Hong Kong Physics Olympiad 2003
2003 香港物理奧林匹克

Education and Manpower Bureau 教育統籌局
Physical Society of Hong Kong 香港物理學會
Hong Kong University of Science and Technology 香港科技大學

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25 May 2003

Multiple-choice Questions 選擇題

Multiple-choice Questions
(2 points for each question, 20 questions in total)
選擇題 (共 20 題，每題 2 分)

The following constants may be useful 可用物理常數：

Specific latent heat of fusion of ice (冰的溶解比潛熱) = 334 KJ/kg
Specific heat capacity of water (水的比熱容量) = 4.2 KJ/kg/K

MC1

Ball A was dropped from the top of a tall building. At the same instant and from the same height ball B was thrown straight downward. Neglecting the effects of air friction, compare their accelerations while they were falling.

A. Their accelerations are equal.
B. Ball A has the greater acceleration.
C. Ball B has the greater acceleration.
D. It is impossible to tell since their accelerations vary greatly.
E. None of the above.

選擇題 1

有一球 A 從一高建築物的頂部落下，與此同時在同一高度另一球 B 被垂直扔下。若不考慮空氣摩擦力的影響，試比較兩物體在下落過程中的加速度？

A. 相等
B. A 球有更大的加速度
C. B 球有更大的加速度
D. 不可能比較兩者的加速度, 因爲兩者的加速度在下落過程中變化很大
E. 以上描述均不正確

MC2

Two objects, A and B, accelerated from rest at the same uniform rate. Object B accelerated for a distance twice as long distance A. Compare to Object A, Object B was moving

A. Twice as fast.
B. 1.414 times as fast
C. Three times as fast.
D. Four times as fast.
E. None of the above.

選擇題 2

兩個物體 A 和 B 由靜止以同樣的加速率開始加速。若 B 加速的距離是 A 的兩倍，那麽與 A 相比，B 的運動速度：

A. 是 A 速率的 2 倍
B. 是 A 速率的 1.414 倍
C. 是 A 速率的 3 倍
D. 是 A 速率的 4 倍
E. 以上描述均不正確

MC3

Five people measured the distance between 2 points by the same ruler with the following results: (1) 5.0cm, (2) 5.2cm, (3) 4.9cm, (4) 5.0cm, (5) 5.5cm. Which of the following is true?

A. We know *for certain* that the distance is between 4.9cm and 5.5cm.
B. We know *for certain* that the distance is 5.12cm.
C. We know *for certain* that the distance is between 4.92 to 5.34cm.
D. The best estimate of the distance is 5.12cm.
E. None of the above.

選擇題 3

五個人以同一把尺量度兩點之間距離的結果是：(1)5.0 公分　(2)5.2 公分　(3)4.9 公分　(4)5.0 公分　(5)5.5 公分。以下哪一項是正確的？

A. 我們確切的知道距離是於 4.9 公分及 5.5 公分之間
B. 我們確切的知道距離是 5.12 公分
C. 我們確切的知道距離是於 4.92 公分至 5.34 公分之間
D. 最佳的估計距離為 5.12 公分
E. 以上均不正確

MC4

Which of the following statements is correct?

A. When you are not moving there is no force exerting on you.
B. When there is no force exerting on you, you cannot move.
C. When you are not moving there is no *net* force exerting on you.
D. There must always be force acting on you when you move.
E. None of the above.

選擇題 4

下列哪一項是正確的？

A. 當你靜止時，不會有任何力施加在你身上
B. 當沒有力施加於你身上時，你便不能移動
C. 當你靜止時，不會有淨力施加於你身上
D. 當你移動時，一定要有力施加於你身上
E. 以上均不正確

MC5

A desk and a book are resting on the ground. Which of the following changes when the book is put on the desk?

A. The *net* force acting on the desk.
B. The gravitational force acting on the desk.
C. The reaction force from the ground acting on the desk.
D. The kinetic energy of the book.
E. The kinetic energy of the desk.

選擇題 5

地上分別有一張書桌和一本書。如把書放到書桌上，以下哪項會有轉變？

A. 施加到書桌的淨力
B. 施加到書桌的地心吸力
C. 施加到書桌的地面反作用力
D. 書本的動能
E. 書桌的動能

MC6

A large negatively charged object was placed on an insulated table. A neutral metallic ball rolled straight towards the object but stopped before touching it. A second neutral metallic ball rolled along the same path as the first ball, struck the first ball driving it a bit closer to the negatively charged object and stopped. After all balls stopped rolling, the first ball was closer to the negatively charged object than the second ball. At no time did either ball touch the charged object. Which statement is correct concerning the final charge on each ball?

A. The first ball is positive and the second negative.
B. The first ball is negative and the second positive.
C. Both balls remain neutral.
D. Both balls are positive.
E. None of the above.

選擇題 6

一絕緣的桌子上有一帶負電荷的大物體。一不帶電的金屬球沿著直線向這物體滾過來，但在兩者接觸之前停了下來。此時有第二個不帶電的金屬球也以相同的路徑滾過來，並和第一個金屬球發生碰撞，從而把第一個金屬球又往前推進了一點並停下來，而第一個金屬球較第二個金屬球接近帶負電荷的物體。這兩個金屬球從未和帶電物體接觸。以下哪項描述是正確的：

A. 第一個球帶正電，而第二個球帶負電
B. 第一個球帶負電，而第二個球帶正電
C. 兩球都不帶電
D. 兩球都帶正電
E. 以上描述均不正確

MC7

The lamps in a string of Christmas tree lights are connected in parallel. What happens if one lamp burns out? Assume negligible resistance in the wires leading to lamps.

A. The brightness of the other lamps will not change appreciably.
B. The other lamps get brighter equally.
C. The other lamps get brighter, but some get brighter than others.
D. The other lamps get dimmer equally.
E. The other lamps get dimmer, but some get dimmer than others.

選擇題 7

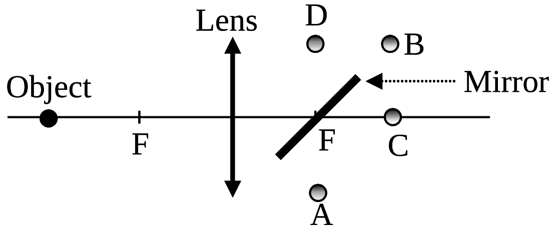
聖誕樹上有一串燈泡被並聯在一起。若其中有一燈泡燒壞了，試問以下哪個描述是正確的：(不考慮電線的電阻)

A. 其他燈泡的亮度不會有明顯變化
B. 其他燈泡都會同等程度地變亮
C. 其他燈泡都會變亮，但變亮的程度不一樣
D. 其他燈泡都會同等程度地變暗
E. 其他燈泡都會變暗，但變暗的程度不一樣

MC8

As shown below, an object is placed on the left side of a lens with its focal points labeled as ‘F’. A flat mirror is placed at the right side focal point, with its reflecting surface facing the lens. The image of the object is at

A. Point A
B. Point B
C. Point C
D. Point D
E. Point F on the right side



選擇題 8

如圖示，有一物件放在一透鏡的左邊，F 爲透鏡的焦點。在透鏡的右邊焦點處放了一平面鏡。平面鏡的反射面向著透鏡。物件的影像會落在哪處？

A. A 點
B. B 點
C. C 點
D. D 點
E. 右邊焦點處

MC9

Which of the following setups can make a small conductor coil that is placed horizontally float on top of an object?

A. The object is a magnet that provides a magnetic field pointing straight downward, while the coil carries an electric current flowing clockwise as viewed from top down.
B. The object is a magnet that provides a magnetic field pointing straight downward, while the coil carries an electric current flowing counterclockwise as viewed from top down.
C. The object carries positive electric charge while the coil carries negative charge.
D. The object carries negative electric charge while the coil carries positive charge.
E. None of the above.

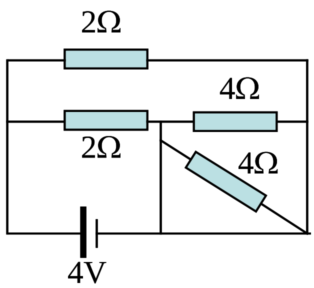
選擇題 9

以下哪一個情況能導致一個水平平放的小導體線圈懸浮於另一物體上？

A. 物體是一塊磁場垂直向下的磁鐵，而線圈帶有順時針方向(從上方觀察)流動的電流
B. 物體是一塊磁場垂直向下的磁鐵，而線圈帶有反時針方向(從上方觀察)流動的電流
C. 物體攜帶正電荷而線圈攜帶負電荷
D. 物體攜帶負電荷而線圈攜帶正電荷
E. 以上均不正確

MC10

In the following circuit, the current through the battery is



A. 4 A
B. 3 A
C. 2 A
D. 1 A
E. None of the above

選擇題 10

在電路圖中，通過電池的電流是

A. 4 安培
B. 3 安培
C. 2 安培
D. 1 安培
E. 以上均不是

MC11

Car A ran the red light and hit Car B which had the right of way at the intersection. Car A was more severely damaged than Car B, but fortunately no one was injured. You are to investigate the accident and decide which of the following statements is correct.

A. Driver of Car A was at fault because Car A collided with Car B before Car B collided with Car A.
B. Driver of Car A was at fault even though both cars collided at the same moment.
C. Both drivers were at fault because both cars collided at the same moment.
D. Driver of Car B was at fault because Car B exerted more force on Car A, and as a result, Car A was damaged more.
E. Driver of Car A was at fault because being the initiator Car A exerted more force on Car B.

選擇題 11

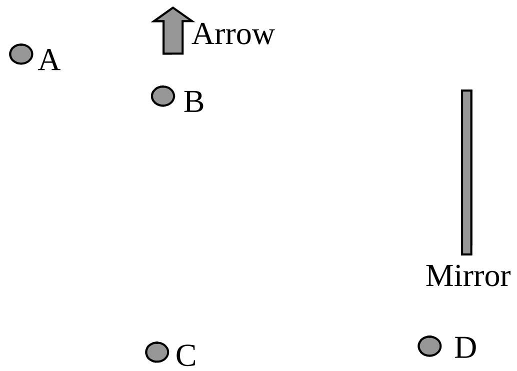
正在十字路口行駛中的車輛乙被衝過紅色交通燈的車輛甲撞到。車輛甲較車輛乙損毀嚴重，但幸虧沒有人受傷。你現在要調查此交通意外及判斷以下哪種是正確的說法：

A. 犯錯的是車輛甲的司機，因為車輛甲碰撞車輛乙的時間早於車輛乙碰撞車輛甲的時間
B. 儘管兩架車輛於同一時刻相撞，犯錯的還是車輛甲的司機
C. 兩位司機都有犯錯，因為兩架車輛於同一時刻相撞
D. 犯錯的是司機乙，因為車輛乙施加較多力在車輛甲上，所以車輛甲的損毀較嚴重
E. 犯錯的是司機甲，因為引致此意外的是車輛甲，所以車輛甲施加較多力在車輛乙上

MC12

From which point indicated here, if any, should you position your eyesight if you wish to see an image of the arrow in the mirror?

A. Point A
B. Point B
C. Point C
D. Point D
E. None of the above



選擇題 12

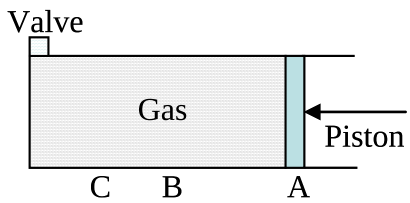
從以下哪一點可以觀看到箭頭在鏡中所成的像？

A. A 點
B. B 點
C. C 點
D. D 點
E. 均看不到

MC13

As shown below, a sealed piston was slowly moving to the left, compressing the (ideal) gas trapped in the chamber. A pressure relieve valve was connected to the chamber. It would let the gas out whenever the gas pressure in the chamber exceeded 3 atm. When the piston was at position A, the chamber volume was V and the gas pressure inside the chamber was 1 atm, same as the air pressure outside the chamber. When the piston reached position B, the chamber volume was reduced to V/2; and at position C, the volume was reduced to V/4. At position B and position C, the gas pressure in the chamber was ___ atm and ____ atm, respectively. The gas temperature remained unchanged throughout the process.

