

(D) It points downward, but increases in magnitude over time.

$\boxed{\text{E}}$ None of the above.

As the string unwinds, the radius r of the string remaining about the axle decreases. Specifically, if the yo-yo has fallen a distance z, and the string has total length ℓ, then

$$\frac{r^2}{R^2} = \frac{\ell - z}{\ell}.$$

This change in r affects how the energy of the yo-yo is divided between translational and rotational motion. If the discs have total mass m, conservation of energy gives

$$mgz = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

where $I = mR^2/2$ but $\omega = v/r$. Solving for v gives

$$v = \sqrt{\frac{2gz}{1 + \frac{\ell}{2(\ell - z)}}}$$

which has a maximum at around $z = 0.6\ell$. Intuitively, in the beginning the yo-yo has roughly constant acceleration downward, but as the string unwinds, more of the energy goes into the yo-yo's rotational motion. In fact, the translational speed of the yo-yo goes to zero when the string completely unwinds, at which point it's just spinning in place! Therefore, during the latter part of its fall, the yo-yo has *upward* acceleration, so the answer is "none of the above".

This remains true regardless of the details of the yo-yo, because it only depends on the fact that the radius r starts equal to R and then becomes very small, since the axle is thin. In particular, changing how r depends on z, or the moment of inertia of the disc, won't change the answer.