A. 2, 3
B. 2, 4
C. 4, 2
D. 3, 2
E. 2, 0



選擇題 13

如圖示，一個密封室的活塞慢慢向左推進，壓縮著裏面一些理想氣體。有一個“壓力釋放閥門”連接著這個密封室。當室內的壓力高過 3 個大氣壓力時，閥門便會打開及排出氣體。當活塞在位置 A 時，室的體積是 V，室內的氣壓是一個大氣壓力，相等於室外的氣壓。當活塞在位置 B 及 C 時，室的體積分別是 V/2 及 V/4。那麼在位置 B 及 C，室內氣壓分別是 __大氣壓力及 _大氣壓力。氣體的溫度於整個過程中維持不變。

A. 2,3
B. 2,4
C. 4,2
D. 3,2
E. 2,0

MC14

1 kg of ice at 0 °C is mixed with 10 kg of water at 5 °C. After thermal equilibrium the water temperature is

A. -3 °C.
B. 0 °C.
C. -5 °C.
D. 5 °C.
E. -8 °C.

選擇題 14

將一千克 0℃的冰塊與十千克 5℃的清水混和。到達熱平衡後，清水的溫度是 _。

A. –3℃
B. 0℃
C. –5℃
D. 5℃
E. -8 °C

MC15

In Newtonian mechanics the upper limit of speed any object could reach under constant action of a force F is ___. In Relativistic mechanics, however, the mass of an object is no longer a constant, but changes with the speed v of the object in the following way: $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$. Here c is the speed of light in vacuum, and m_0 is the mass when the object is at rest. Therefore, the maximum speed the object under constant action of force could reach is ___.

A. ∞, ∞
B. F/m, F/m_0
C. F/ m_0, F/m
D. c, ∞
E. ∞, c

選擇題 15

根據牛頓力學，當物件在受到恆常力(F)作用時，它的速度上限是____。但在相對論力學中，根據以下公式：$m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$ 物件的質量會隨著它的速度 v 而改變。註：c 是光在真空的速度；m_0 是物件靜止時的質量。因此，當物件在受到恆常力作用時，它的速度上限是 ________ 。

A. ∞, ∞
B. F/m, F/m_0
C. F/ m_0, F/m
D. c, ∞
E. ∞, c

MC16

Which of the following is *impossible* in electromagnetism?

A. Breaking a magnet in half, with one piece having only north pole and the other having only south pole.
B. Breaking a charged conductor in half with one piece carrying positive charge and the other carrying negative charge.
C. Detecting static electric charges with a compass.
D. Detecting steady electric currents with a compass.
E. Detecting static magnetic field with a voltmeter.

選擇題 16

根據電磁學，以下哪項是不可能做到的？

A. 將一磁鐵分成兩塊，其中一塊只有北極，另一塊則只有南極
B. 將一帶電導體分成兩塊，其中一塊帶正電，另一塊則帶負電
C. 利用指南針探測靜止的電荷
D. 利用指南針探測穩定的電流
E. 利用伏特計探測穩定的磁場

MC17

Which of the following describes what we now believe about our universe?

A. Our universe is expanding.
B. Our universe is contracting.
C. The size of our universe is infinite.
D. Newton predicted the expansion of our universe.
E. Newton predicted the contraction of our universe.

選擇題 17

以下哪一項正確描述我們目前對宇宙的看法？

A. 我們的宇宙正在擴張中
B. 我們的宇宙正在收縮中
C. 宇宙是無限大的
D. 牛頓預言到宇宙的擴張
E. 牛頓預言到宇宙的收縮

<u>MC18</u>

Which of the following describes most closely the color of the sky on the moon?

A. Blue
B. Red
C. White
D. Dark
E. Yellow

選擇題 18

以下哪項最能貼切形容月亮的天空顏色？

A. 藍色
B. 紅色
C. 白色
D. 黑暗
E. 黃色

<u>MC19</u>

Which of the following processes does not involve the transformation of energy from one form to another?

A. You lift up a cup of water.
B. You turn on an air-conditioner.
C. Plants grow under sunlight.
D. You light up a match.
E. A planet moves around a star in a circular orbit.

選擇題 19

以下哪一個過程沒有能量轉換？

A. 提起一杯水
B. 啓動空調
C. 陽光下植物的生長
D. 燃點一根火柴
E. 行星圍繞恆星循圓形軌道運行

<u>MC20</u>

Which of the following describes correctly the difference between microwave and radio wave?

A. One of them is an electromagnetic wave, while the other is a sound wave.
B. Microwave travels faster in vacuum than radio wave.
C. Microwave carries energy but radio wave does not.
D. Water molecules absorb energy from microwave but not from radio wave.
E. Radio waves can travel longer distance than microwave in vacuum.

選擇題 20

以下哪一項正確地形容微波及無線電波的分別？

A. 一個是電磁波，另一個是聲波。
B. 在真空中，微波行走速度比無線電波快。
C. 微波帶有能量而無線電波沒有。
D. 水分子能從微波中吸收能量，卻不能從無線電波中吸取。
E. 在真空中，無線電波能行走的距離比微波遠。

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25 May 2003

Open-ended Questions 開放題

Open-ended Questions (5 questions in total)
開放題 (共 5 題)

Short Questions 短題目 (10 points each 每題 10 分)

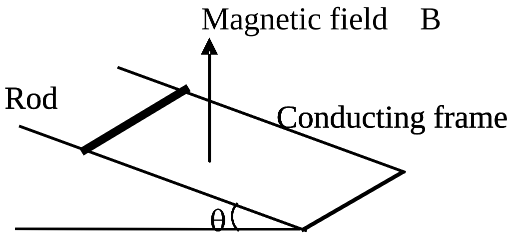
Q.1

As shown in the figure, a conducting slope frame with inclining angle θ is placed in a uniform magnetic field B pointing upwards. A conducting rod of length L, mass M, resistance R, is moving down the frame at steady speed v. (Hint: the e.m.f. of a rod moving in magnetic field is BLv_p, where v_p is the velocity component perpendicular to the field. If the rod carries electric current 'I' which is flowing in a direction perpendicular to the field, then the magnetic force on the rod is BLI.)

(a) Find v in terms of θ, L, M, R, B, and any other physics constants you think appropriate. (5 points)

(b) If the direction of B is reversed, what will be the direction of the steady velocity when the rod is released from rest? (2 points)

(c) If R = ∞, describe the motion of the rod after it is released from rest. (3 points)



題 1

如圖所示，一個導電金屬框架以傾角 θ 放在一個向上的均勻磁場 B 中。一個長短爲 L，質量爲 M，電阻爲 R 的棒形導體以勻速 v 從框架上滑下。(提示：在磁場中運動的棒的電動勢爲 BLv_p，其中 v_p 爲垂直於磁場的速度分量。如果導體棒中的電流是 I 並且垂直於磁場，磁場的力則爲 BLI)

(a) 用 θ，L，M，R，B 和其他適當的物理常數，寫出 v 的表示式 （5 分）

(b) 如果磁場 B 的方向倒轉，求該棒從靜止狀態被釋放後勻速滑動的方向？ （2 分）

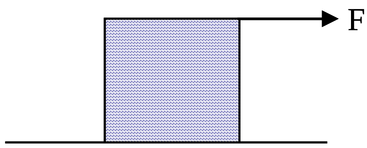
(c) 如果 R = ∞ ，描述棒形導體從靜止狀態被釋放後的運動 （3 分）

Q.2

Suppose you are to move a piece of heavy furniture at home. Let us approximate it by a cubic block of uniform mass M on a rough floor surface with friction coefficient μ=1. Your strength can only provide a force F = 0.8Mg, where g is the gravity acceleration, so you cannot simply lift the block and move it around.

(a) If you apply the force as shown in the figure, what will happen to the block? (3 points)

(b) You are free to apply the force F in any direction to any point of the block, how to apply the force so the block will have maximum acceleration in sliding motion but without tipping over? (7 points)



[There may be many ways and you are only required to find one. You may also need this: $\sin\theta + \cos\theta = 1.414\sin(\theta + 45^\circ)$]

Hint: The magnitude of the friction is the smaller of the two: F or μN, where N is the reaction force of the floor perpendicular to the surface. The effect of the friction is to keep the block from sliding until it reaches its maximum value μN.

題 2

假定你在家裏移動一件很重的家具。我們可把它近似看成一個質量爲 M 的均勻立方塊，它放在摩擦係數爲 1 的粗糙地面上。而你能提供的最大施力爲 F = 0.8Mg，g 爲重力加速度，因此你不能把該家具提起隨意移動。

(a) 如果你施加一個如圖所示的力，這件家具將如何運動？（3 分）

(b) 你可以在家具的任何位置施加任何方向的力 F。如何施以力 F 才能獲得最大的滑動加速度而不會翻倒？（7 分）

【方法可以很多，但你只需展示其中一種解決方法。你可能會用到下列等式：$\sin\theta + \cos\theta = 1.414\sin(\theta + 45^\circ)$】

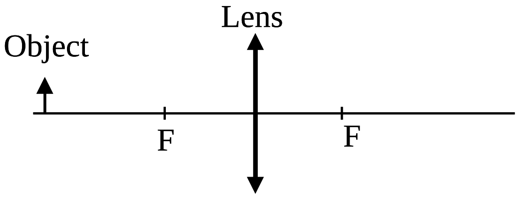
提示：摩擦力的量是力 F 和 μN 兩者中較小者，N 是地面對家具的垂直地面反作用力。摩擦力的作用是阻止家具滑動，直至達到最大值 μN 爲止。

Q.3

(a) Complete the drawing below to show where the image of the object formed by the lens is. (3 points)

(b) Use the drawing you have completed, prove that $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. Here *s* is the distance between the object and the lens, *s'* is the distance between the image and the lens, and *f* is the distance between either of the focal points F and the lens. (5 points)

(c) Prove that the image magnification is $M = s'/s$. (2 points)



題 3

(a) 在圖中畫出光線通過透鏡折射後物體所成的影像。 (3 分)

(b) 根據你畫出的影像，證明$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$。***s*** 是物體和透鏡之間的距離，***s'*** 是影像和透鏡之間的距離，***f*** 是焦點 F 到透鏡之間的距離。 (5 分)

(c) 證明影像的放大倍數是 $M = s'/s$ 。 (2 分)

Long Questions 長題目 (15 points each 每題 15 分)

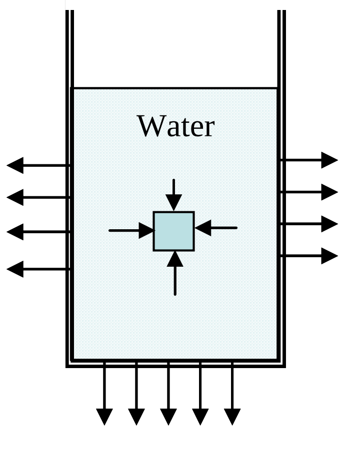
Q.4

Stationary liquid on Earth exerts pressure on any surface in contact, and the resulting net force is always perpendicular to the surface and pointing away from liquid. For example, as shown by the arrows in the figure, the pressure force of water on the sidewalls of the container is horizontal, and the force on the bottom of the container is straight downwards. The same is applicable to any object in liquid. The pressure force of water on the bottom surface of the cubic object shown in the figure is straight upwards; the force on the top surface of the cubic object is straight downwards; and the force on the side surfaces of the cubic object is horizontal and pointing towards the cubic object. (The lengthes of the arrows have no relevance to the strength of the forces.)

(a) Use the above concept, derive an expression for the pressure at any point in water in terms of the depth h of the point (the distance from the point to the water surface in contact with air), mass density of water ρ, and gravity acceleration g on Earth surface. (6 points) (1 point will be deducted in exchange for hint)

(b) Use the results in (a), prove that the net force of water on the cubic object is equal to ρVg and pointing straight upwards, where V is the volume of the cubic object. (5 points)

(c) If the container is placed in a weightless outer space environment, what is the pressure of water on the bottom of the container? (1 point)



(d) We now replace the water and the cubic object with some ideal gas and seal the container. When it is in a weightless outer space environment, is the pressure of the gas on the container the same as it is on Earth? Explain your answer. (3 points)

題 4

地球上靜態液體會向任何接觸面施加壓強，從而產生的合力總是垂直於液面並朝外。例如：圖中箭頭所示，水對容器側壁的壓力是在水平方向，而對底部的壓力則是垂直向下。這原理也適用於液體中的任何物體。水對圖中所示立方體底部的壓力是垂直向上；對頂部的壓力是垂直向下；對立方體側壁的壓力則是水平指向立方體。(箭頭長度與力的大小無關)

(a) 利用以上論述，試求出水中任意一點深度 h 的壓強表示式 (h 是該點與空氣接觸的水面距離)，並以水的密度 ρ，和地球的重力加速度 g 來表示。 (6 分)(換取提示扣 1 分)

(b) 利用(a)所得結果，證明水對立方體的淨力等於 ρVg，並且垂直向上。V 是立方體的體積。(5 分)

(c) 如果容器是放在失重的外太空，那麼水對容器底部的壓強是多少？ (1 分)

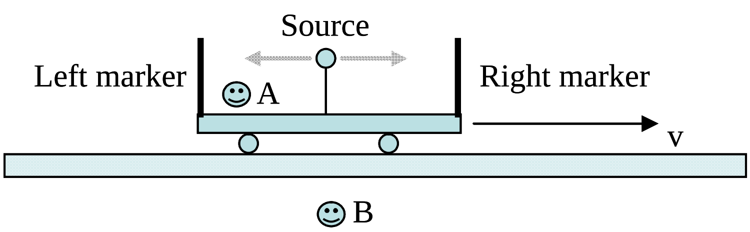
(d) 現在我們用理想氣體代替容器中的水和立方體，並將容器密封。當它處於失重的外太空時，理想氣體對容器的壓強是否和地球上相同？請解釋你的答案。 (3 分)

Q.5

In the following discussions, we assume that all people and objects involved are in vacuum. They can wear oxygen masks when necessary.

One of the principles of Einstein’s Relativity is that the speed of light in vacuum is constant c (c = 2.99792458 x 10^8 m/s), regardless of the motion of the light source or the observer. That is to say, once light leaves its source, it ‘forgets’ all about the motion of the source, and moves at speed c. For example, if the source moves at c/2, the speed of the light emitted by the source in the same motion direction is not c+c/2 = 3c/2, but c. And the speed of the light emitted in the opposite direction is not c–c/2 = c/2, but also c. Furthermore, to someone who is moving with the light source the speed of both light beams is c too. This might sound strange, because common sense tells us that if an air gun moving at speed v relative to you shoots out a bullet at speed u relative to the gun, the speed of the bullet you see is either v+u if it was shot forward, or u–v if it was shot backwards. It turns out, as you may learn later in a university physics course, that the common sense answer is correct only when u and v are much smaller than c, which is the case in ordinary situations such as air gun shooting out bullets. Einstein’s principle of speed of light, which has been proven by numerous experiments, has profound implications to the most fundamentals of physics. Here you are asked to study a particular case, as given below.

A cart of length 2L with one marker at each end is moving at constant speed v along a straight rail. On board is Observer A. Right in the middle is a light source. On the ground is Observer B. Both observers are able to measure the time instantly whenever a light pulse (you can treat it as a small ball of light) hits a marker*.



(a) The light source sent out two light pulses simultaneously, one to the left and one to the right. The pulses eventually hit the markers. The time for the right pulse to travel from the source to the right marker is T_r, and the time for the left pulse to travel from the source to the left marker is T_l. Find T_r and T_l in terms of c, L and v if necessary, as recorded by Observer A. According to her, did the two pulses hit their markers simultaneously? (2 points)

(b) Find T_r and T_l as recorded by Observer B. According to him did the two pulses hit their markers simultaneously? (4 points)

(c) Along the same way of thinking as (a) and (b), design an experiment to demonstrate that it is possible for Observer A to see a pulse hits the right marker (let us call it event-1) **before** another pulse hits the left marker (event-2), while according to Observer B the same pulse hits the right marker (event-1) **after** the other pulse hits the left marker (event-2). (5 points)

(d) What has been shown in (c) is that the sequence of two events can be reversed, i.e., according to Observer A **event-1 took place before event-2**, while to Observer B **event-2 took place before event-1**. However, reversing the sequence of two events is not always possible. Use the results in (c), prove that the two events are reversible if the distance between the two events is larger than $|c\Delta t|$, where $\Delta t = T_l - T_r$ is the difference in time between event-1 and event-2. The maximum value of v is c. (4 points)

*Both observers have as many assistants as needed. Any one of them is capable of recording a hit by a pulse on a marker instantly if the marker is right where the assistant is. Observer A can have one assistant at each marker for such purpose. Observer B can have many assistants aligned side by side along the rail so that when a pulse hits a marker, there is always one assistant right at where the marker is to instantly record the event.

題 5

在以下討論中，我們假設所有涉及的人和物均處於真空中。如有必要，他們可以使用氧氣面罩。

愛因斯坦相對論的一個基本原理是：無論光源或觀察者的運動狀態如何，光在真空中的速度是一常數 c ($c = 2.99792458 \times 10^8$ m/s)。也就是說，一旦光離開了光源，它就會失去對光源運動狀態的所有記憶，而以一恆速 c 運動。例如，如果光源以 c/2 運動，朝同一方向由此光源發射的光的速度不是 c+c/2=3c/2，而是 c。同樣朝相反方向發射的光的速度不是 c-c/2=c/2，而同樣是 c。此外，從與光源同步移動的人來看，此兩束光的速度也都是 c。這也許看起來是挺奇怪的，因爲一般經驗告訴我們，如果以速度 v 移動的氣槍，射出一發相對於氣槍以速度 u 運動的子彈，而子彈是向前發射的話，你所看到的子彈的速度應是 v+u；如果它是向後發射的話則應是 u-v。日後通過學習大學物理課程，你就會知道此種憑經驗得出的結果只有在 u 和 v 比 c 小很多的條件下才能成立，例如氣槍發射子彈的情況。經大量實驗證實的愛因斯坦光速原理對物理學的基本法則有著深遠的影響。下面有一特定例子供你研習：

長爲 2L 及兩端各有一標識屏的小車，以恆定速度 v 沿筆直軌道運動。車上有一位觀察者 A。一光源處於車的正中間。地上也有一位觀察者 B。兩位觀察者都能立刻測得光脈衝(可視作是一小光球)打在標識屏的時間*。

(a) 光源同時射出兩個脈衝，一個朝左，一個朝右。光脈衝最終打在標識屏上。右邊的脈衝從光源到右邊標識屏的時間是 T_r，而左邊的脈衝從光源到左邊標識屏的時間是 T_l。以觀察者 A 的角度，用 c，L 和 v(如有需要)表示出 T_r 和 T_l。 對她而言，兩個脈衝是否同時打在標識屏上？ (2 分)

(b) 對觀察者 B 而言，T_r 和 T_l 又是如何呢？兩個脈衝是否同時打在標識屏上？ (4 分)

(c) 以(a)和(b)同樣的思路，設計一個實驗來演示觀察者 A 能在一個脈衝打到左邊的標識屏(事件 2) <u>之前</u> 看到另一個脈衝先打在右邊的標識屏上(事件 1)，而觀察者 B 則在同樣一個脈衝打到左邊的標識屏(事件 2) <u>之後</u> 才看到另一個脈衝打在右邊的標識屏上(事件 1)。 (5 分)

(d) (c)說明兩件事件的時序是可逆的。對觀察者 A 而言，<u>**事件 1 早於事件 2**</u> 發生，而對觀察者 B 來說，則<u>**事件 2 早於事件 1**</u> 發生。然而，改變兩件事件的時序不是在任何條件下均能做成。由(c)的結果，證明如果兩事件可逆，則兩事件的距離要大於 $|c\Delta t|$，其中 $\Delta t = T_l - T_r$ 是兩事件相隔時間。v 的最大值是 c。 (4 分)

*兩位觀察者均有足夠助手。當他們處於脈衝撞擊標識屏的位置時，他們都能立刻記錄脈衝對標識屏的撞擊。故此，觀察者 A 在每個標識屏處各有一位助手作出記錄。觀察者 B 亦有許多肩並肩沿著軌道排列的助手，所以每當一個脈衝擊中標識屏時，在標識屏處的助手便可以立即作出記錄。

Take Home Question
家中練習題

The following is an open question that you are encouraged to take home to study. How you answer it will NOT affect in any way your grade and standing in HKPhO.
下列開放題希望你帶回家好好研究。你的答案不會對你的成績有任何影響。

The battle to overcome SARS is still raging, and everyone must maintain high vigilance. Infrared thermometers are used widely in schools now. Due to shortage of disposable lens filters, some schools use plastic wrap sheet instead. But experts warned that wrap sheet will distort the temperature readings.

(a) Design a simple experiment to test whether experts' warning is justified.
(b) If the result of your experiment shows that there is indeed small (~ 1°C) deviation of reading when using wrap sheet, design a way to correct the reading.

對抗 SARS 的戰鬥仍在進行。大家還要保持高度警惕。紅外探熱器在各學校廣泛使用。由於即棄耳套短缺，有時會用保鮮紙來代替。但有專家警告保鮮紙會影響讀數。

(a) 設計一簡單試驗看看專家的警告對不對。
(b) 如果試驗發現確有約 1°C 讀數偏差，設計一糾正方法。

Hong Kong Physics Olympiad 2003
25 May 2003
<u>Answers and Suggested Solutions</u>

Answers to Multiple-choice Questions:

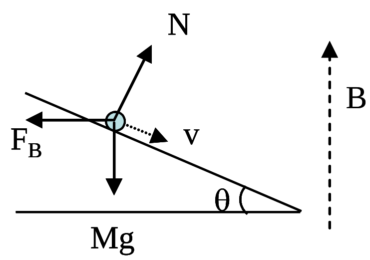
1. A	**2.** B	**3.** D	**4.** C	**5.** C	**6.** A	**7.** A
8. D	**9.** E	**10.** A	**11.** B	**12.** C	**13.** A	**14.** B
15. E	**16.** A	**17.** A	**18.** D	**19.** E	**20.** D	

(Note for MC9: The magnetic field force on the coil is horizontal)

Suggested Solutions to Open-ended Questions:

Q.1

(a) As shown, the forces on the rob are the gravity Mg, the reaction force from the frame N, and the magnetic field force F_B.
The emf is BLvcosθ, so the electric current is
$I = BLv\cos\theta/R$ …………….(1)
The magnetic field force on the rod is
$F_b = BLI$………………(2)
The balance of force along the slope implies

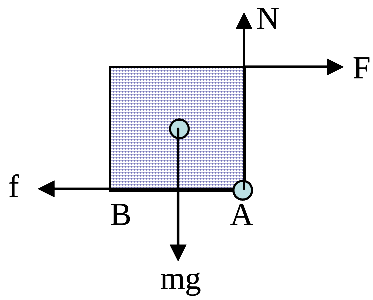


$Mg\sin\theta = F_b\cos\theta = v(BL\cos\theta)^2/R$
So $v = RMg\sin\theta/(BL\cos\theta)^2$

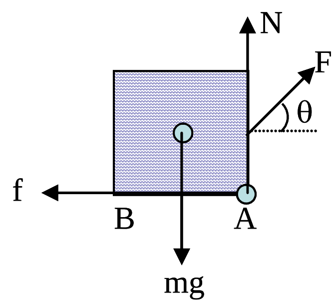
(b) v is the same, but the direction of current is reversed.
(c) Then I = 0 so $F_b = 0$.
The rod will slide down under constant acceleration gsinθ.

Q.2

(a) Taking point-A as the pivot, both the floor reaction force N and the friction force f are acting upon the point because it is where the block touches the floor if it is about to turn. The torque (moment) due to F = 0.8mg is FL and clockwise, where L is the length of the cubic side. The torque due to gravity is mgL/2 and counterclockwise. So the block will tip over around point-A.
On the other hand, the maximum friction force is
$\mu N = \mu mg = mg > F$
So the block will not slide.



(b) With the force applied as shown in the figure, the reaction force of floor $N = mg - F\sin\theta$,
So the maximum $f = (mg - F\sin\theta)$.
The sliding acceleration is
$ma = F\cos\theta - (mg - F\sin\theta)$
or
$a/g = 0.8(\cos\theta + \sin\theta) - 1$, and its maximum is 0.13 when θ is 45 degrees.



Now, will the block tip over around point-A or point-B?
The torque due to F is 0.5FLcos45° < 0.5mgL so it will not tip over.

Note that if force F is applied on point-A at 45°, the block will tip over around point-B. To prevent tipping over around either point-A and point-B, force F must be applied near the middle point. Maximum distance x away from the middle point is determined by (0.5L + x)Fcos45° = 0.5mgL, which leads to x = 0.384L.

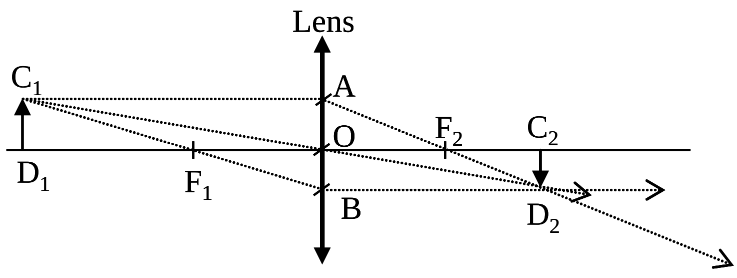
Now let us go back to part (a), and consider F in general term. One type of answer I often encountered during grading is ‘The friction and F form a clockwise torque that tips off the block’. This, however, cannot be true because it implies that no matter how small F is, it can always tip off the block, with the help of the friction. The key element missing in the consideration is the illusive reaction force of the floor on the block. Its direction must be upwards and strength equals to mg to balance the gravity force along the vertical direction. The question is, where is the reaction force N (= mg) acting upon? Or more precisely, what is the torque of N? Just for the consideration of torque we can view N as acting upon a point on the bottom of the block, a distance ‘X’ from the center. The reaction force N is of course acting upon all the places the block is in touch with the floor but the force is NOT uniformly distributed on the bottom surface. The net effect of it is to produce a torque to keep the block in balance.

Choose the center of mass as the pivot point, the torque due to F and f (= F) is FL. This must be balanced off by N so N must acting upon a point at X distance to the right of the center, and Xmg = FL, so X = L(F/mg). Note that X cannot exceed L/2 so when F > 0.5 mg the block will tip over.

One can also choose point-A as pivot. In that case f produces no torque. The clockwise torque is now FL + N(L/2-X), and the gravity produces a counter-torque 0.5mgL. So we have FL + mg(L/2-X) = mgL/2. Again we get X = L(F/mg).

Q.3

(a) Complete the graph



(b) Define:

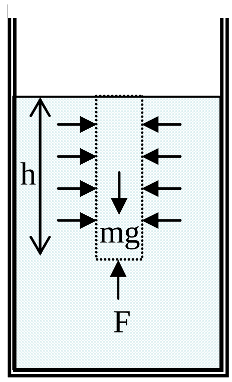
$\underline{C_1D_1}$ = h = $\underline{AO}$, $\underline{C_2D_2}$ = h’ = $\underline{OB}$, $\underline{OD_1}$ = $\underline{AC_1}$ = s, $\underline{C_2O}$ = $\underline{BD_2}$= s’, $\underline{OF_1}$ = $\underline{OF_2}$ = f,
Angle α = AC_1B = OF_1B, Angle β = AF_2O = AD_2B (1 point)
Then $\underline{OF_1}/\underline{OB}$ = h’/f = tanα = $\underline{AB}/\underline{AC_1}$ = (h + h’)/s …………. (1)
$\underline{OF_2}/\underline{AO}$ = h/f = tanβ = $\underline{AB}/\underline{BD_2}$ = (h + h’)/s’ ………… (2)
(1) + (2) (h + h’)/f = (h + h’)/s + (h + h’)/s’, cancel (h + h’).

Note that there are many ways to prove the formula, and any one of them is as good as the others.

(c) Angle C_1OD_1 = Angle C_2OD_2 (1 point), so M = h'/h = s'/s.

Q.4

(a) Consider the column of water of height h and cross area A shown in the figure. The force the rest of the water on the water column must balance the gravity force of the column. Ignore the air pressure on the surface of water, the force on the bottom surface of the column must equal to mg, where m is the mass of the water column.
So
$m = \rho hA$
$F = mg$, and
pressure $P = F/A$
Combining the three, one gets $P = \rho gh$



(b) Using the result in (a), the force from water on the top surface of the cubic is $A\rho gh_1$, where h_1 is the depth of the top surface, and is pointing downwards. Likewise, the force on the bottom surface of the cubic is $A\rho gh_2$, where h_2 is the depth of the bottom surface, and is pointing upwards. The net force of water on the side walls of the cubic is zero. So the total net force is
$F = A\rho gh_2 - A\rho gh_1$
$= A\rho g(h_2 - h_1) = \rho gV$

(c) Pressure is zero (not counting the water vapor) because g = 0 in weightless environment.

(d) The same, because pressure of gas is a result of the thermal motion of the gas molecules that keep hitting on the container wall. Acceptable answers also include 'the pressure is smaller because the outer space is cold', or 'the balance of force on the wall is changed because there is no gas pressure outside the container, so it will explode', etc., although the mechanical strength of the container may well sustain such difference in pressure, just as a spaceship can keep normal pressure inside while there is vacuum outside.

Q.5

(a) Observer-A sees the following. The two markers are stationary. The two light pulses are moving at c towards their markers at distance L away. So
$T_r = T_l = L/c$

(b) Observer-B sees the following. The two markers are moving to the right at speed v. The two light pulses are moving at c towards their markers, initially (right after the pulses are emitted) at distance L away.
For the left pulse $L = cT_l + vT_l$, so $T_l = L/(c + v)$
For the right pulse $L = cT_r - vT_r$, so $T_r = L/(c - v) > T_l$
So Observer-B sees that the two pulses hit markers at different times. This means that simultaneity is no longer absolute. It depends on the motion of the observers.

(c) Relocate the light source to the right, so that its distance to the right marker is L_r, and its distance to the left marker is L_l. $2L = L_l + L_r$. Since $L_r < L_l$, $T_r < T_l$, i. e., event-1 takes place ***before*** event-2, according to Observer-A.
According to Observer-B,

$$T_r' = \frac{L_r}{c - v} \qquad \text{and} \qquad T_l' = \frac{L_l}{c + v}$$

If $T_r' > T_l'$, then Observer-B sees event-1 takes place ***after*** event-2.

$$\Delta t' \equiv T_r' - T_l' = \frac{L_r}{c-v} - \frac{L_l}{c+v} = \frac{v(L_r + L_l) - c(L_l - L_r)}{c^2 - v^2} = \frac{2vL - c(L_l - L_r)}{c^2 - v^2}$$

$\Delta t' > 0$ if $2vL > c(L_l - L_r)$. This can be easily achieved since $(L_l - L_r)$ can be as small as we want. For example, let $(L_l - L_r) = 0.1L$, then all we need is $v > 0.05c$.

(d) From the expression of Δt' in (c) we see that 2vL is maximum if v = c (To save the denominator from being zero we can let v very close to but not equal to c).
The inequality becomes
$2L > (L_l - L_r) = c\Delta t$, where Δt is the time difference of event-1 and event-2 seen by Observer-A. Note also that 2L is the distance between the two events.
So the general criteria is (distance) > (speed of light) x (time difference).
To put it into words, suppose Observer-A sees that event-1 takes place at time Δt before event-2 at a distance S away from event-1, then it is possible for Observer-B, who is moving relative to Observer-A, to see event-2 takes place before event-1 if $S > c\Delta t$. But if $S < c\Delta t$ then it is not possible given the restriction that he cannot move faster than light.

Take-Home Question

Answer to (a)

Take two measurements on the same person, one with the filter and the other with the wrap, and compare the readings.

Answer to (b)

I would like to ask a more general question: Is the reading of infrared thermometer reliable? There are several aspects to be considered.

First, what is the random error of the reading? This is the spread of the readings when you use the thermometer to measure a person (perhaps yourself) many (say 20 +) times within a short period of time, and do statistics of the 20+ data to find out the Mean (say 36 ˚C) and the Standard Deviation (say 0.2 ˚C), which is the random error of the thermometer you operate. The Mean may not necessarily be the same as your true body temperature. This brings out the second point: the systematic error of the thermometer.

The body temperature can be most reliably measured by the old fashioned mercury/glass thermometer. In the following, when I say 'temperature' I mean that measured by a mercury/glass thermometer. When I say 'reading' I mean the temperature reading displayed by the infrared thermometer. Obviously, to find out the systematic error, you need to compare your body temperature with the reading. Now, what if the reading is different from the temperature but the difference is smaller than the random error? Then your thermometer is fine. But what if the difference is larger than the random error? In that case you need to do a 'calibration' of your infrared thermometer using the 'standard' mercury/glass thermometer. First, you need to find an object that you can change its temperature with relative ease. Second, such object must give out the same reading as your body when its temperature is the same as your body. This ensures that the emissivity (I will explain what it really means later) of the object is the same as your body. Third, change and record the temperature of the object within the normal range of human being (say 34 to 42 ˚C) and record the reading at each temperature. The data you obtain (temperature versus reading) then give you the correction you need. You can even fit the data by a function with temperature as the Y value and reading as the X value. Suppose you find that the temperature (Y) versus reading (X) data can be best described by Y =

1.2 + 0.9*X, then when you get a reading of X = 39, you know the true temperature is 36.3 ˚C.

Some general information

The working principle of an infrared thermometer is that it measures the amount of infrared light emitted from a particular part of a human body, say inside an ear, the forehead, etc, and from the amount determines the temperature. Every object at non-zero degree Kelvin emits light, or electromagnetic waves. This is the so called blackbody radiation. The total amount of emitted energy is proportional to $(\text{Temperature})^4$. So a little bit of change in temperature brings a much larger change in emission power. An ideal blackbody (totally black object) does not reflect any light, and in return it emits the highest amount of light among all other non-black objects held at the same temperature. Hence comes the term emissivity e. Blackbody's e is 1. Other objects have $e < 1$, and their e's vary from object to object. But some objects may happen to have almost the same e's. In general, objects that look shinny (high reflection) tend to have low e's. A piece of charcoal for barbeque has $e \sim 0.95$, while a piece of gold has $e < 0.05$. Therefore, at the same temperature, a piece of charcoal emits 19 times the amount of energy of a piece of gold. An infrared thermometer should have been calibrated for emissivity of human skin. But skin covered by other substances such as sun cream or water may have its emissivity modified. All these variations, however, can be tested by yourself. Different parts of the human body have different temperatures. How do you deal with that?

Hong Kong Physics Olympiad 2004
2004 年香港物理奧林匹克競賽

Written Examination
筆試

Jointly Organized by

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香港科技大學

共同舉辦
May 30, 2004
2004 年 5 月 30 日

The following symbols will be used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitation constant, 6.67 x 10^{-11} (N m^2/kg^2)
e – charge of an electron, -1.6 x 10^{-19} (A s)
ε_0 – electrostatic constant, 8.85 x 10^{-12} (A s)/(V m)
m_e – electron mass, 9.11 x 10^{-31} kg
c – speed of light in vacuum, 3.0 x 10^8 m/s

除非特別說明，本卷將使用下列符號：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 重力常數, 6.67 x 10^{-11} (N m^2/kg^2)
e – 電子電荷, –1.6 x 10^{-19} (A s)
ε_0 – 靜電常數, 8.85 x 10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11 x 10^{-31} kg
c – 真空光速, 3.0 x 10^8 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much lower than the speed of light.

除非特別說明，本卷將使用下列條件：

1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠低於光速.

Multiple choice questions (2 points each. Select one answer in each question.)
選擇題（每道題二分，每道題選擇一個答案）

MC-1
Which of the following provides the largest buoyancy to a totally submerged object?

(A) shallow sea water (B) shallow fresh water (C) deep fresh water
(D) oil (E) oil and fresh water mixture

選擇題 1
下面哪一種情況會使完全浸沒物體受到最大浮力？

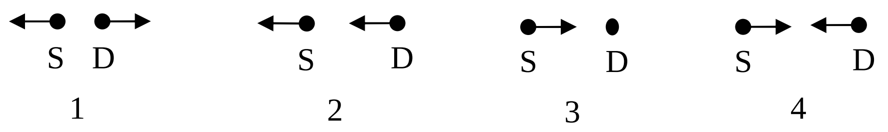
(A) 淺海水 （B）淺淡水 （C） 深淡水 （D）油 (E）油與淡水混合物

MC-2
A sinusoidal wave is traveling along a string. Any point on the string:

(A) moves in the same direction as the wave
(B) moves periodically with a different frequency from that of the wave
(C) moves periodically with the same frequency as the wave
(D) moves circularly with a different speed from that of the wave
(E) moves circularly with the same speed as the wave

選擇題 2
一正弦波在繩上傳播，繩上任意一點都_____.
（A）沿著波傳播的方向運動 （B）作與波不同頻率的周期性運動
（C）作與波相同頻率的周期性運動 （D）作與波不同速率的圓周運動
（E）作與波相同速率的圓周運動

MC-3
The diagram shows four situations in which a source of sound S and a detector D are either moving or stationary. The arrows indicate the direction of motion. The speeds are all the same. Detector 3 is stationary. Rank the situations according to the frequency detected, from lowest to highest.



(A) 1, 2, 3, 4 (B) 4, 3, 2, 1 (C) 1, 3, 4, 2
(D) 2, 1, 4, 3 (E) None of the above

選擇題 3
下圖給出運動或靜止聲源 S 與探測器 D 的四種情況。箭頭表明運動的方向。運動速率是相同的。圖 3 中探測器是靜止的。探測到的頻率由低到高的順序是

（A） 1，2，3，4 （B） 4，3，2，1 （C） 1，3，4，2
（D） 2，1，4，3 （E） 以上皆不是。

MC-4
In constructing a thermometer, one *must* use a substance that:

(A) expands with rising temperature
(B) expands linearly with rising temperature
(C) will not freeze
(D) will not boil
(E) undergoes some changes when heated or cooled

選擇題 4
在組裝一個溫度計時，選用的材料**必須**是：

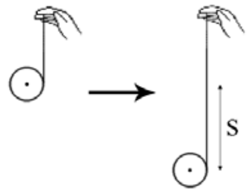
（A） 隨溫度上升而膨脹 （B）隨溫度上升而線性膨脹 （C）不會凝固
（D）不會沸騰 （E）加熱或冷卻時會發生變化

MC-5
Two identical rooms in a house are connected by an open doorway. The temperatures in the two rooms are maintained at different values. Which room contains more air?

(A) the room with higher temperature
(B) the room with lower temperature
(C) the room with higher pressure
(D) neither, because both have the same pressure
(E) neither, because both have the same volume

選擇題 5
一座房子裏有兩間由通道相連的完全相同的房間，如果兩間房間的溫度不相同，那麼哪個房間含有較多的空氣？
（A） 溫度較高的房間 （B）溫度較低的房間 （C）氣壓較高的房間
（D）都不是，因爲它們具有相同的氣壓
（E）都不是，因爲它們具有相同的體積。

MC-6
In the right figure, a light string is wound round the rim of a yo-yo of mass *m* and radius *r*. One end of the string is held by a person. When the yo-yo is released from rest, it falls and rotates at a linear acceleration of 0.8g. What is the tension in the string?

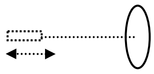


(A) 0 (B) 0.2 mg (C) 0.4 mg (D) 0.8 mg (E) mg

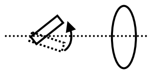
選擇題 6
右圖中一根細繩纏繞在質量爲 *m* 和半徑爲 *r* 的溜溜球的軸上。繩的一端用人手執著。當溜溜球從靜止狀態放下時，它以 0.8g 的線性加速度旋轉落下。那麼繩上的張力爲：

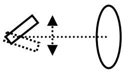
(A) 0 (B) 0.2 mg (C) 0.4 mg (D) 0.8 mg (E) mg

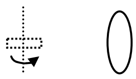
MC-7
A magnetic bar is in front of a coil as shown below. The line joining the center of the bar and the center of the coil (central axis) is perpendicular to the plane of the coil. Which of the following motions of the bar will **NOT** induce electric current in the coil?

(A) Translational motion back and forth. 

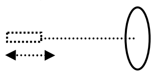
(B) Translational motion up and down. 

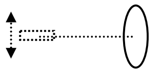
(C) The bar axis is at an angle to the central axis and spins around it. 

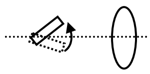
(D) The bar axis swings back and forth about the central axis within the paper plane. 

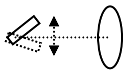
(E) The bar spins around an axis in the paper plane. 

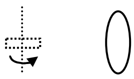
選擇題 7
一磁棒放在如下圖所示的線圈前。磁棒中心與軸對稱的線圈中心的連線垂直於線圈的平面。下面哪種磁棒運動將**不會**產生感應電流？

(A) 前後移動. 

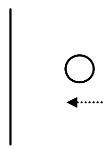
(B) 上下移動. 

(C) 磁棒的軸與中心軸成一角度，並圍繞它旋轉. 

(D) 磁棒的軸在紙的平面內以中心軸上下擺動. 

(E) 磁棒以在紙的平面內的軸旋轉. 

MC-8
A coil is moving towards a straight long wire carrying a steady electric current. The wire and the motion are within the plane of the coil. The force exerted by the wire on the coil is in the direction ___.



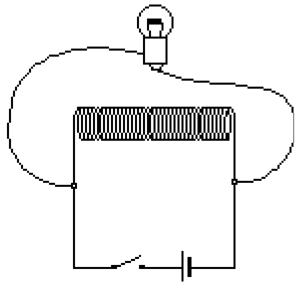
(A) away from the wire　　(B) towards the wire
(C) into the paper plane　　(D) out of the paper plane
(E) upwards

選擇題 8
線圈靠近載有穩定電流的長直導線，導線和運動方向都在線圈的平面內。那麼導線對線圈施加的力的方向是：

（A） 排斥　（B）吸引　（C）朝紙內方向　（D）朝紙外方向　（E） 向上

MC-9

A battery is connected to a solenoid and a light bulb in parallel. When the switch is opened, the light bulb ___.



(A) remains off
(B) instantly goes off
(C) slowly dims out
(D) keeps burning as brightly
(E) flares up brightly, then dims and goes off

選擇題 9

一電池與線圈和燈泡並聯。當打開開關時，燈泡將___ 。

(A) 仍然不亮　（B）立刻熄滅　（C）慢慢熄滅
（D） 保持點亮　（E）突然變亮，然後慢慢熄滅

MC-10

We have seen on TV how the astronauts were trained in weightless condition in a large airplane. To achieve weightless condition the plane should ___.

(A) Dive downwards at constant velocity
(B) Dive downwards at constant acceleration that is equal to *g*
(C) Accelerate in horizontal direction
(D) Move upwards at constant velocity
(E) Move upwards at constant acceleration that is equal to *g*

選擇題10

我們在電視上看到宇航員在一架大飛機裏處於失重狀態下訓練。爲達失重狀態，飛機應該___ 。

(A) 以勻速向下俯衝
(B) 以勻加速度*g*向下俯衝
(C) 沿水平方向加速
(D) 以勻速上升
(E) 以勻加速度*g*上升

MC-11

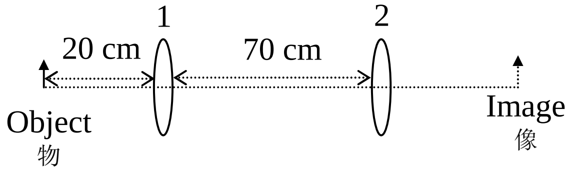
You have a manual camera with a focal length of 5cm. It is "focused" at infinity, but you want to take a picture of an object that is only 30cm away. What should you do?

(A) Move the lens out by about 1 cm
(B) Move the lens out by about 5 cm
(C) Decrease the distance between the lens and the film by about 1 cm (move the lens in)
(D) Decrease the distance between the lens and the film by about 5 cm (move the lens in more)
(E) None of the above

選擇題 11
你有一架焦距爲 5 釐米的手動相機。如果現在相機聚焦在無窮遠處，而你想取景的物體在 30 釐米處，那麼你應該_____

(A) 將透鏡移出約 1 釐米。
(B) 將透鏡移出約 5 釐米。
(C) 減小透鏡與膠片的位置約 1 釐米（將透鏡移入）。
(D) 減小透鏡與膠片的位置約 5 釐米（將透鏡移入很多）。
(E) 以上皆不是。

MC-12
As shown below (not drawn to the proportion), the focal lengths of both lens-1 and lens-2 are 10 cm. If a third lens identical to lens-1 is added while maintaining the image position unchanged, where should the lens be placed?

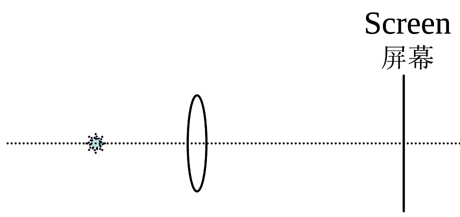


(A) Between object and lens-1, 10 cm from lens-1
(B) At the middle point between lens-1 and lens-2
(C) Between lens-1 and lens-2, 20 cm from lens-1
(D) Between lens-1 and lens-2, 20 cm from lens-2
(E) Between lens-2 and image, 10 cm from lens-2

選擇題 12
如上圖所示（沒有按比例畫圖）透鏡 1 與透鏡 2 的焦距均爲 10 釐米。如果要加入第三個與透鏡 1 完全相同的透鏡，而保持成像位置不變，那麼這個透鏡應該放在__。

(A) 物體與透鏡 1 之間，且與透鏡 1 相距 10 釐米
(B) 透鏡 1 與透鏡 2 的中間位置
(C) 透鏡 1 與透鏡 2 之間，且與透鏡 1 相距 20 釐米
(D) 透鏡 1 與透鏡 2 之間，且與透鏡 2 相距 20 釐米
(E) 透鏡 2 與像之間，且與透鏡 2 相距 10 釐米

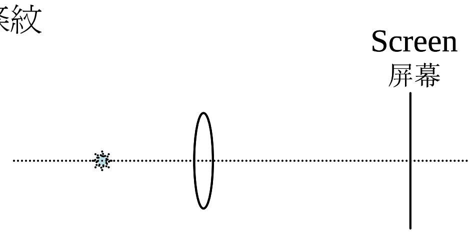
MC-13
As shown in the right figure, a point light source is placed at distance $2f$ from a lens with focus length f, and a screen is placed at $4f$ from the lens. The lens is then cut at the middle into two equal portions: upper half and lower half. The upper half is moved upwards by a small distance d comparable to the light wavelength, and the lower half is moved downwards by the same distance d. What is the light pattern on the screen?



(A) Bright and dark stripes similar to the pattern seen in Young's double slit experiment
(B) Bright and dark concentric rings
(C) Two large, bright, and partly overlapping patches
(D) Two separate bright spots
(E) A large and nearly uniform light patch

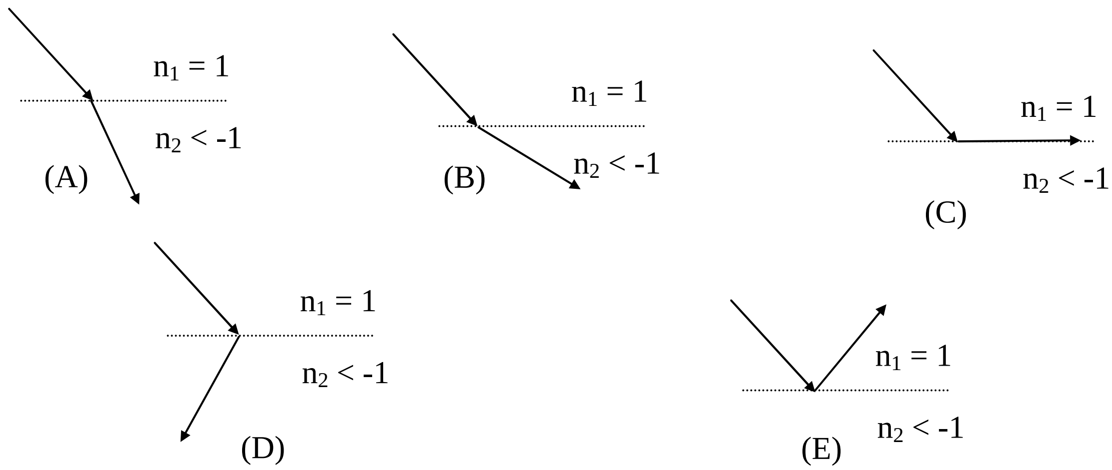
選擇題 13
如右圖所示，一個點光源放在距離焦距爲 f 的透鏡 $2f$ 的位置，而螢幕放在距離透鏡 $4f$ 的位置。然後將透鏡中間切開成上下兩等份。上半個透鏡向上移動與光的波長相近的距離 d，而下半個透鏡向下移動相同的距離 d，那麼光在螢幕上的圖像是什麼？

(A) 類似在楊氏雙縫實驗中螢幕上所看到的明暗條紋
(B) 明暗相間的同心環
(C) 兩塊較大且部分重疊的明亮光斑
(D) 兩個分開的亮點
(E) 一塊較大和近似均勻的光斑。



MC-14
It has been shown recently that there exists media with negative refractive index ($n_2 < -1$). If a light beam is incident on the surface of such a medium from air, which of the following describes correctly the beam entering the medium? Choice-E means that the beam cannot enter the medium. In all the others, the reflected beam is not shown, but is present.

選擇題 14
最近發現一種具有負折射率($n_2 < -1$)的介質。如果一束光從空氣入射到該介質的表面，那麼下列哪一個圖正確描述進入介質的光束? 選擇(E)表明光束不會進入介質。在所有其他圖中沒有給出反射光，但它是存在的。

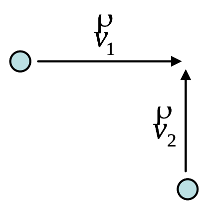


MC-15
Two balls of masses m_1, m_2 and speeds v_1 and v_2 collide at right angle. The maximum amount of kinetic energy loss due to inelastic collision is ____.

(A) $\frac{1}{2}(m_1v_1^2 + m_2v_2^2)$　　(B) $\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(v_1^2+v_2^2)$

(C) $\frac{1}{2}\frac{1}{m_1+m_2}(m_1^2v_1^2+m_2^2v_2^2)$　　(D) $\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(v_1+v_2)^2$

(E) None of the above



選擇題 15

兩個質量爲 m_1, m_2 的球分別以速率 v_1 和 v_2 垂直碰撞。那麼由於非彈性碰撞引起的最大動能損失是：

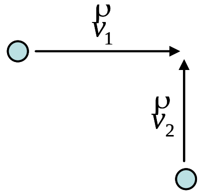
(A) $\frac{1}{2}(m_1v_1^2 + m_2v_2^2)$

(B) $\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(v_1^2 + v_2^2)$

(C) $\frac{1}{2}\frac{1}{m_1+m_2}(m_1^2v_1^2 + m_2^2v_2^2)$

(D) $\frac{1}{2}\frac{m_1m_2}{m_1+m_2}(v_1+v_2)^2$

(E) 以上皆不是



MC-16

The rebound coefficient between a tennis ball and a racket is defined as $\gamma = v_2/ v_1$, where v_1 is the incoming speed of the ball and v_2 is the speed of the ball after rebound while the racket is at rest. A tennis ball falls from height H to a racket at rest and bounces back to 0.8 H. A tennis player is using the racket to hit an incoming tennis ball traveling at 150 km/hr and the racket is moving at 100 km/hr. What is the speed of the ball after being hit? (Assume the mass of the racket >> that of the ball)

(A) 323.6 km/hr (B) 350 km/hr (C) 150 km/hr (D) 250 km/hr
(E) 234 km/hr

選擇題 16

網球與球拍的反彈係數定義爲$\gamma = v_2/ v_1$，其中 v_1 爲網球碰撞前的速率，v_2 爲網球碰撞後的速率，而球拍不動。網球從高度 H 落到靜止的球拍上，反彈高度爲 0.8 H。一個網球手用球拍以 100 km/hr 的速率撞擊以速率爲 150 km/hr 迎面而來的網球，那麼撞擊後的網球的速率是什麼？(假設球拍的質量>>網球的質量)

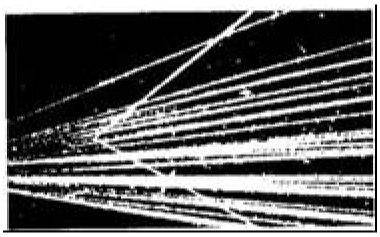
(A) 323.6 km/hr (B) 350 km/hr (C) 150 km/hr (D) 250 km/hr
(E) 234 km/hr

MC-17

As shown to the right, in a ^{4}He (the number on the left-up corner is the number of nucleons, i. e., protons plus neutrons, in the nucleus) cloud chamber photograph, an unknown nucleus collides with a ^{4}He nucleus, and after the collision the two nuclei travel in perpendicular directions relative to each other. If kinetic energy is lost in the collision, the unknown nucleus **must** be ___.

(A) ^{1}H (B) ^{4}He (C) ^{12}C (D) a nucleus with mass lighter than ^{4}He
(E) a nucleus with mass heavier than ^{4}He

選擇題 17



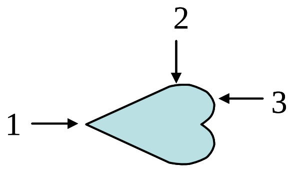
如右圖所示，在 ^{4}He（左上角的數位爲核子數，即在核子中的質子和中子數之和）雲室照片中，一個未知核子與 ^{4}He 核子碰撞，碰撞後兩個核子以彼此垂直的方向運動。如果在碰撞中動能損失了， 那麽未知的核子一定是：

(A) ^{1}H　(B) ^{4}He　(C) ^{12}C　(D)質量輕於 ^{4}He 的核子
(E) 質量重於 ^{4}He 的核子

MC-18

A heart shaped conductor shown below carries net charge Q. Which of the statement about the electric field E and the surface charge density σ below is correct?

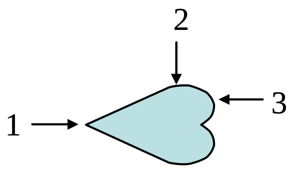
(A) E strongest and σ smallest at position-1
(B) E strongest and σ highest at position-1
(C) E weakest and σ highest at position-2
(D) E strongest and σ highest at position-3
(E) E strongest and σ highest at position-2



選擇題 18

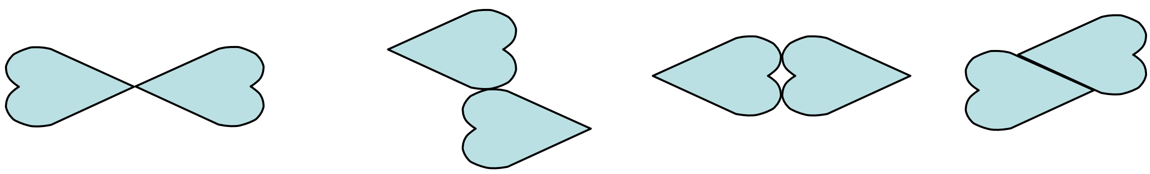
一個心形的導體如右圖所示，淨電荷爲 Q。下列哪個陳述電場 E 和表面電荷密度σ是正確的？

(A) 在位置 1 的 E 最強，σ最小
(B) 在位置 1 的 E 最強，σ最高
(C) 在位置 2 的 E 最弱，σ最高
(D) 在位置 3 的 E 最強，σ最高
(E) 在位置 2 的 E 最強，σ最高



MC-19

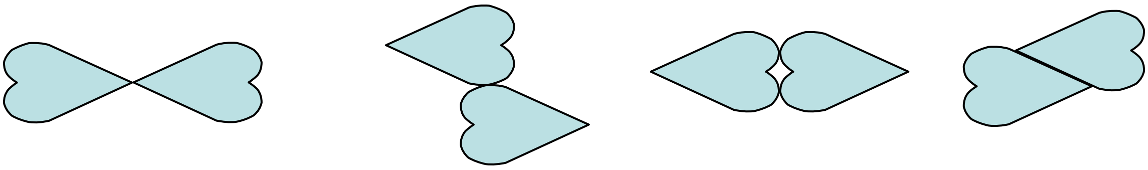
A second conductor identical to the first one, originally carrying no charge, is brought to contact with the first one which carries charge Q. In which way will the second conductor receive most amount of charge?



(A)　(B)　(C)　(D)

(E) none of the above

選擇題 19
第二個導體與第一個完全一樣，最初並沒有帶電荷，然後與第一個帶電導體接觸。那麼按照哪種方式第二個導體可以收到最多的電荷？



(A) (B) (C) (D)

(E) 以上皆不是

MC-20
A piece of ice at 0 °C is mixed with some water also at 0 °C. If there is no heat exchange of the ice-/water mixture with the outside world, what will happen when the ice-water mixture reaches equilibrium?

(A) All ice melted (B) All water becomes ice (C) Portion of ice melted
(D) Portion of water becomes ice (E) No change in the ice/water ratio

選擇題 20
一塊處於 0 °C 的冰和一些處於 0 °C 的水混合。如果冰-水混合物與外部環境沒有熱量交換，那麼當冰-水混合物達到平衡時，會發生什麼事？
(A) 所有的冰都融化了
(B) 所有的水結冰了
(C) 部分的冰融化了
(D) 部分的水結冰了
(E) 冰水比例沒有發生改變

Hong Kong Physics Olympiad 2004

Answers and Suggested Solutions

Answers to Multiple-choice Questions:

1. A	2. C	3. A	4. E	5. B	6. B
7. C	8. A	9. E	10. B	11. A	12. C
13. A	14. D	15. B	16. A	17. D	18. B
19. E	20. E				

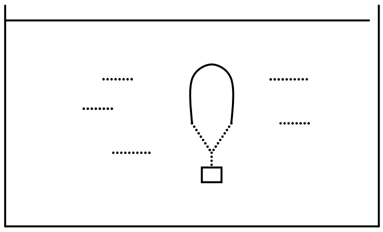
Open Questions Total 7 questions

Q-1 (5 points)
One kilogram of coarse sand is placed in a can, as shown to the right. Is the total force of the sand directly acting upon the bottom of the can larger or smaller than 9.8 N? Limit your argument to half a page.

Solution:

Due to the friction between sand grains and wall, the force on the bottom is less than 9.8 N.

Q-2 (5 points)
A rigid can with an open end is inserted into water upside down, as shown in the figure. A mass m is attached to the end of the can so that the can becomes stable when it is at a depth *h* in water. Assume air cannot escape, what happens to the motion of the can if (i) the can is moved a little bit upward from the depth *h*, (ii) downward a little bit from *h* and (iii) the can is being heated?



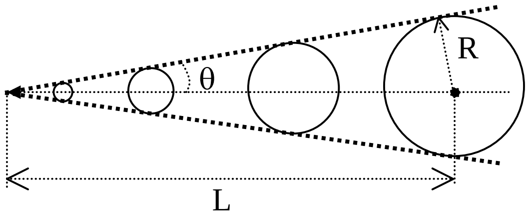
Solution:

The buoyancy of water is determined by the volume of air trapped inside the can. The volume in term is determined by PV = RT. In equilibrium at depth *h*,

$P_0V_0 = (P_0 + \rho gh)V$, and $mg = \rho gV$, where ρ is the water density, P_0 is atmosphere pressure, V_0 is volume of the can. (2 points)

(i) Decrease *h* by a little, V increases, so does buoyancy. The can will keep rising faster and faster. (1 point)
(ii) Increase *h* by a little, V decreases, so does buoyancy. The can will sink faster and faster. (1 point)
(iii) T rise causes V increase. Same motion as (i). (1 point)

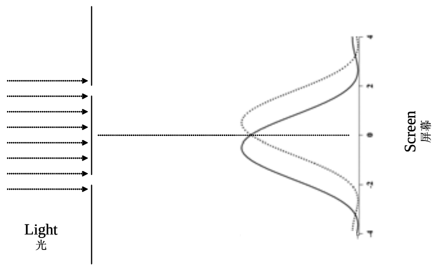
Q-3 (7 points)
The Cherenkov radiation is caused by an electron moving faster than the speed of light in a medium, such as gas. At any instance, the electron can be considered as a point source emitting spherical electromagnetic wave. Suppose the electron is not slowing down, find the shape of the wave front and its relation with the electron trajectory, in terms of the electron speed *v*, refractive index of the medium *n*, and speed of light in vacuum *c*.



(3 points)

A series of spheres form a cone-shaped wave front, as shown in the above figure. Within duration t, the radius of the wave sphere emitted at t = 0 is R = ct/n (1 point),
and the electron has traveled distance L = vt (1 point).
So sinθ = R/L = c/nv (2 points).

Q-4 (8 points)
A typical Young's double spit experiment setting is shown to the right (not drawn to the proportion). The solid curve represents the light intensity on the screen when only the lower slit is open, and the dashed curve represents the light intensity observed when only the upper slit is open. Draw a curve that qualitatively represents the light intensity on the screen when both slits are open, and briefly explain your answer (< half a page).



Solutions:

Take the coordinate x on the screen along the direction perpendicular to the slits, then at any point on the screen, the E-field from the wave through the upper slit is
$E_{upper} = A_{upper}(x)\cos(kd_1 - \omega t)$, where d_1 is the distance from point-x to the upper slit, and $A_{upper}(x)$ is given by the dashed curve. (1 point)

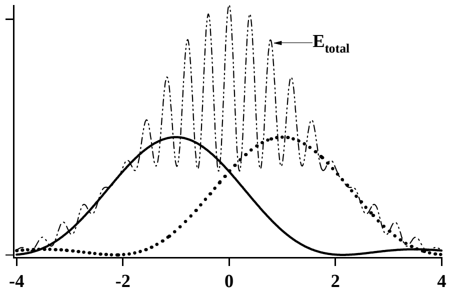
Similarly, $E_{lower} = A_{lower}(x)\cos(kd_2 - \omega t)$, where A_{lower} is given by the solid curve, and d_2 is the distance between x-point and lower slit. (1 point)

Note that $d_2 - d_1 = xd/L$, where L is the distance between slit and screen, and d is the distance between the two slits, and x is measured from the mid-point on the screen. (1 point)

When both slits are open, the total E-field at any point on the screen is then
$E = A_{upper}(x)\cos(kd_1 - \omega t) + A_{lower}(x)\cos(kd_2 - \omega t)$ (1 point)

The light intensity is $I(x) = \langle E^2 \rangle = A^2_{upper}(x) + A^2_{lower}(x) + A_{upper}(x)A_{lower}(x)\cos(kxd/L)$.
(2 points)

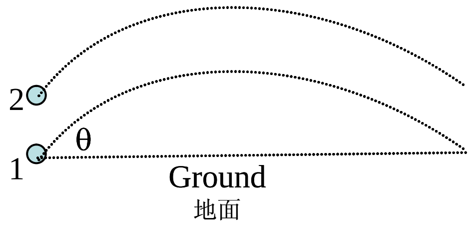
The actual light curve is show below.



(2 points)

Q-5 (10 points)

Two small solid spheres of mass *m* are thrown at the same time with the same initial velocity *v* and angle θ. Sphere-1 is thrown from the ground level while Sphere-2 is thrown at a height *h* above Sphere-1. Counting the gravitational force between the two spheres, calculate the amount of distance change δh between the two spheres at the moment Sphere-1 hits the ground. You may take the distance between the two spheres as constant *h* in your derivation for the gravitational force because $\delta h \ll h$. With v = 200 m/s, θ = 30 °, h = 1 m, mass of both spheres m = 1 kg, find the value of δh, and compare that to the size of an atom.



Solution:

Take the center of mass of the two balls as the reference frame, the two balls can be considered as attracting to each other via their gravity $F = Gm^2/h^2$. (2 points)

The flying time is $t = 2v\sin\theta/g$. (1 point)

During that time the distance between the two balls has reduced by

$$\delta h = 2\times\frac{1}{2}(F/m)t^2 \quad \text{(2 points)}$$

$$= \frac{Gm}{h^2}\left(\frac{2v\sin\theta}{g}\right)^2 \quad \text{(2 points)}$$

$$= 6.67\times10^{-11}\times4\left(\frac{0.5\times200}{9.8}\right)^2 = 2.8\times10^{-8}(meters) \quad \text{(2 points)}$$

An atom is about $10^{-10}(meters)$ so the distance has reduced by an order of magnitude of about 200 atoms length. (1 point)

Q-6 (10 points)

In classical hydrogen atom model an electron is circling around the nucleus like Earth revolving around the sun, except that the force between the electron and the nucleus is due to the electric field. However, an electron under acceleration also emits electromagnetic waves, and its radiation power is given by $W = \dfrac{e^2 a^2}{6\pi c^3 \varepsilon_0}$, where a is the acceleration of the electron. Given the radius of the circular orbit R, find the radiation power W, and compare that with the kinetic energy E of the electron. Assuming that it roughly takes $t = E/W$ for the electron to loose all its energy and R = 5.0 x 10^{-11} m, what would be the lifetime of a hydrogen atom before it becomes a neutron?

Solution:

$$a = \frac{e^2}{4\pi R^2 m\varepsilon_0},\ W = \frac{e^2 a^2}{6\pi c^3 \varepsilon_0} = \ldots \text{ (3 points)}$$

$$a = \frac{v^2}{R} = \frac{2E}{Rm}, \text{ (2 points)}$$

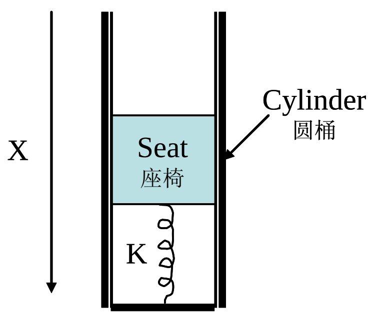
$$t = \frac{E}{W} = \frac{12\pi^2 c^3 \varepsilon_0^2 m^2 R^3}{e^4} \text{ (3 points)}$$

$$= \frac{12\times 3.14^2 \times 27\times 10^{24} \times 8.85^2 \times 10^{-24} \times 9.1^2 \times 10^{-62} \times 125\times 10^{-33}}{1.6^4 \times 10^{-76}} = 4\times 10^{-11} s \text{ (2 points)}$$

Note that because of a typo in the examination paper some students may use m = 9.1 x 10^{31} kg, and *t* so obtained is off by a factor of 10^{62}. Take such answer as correct one.

Q-7 (15 points)

An automatic mechanical damping system is shown to the right. The seat of mass M can move vertically inside the cylinder. The wall of the cylinder is coated with a thin layer of electro-rheological (ER) fluid. The friction force between the seat and the wall is $f = -\gamma v$, where v is the relative speed between wall and seat, and γ is the viscosity constant. The force f is always in the opposite direction of relative motion. The γ of the ER fluid can be adjusted by a voltage applied between the seat and the wall (both are metallic). At the bottom of the cylinder there is an ideal spring with force constant K and natural length d.



(a) Choose your own x = 0 initial position, write down the general expression for the acceleration of the seat when it is moving at velocity v at an arbitrary position x. (2 points)
(b) Find the position of the seat when everything is stationary. (2 points)
(c) An impulse then hits the cylinder from below at t = 0, instantly giving it an upward velocity v_0. The viscosity at this moment is γ_0. Using the cylinder as the reference frame, find the acceleration a_0 of the seat at this moment. (2 points)
(d) The active damping is now turned on and γ is controlled by the applied voltage. Find γ as a function of time such that the acceleration at t (> 0) remains to be a_0 until the seat comes to rest. (5 points)
(e) Find the distance traveled by the seat in (d). (2 points)
(f) Find the amount of energy converted to heat due to friction in (d). (2 points)

Solution:

(a) Choose x = 0 point when the spring is at its natural length, i. e., when the seat is at distance d from the bottom, and downward as positive. $a = \dfrac{mg - \gamma v - kx}{m}$.

(b) $mg - kx_0 = 0$, $x_0 = mg/k$.

(c) In cylinder reference frame, the seat suddenly acquires v_0 downwards at t = 0. $a_0 = \dfrac{-\gamma_0 v_0}{m}$

(d) The seat is decelerating at constant a_0, so the velocity at any time t is

$v(t) = v_0 - a_0 t$ (1 point)

$x(t) = x_0 + v_0 t - \dfrac{1}{2} a_0 t^2 = \dfrac{mg}{k} + v_0 t - \dfrac{1}{2} a_0 t^2$, (1 point)

Using the answer in (b), $\gamma(t) = \dfrac{mg - ma_0 - kx(t)}{v(t)}$, with a_0 given in (c), $x(t)$ and $v(t)$ given above. (3 points)

(e) Set $v(t) = 0$ in (d), we get $T = \dfrac{v_0}{a_0}$, (1 point)

and using the results in (d) we get $x(T) = \dfrac{v_0^2}{2a_0}$. (1 point)

(f) Using energy conservation. At $t = 0$ the total mechanical energy is (choose the gravitational potential energy to be zero at this point)

$E_i = \dfrac{1}{2} m v_0^2 + \dfrac{1}{2} k x_0^2$ (1 point)

The final energy is $E_f = \dfrac{1}{2} k x(T)^2 - mgx(T)$, and energy loss to friction is

$\Delta E = E_i - E_f =$… (putting v_0, x_0, $x(T)$ in) (1 point)

Hong Kong Physics Olympiad 2005
2005 年香港物理奧林匹克
Written Examination 筆試

Jointly Organized by
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Physical Society of Hong Kong 香港物理學會
HKUST 香港科技大學
共同舉辦
May 29, 2005
2005 年 5 月 29 日

The following symbols and constants will be used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitation constant, 6.67 x 10^{-11} (N m^2/kg^2)
e – charge of an electron, –1.6 x 10^{-19} (A s)
ε_0 – electrostatic constant, 8.85 x 10^{-12} (A s)/(V m)
m_e – electron mass = 9.11 x 10^{-31} kg
c – speed of light in vacuum, 3.0 x 10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance = 1.5 x 10^{11} m
Density of water = 1.0 x 10^3 kg/m^3
Density of iron = 7.7 x 10^3 kg/m^3
Density of mercury = 13.6 x 10^3 kg/m^3
Speed of sound in air = 340 m/s

除非特別說明，本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 重力常數, 6.67 x 10^{-11} (N m^2/kg^2)
e – 電子電荷, –1.6 x 10^{-19} (A s)
ε_0 – 靜電常數, 8.85 x 10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11 x 10^{-31} kg
c – 真空光速, 3.0 x 10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 = 1.5 x 10^{11} m
水的密度 = 1.0 x 10^3 kg/m^3
鐵的密度 = 7.7 x 10^3 kg/m^3
水銀的密度= 13.6 x 10^3 kg/m^3
空氣中聲速= 340 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much lower than the speed of light.

除非特別說明，本卷將使用下列條件：

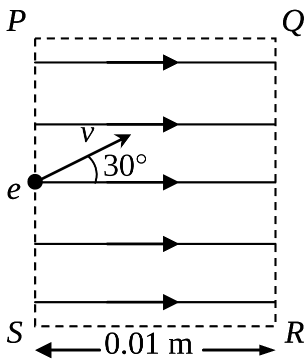
1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠低於光速.

Multiple choice questions (2 points each. Select one answer in each question.)
選擇題（每道題二分，每道題選擇一個答案）

[1] A car of mass m is slipping down a slope of inclination angle θ at a constant acceleration a. The static friction coefficient between the wheels and the slope is μ. What is the friction force between the wheels and the slope?
一質量爲 m 的小車從斜度爲θ的斜坡以勻加速度 a 滑落下來。其輪與坡面的靜磨擦係數爲μ。求輪與坡面的磨擦力。

(a) $\mu mg\cos\theta$. (b) μmg. (c) $mg(\sin\theta - \mu)$. (d) $m(g - a)$.
(e) $mg\sin\theta - ma$.

[2] Refer to the figure, rectangle $PQRS$ represents the cross-section of a uniform magnetic field region of 0.20 T. An electron is projected at a speed of $v = 2.0 \times 10^6$ m/s into the region at an angle of 30° to the direction of the magnetic field. The length of the magnetic field region is 0.01 m. Find the number of revolutions made by the electron before it leaves the magnetic field region.



如圖所示。長方型區域 $PQRS$ 爲一 0.20 T 的均勻磁場的橫截面。區域長度爲 0.01 m。一電子以 $v = 2.0 \times 10^6$ m/s 的速度和與磁場成 30°的角度射入。求電子離開區域前轉的圈數。

(a) 28 (b) 16 (c) 9 (d) 6 (e) 32

[3] Two weights, both of mass m, are joined by a weightless spring of natural length l and force constant k. They are placed on a smooth surface and at rest. One weight is suddenly given an impulse and acquires an initial velocity v towards the other weight. What is the speed of the center of mass of the weights-spring system?

在光滑平面上有兩質量均爲 m 的物體，中間由一自然長度爲 l 、力常數爲 k 的輕彈簧相連。現突然給其中一物體一衝量，使它具有指向另一物體的初速度 v。求兩物體的共同質心的速度。

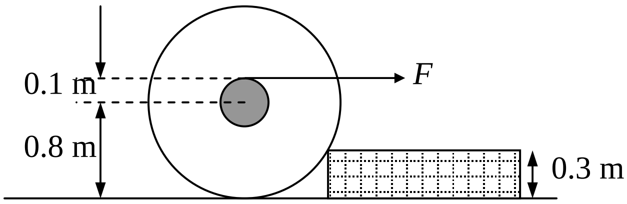
(a) $0.5v$. (b) $0.5v - \sqrt{kl^2/2m}$. (c) $\sqrt{kl^2/2m} - 0.5v$.
(d) v. (e) $0.5v - \sqrt{kl^2/m}$.

[4] Following the above MC. What is the minimum distance between the two weights?
接上題。求兩物體間最小距離。

(a) $l - \frac{v}{2}\sqrt{\frac{m}{k}}$. (b) $l - v\sqrt{\frac{m}{2k}}$. (c) $l - v\sqrt{\frac{m}{k}}$. (d) $v\sqrt{\frac{m}{k}}$.
(e) $\frac{v}{2}\sqrt{\frac{m}{k}}$.

[5] As shown, a wheel of weight W and radius 0.8 m is placed against a 0.3 m height rectangular block fixed on the ground. The wheel has an axle of radius 0.1 m. A force F is applied tangentially to the axle to lift the wheel. The minimum value of F is___.

如圖所示。一重量爲 W 半徑爲 0.8 m 的輪子放在 0.3 m 高的固定方磚前。輪軸的半徑爲 0.1 m。現沿輪軸切向施力 F 以拉起輪子。求所需最小力。



(a) 1.05W (b) 0.86W (c) 0.69W (d) 0.32W
(e) 2.45W

[6] A helicopter is trying to land on a ship deck which is drifting south (unit vector $\vec{y}_0$) at 17m/s. A 12m/s wind is blowing from east (unit vector $\vec{x}_0$). The ship crew sees the helicopter descending at 5 m/s. Take the downwards direction as unit vector $\vec{z}_0$. What is its velocity relative to water and air?
一直升機要降落在以 17m/s 向南(單位向量 $\vec{y}_0$)飄流的船上。此時刮的是風速爲 12m/s 的東風(單位向量 $\vec{x}_0$)。船員見到直升機以 5 m/s 的速度垂直降落下來。取向下方向爲單位向量 $\vec{z}_0$。求直升機相對于水和空氣的速度。

(a) $(5\vec{y}_0 - 17\vec{z}_0)$ m/s; $(-12\vec{x}_0 + 17\vec{y}_0 + 5\vec{z}_0)$ m/s

(b) $(-12\vec{x}_0 + 17\vec{y}_0 + 5\vec{z}_0)$ m/s; $(17\vec{y}_0 + 5\vec{z}_0)$ m/s

(c) $(5\vec{z}_0)$ m/s; $(-12\vec{x}_0 + 17\vec{y}_0 + 5\vec{z}_0)$ m/s

(d) $(17\vec{y}_0 + 5\vec{z}_0)$ m/s; $(-12\vec{x}_0 + 5\vec{z}_0)$ m/s

(e) $17\vec{y}_0$ m/s; $(-12\vec{x}_0 + 5\vec{z}_0)$ m/s

[7] Suppose the force by air to a plane is always perpendicular to its wings' surfaces. The plane is moving in a circle of radius R at speed v. The inclination angle θ of the wings should satisfy ________.
假設空氣對機翼的力總是與機翼面垂直，當飛機以速度 v 作半徑爲 R 的圓周運動時，機翼面與水平線的角度應該滿足________。

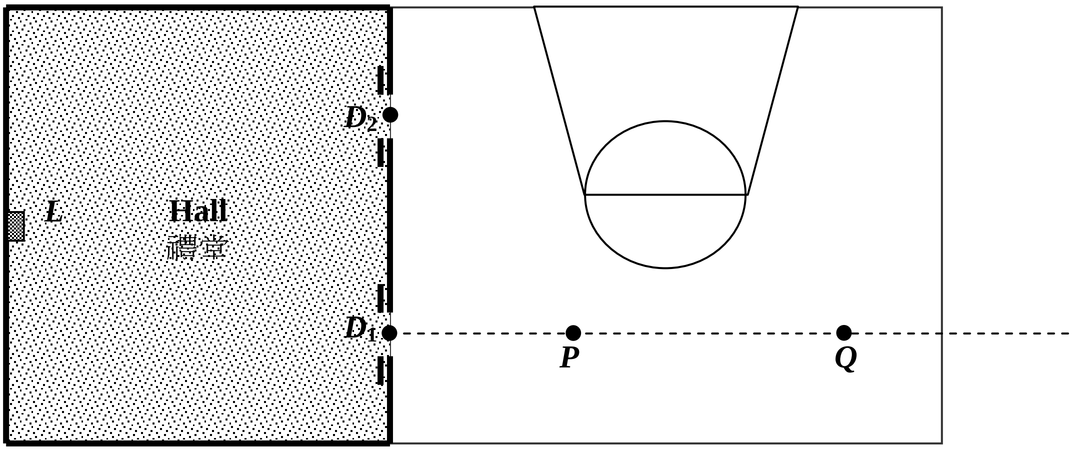
(a) $\sin\theta = \dfrac{v^2}{Rg}$ (b) $\cos\theta = \dfrac{v^2}{Rg}$ (c) $\tan\theta = \dfrac{v^2}{Rg}$

(d) $\tan\theta = \dfrac{Rg}{v^2}$ (e) $\theta = \dfrac{Rg}{v^2}$

[8] The mass of the sun is______.
太陽的質量爲______。

(a) 2×10^{27} kg (b) 2×10^{28} kg (c) 2×10^{29} kg
(d) 2×10^{30} kg (e) 2×10^{31} kg

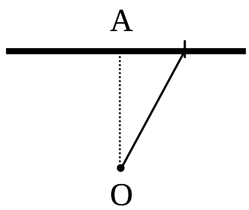
[9] A loudspeaker L is placed in the hall with two doors D_1 and D_2 open to the playground, as shown below. The distance between D_1 and D_2 is 8.5 m. The loudspeaker L is at equidistance from D_1 and D_2. Monotonic sound waves are emitted from the loudspeaker, and it is found that at point P which is 6.0 m from D_1 and at point Q the sound intensities are minimum. The line joining D_1, P and Q is perpendicular to the line joining D_1 and D_2. No other minimum intensity locations can be found between PQ and beyond Q along the PQ line. Find the frequency of the sound wave generated by the loudspeaker.



如圖，禮堂裏有一擴音器 L，離兩門 D_1 和 D_2 等距，並發出單頻聲波。兩門間距離爲 8.5 m。門外是操場。在離 D_1 6.0 m 處的 P 點和 Q 點發現聲波的強度最低。D_1PQ 連線與 D_1D_2 連線垂直。PQ 連線上 PQ 點之間和過了 Q 點後再無最弱聲波點。求聲波頻率。

(a) 17 Hz (b) 41 Hz (c) 52 Hz (d) 116 Hz
(e) 123 Hz

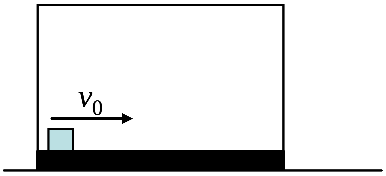
[10] As shown in the figure, a smooth rod is mounted horizontally on a tabletop. A 10-kg collar, which is able to slide on the rod without friction, is fastened to a spring whose other end is fixed at point-O. The nearest point of the rod to point-O is point-A, and the distance is 20 cm. The spring has a natural length of 10 cm and of negligible mass, and a spring constant of 500 N/m. The collar is released at 15 cm from point-A. Find its speed when reaching point-A.



如圖，水平桌面上有一固定的光滑杆，上套一重 10-kg 的環。一彈簧一端連著環，另一端固定在距離杆 20 cm 的 O 點。彈簧自然長度爲 10 cm，力常數 500 N/m。A 點是杆上離 O 點最近點。現將環拉到離 A 點 15 cm 處放開。求環到 A 點時的速度。

(a) 0.59 m/s (b) 0.791 m/s (c) 1.04 m/s (d) 0.88 m/s
(e) 1.24 m/s

[11] As shown, a big box of mass M is resting on a horizontal smooth floor. On the bottom of the box there is a small block of mass m. The block is given an initial speed v_0 relative to the floor, and starts to bounce back and forth between the two walls of the box. Find the final speed of the box when the block has finally come to rest in the box.



一質量爲 M 的大盒放在光滑地板上。盒底有一質量爲 m 的物體。現給該物體一初速度 v_0，使它在盒的兩壁來回碰撞。求最後物體在盒裏停下後盒的速度。

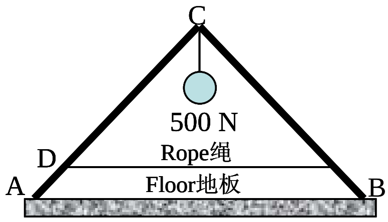
(a) 0 (b) v_0 (c) $\frac{m}{M}v_0$ (d) $\frac{M}{m+M}v_0$

(e) $\frac{m}{m+M}v_0$

[12] A jet of water from the 30cm-diameter nozzle of a fire hose can reach the maximum height of 25 meters. How large is the force from the water jet to the hose?
從直徑爲 30 cm 的消防水管射出的水柱最高可達 25 米。求水柱對水管的力。

(a) 15 kN (b) 3.46 kN (c) 346 N (d) 3.46 N
(e) 34.6 kN

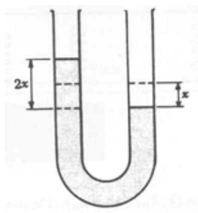
[13] As shown in the figure, AB = 3.5 m, AC = 3.0 m, AD = 0.5 m. The two rods AC and BC weight 150 N each. The floor is frictionless. Find the tension in the rope.



如圖，AB = 3.5 m, AC = 3.0 m, AD = 0.5 m。杆 AC 和 BC 各重 150 N。地板與杆間無磨擦。求繩的張力。

(a) 280 N (b) 500 N (c) 150 N
(d) 300 N (e) 180 N

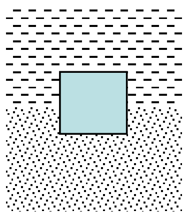
[14] 9 kg of mercury is poured into a glass U-tube with inner diameter of 1.2 cm. The mercury can flow without friction within the tube. Find the oscillation period.



一內徑爲 1.2 cm 的 U 型玻璃管裏裝有 9 kg 的水銀。設水銀可無磨擦地在管裏流動。求振盪週期。

(a) 1.2 s (b) 3.4 s (c) 5.6 s (d) 7.8 s
(e) 8.9 s

[15] A tank contains water on top of mercury. A cube of iron is sitting upright in equilibrium in the liquids. Find the fraction of its total volume in mercury.



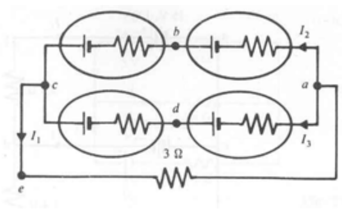
一水缸的下部是水銀，上部是水。一立方體的鐵塊正放在液體裏。求鐵塊在水銀裏的體積與總體積之比。
(a) 0.35 (b) 0.53 (c) 0.1 (d) 0.62 (e) 0.73

[16] The average density of Earth is _____ x 10^3 kg/m^3.

地球的平均密度爲____ x 10^3 kg/m^3。

(a) 3.1 (b) 2.2 (c) 5.5 (d) 1.1 (e) 4.1

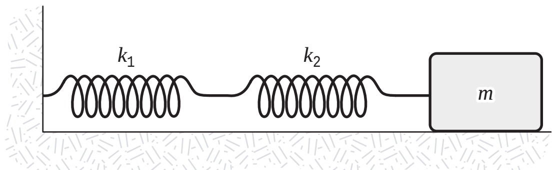
[17] Each of the four cells shown has an emf of 3.0 V and a 0.0075-Ω internal resistance. Find the current through the 3-Ω resistor.



如圖，每個電池的電動勢爲 3.0 V ，內阻爲 0.0075-Ω 。求通過 3-Ω 電阻的電流。

(a) 0.97 A (b) 0 (c) 1.95 A (d) 0.48 A
(e) 3.23 A

[18] The mass in the figure below slides on a frictionless surface. When the mass is pulled out, spring 1 is stretched a distance x_1 from its equilibrium position and spring 2 is stretched a distance x_2. The spring constants are k_1 and k_2 respectively. Find the force pulling back on the mass.



如圖，物塊可在光滑平面滑行。現將物塊拉出，使彈簧-1 拉長了 x_1 ，彈簧-2 拉長了 x_2。彈簧-1 和彈簧-2 的力常數分別爲 k_1 和 k_2 。求彈簧對物塊的拉力。

(a) $-k_2x_1$. (b) $-k_2x_2$. (c) $-(k_1x_1+k_2x_2)$

(d) $-\dfrac{k_1+k_2}{2}(x_1+x_2)$ (e) $-\dfrac{k_1k_2}{k_1+k_2}(x_1+x_2)$.

[19] An empty open bottle has an inner volume of 1.31 x 10^{-4} m^3. It has a mass of 112 g when filled with air at 1 atm, and it displaces 1.63 x 10^{-4} m^3 of water when fully submerged. What fraction of the total volume of the bottle will be beneath the surface when it floats on water but without water inside the bottle?

一開口小瓶的內體積爲 1.31 x 10^{-4} m^3。裝滿 1 個大氣壓時的總質量爲 112 g。當完全浸在水裏時小瓶排開的水體積爲 1.63 x 10^{-4} m^3。現將小瓶口向上浮在水面上，瓶裏無水，求在水面下的體積部分與總體積之比。

(a) 69% (b) 18% (c) 38% (d) 100%
(e) 46%

[20] A parallel plate capacitor of capacitance *C* is charged to potential *V* by a battery. The battery is then disconnected. Which statement is correct?

一平行板電容器被加上電壓 *V* 後與電池斷開。以下哪個說法是正確的？

(a) There is no charge on either plate of the capacitor. 電容器板上無電荷。

(b) The capacitor can be discharged by grounding any one of its two plates. 如要電容器放電可將其中任意一板接地。

(c) Charge is distributed evenly over both the inner and outer surfaces of the plates. 電荷均勻分佈在兩板的內外表面上。

(d) The magnitude of the electric field outside the space between the plates is approximately zero. 電場在兩板間以外的空間幾乎為零。

(e) The capacitance increases when the distance between the plates increases. 電容器的電容會因板間距離增加而增加。

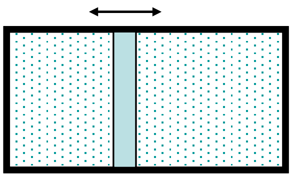
Open Problems 開放題
Total 6 problems 共 6 題

Q1 (8 points)

On a smooth and insulating ring of radius R there is a small ring of mass m and carrying charge q. The large ring is placed horizontally and in a uniform magnetic field of strength B_0 and perpendicular to the ring plane. Starting from $t = 0$, the magnetic field is changed to $B(t) = B_0 + \alpha t$. Find the force of the small ring on the big ring afterwards.

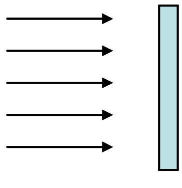
題 1 （8 分）
一半徑爲 R 的光滑絕緣大圓環上有一質量爲 m 帶電爲 q 的小環。大圓環水平放置，與一強度爲 B_0 的均勻恒定磁場垂直。從時間 $t = 0$ 開始該磁場變爲 $B(t) = B_0 + \alpha t$ 。求之後小環對大圓環的力。

Q2 (8 points)
As shown in the figure, separating two sealed gas chambers is a piston of mass m and area A that can move horizontally without friction. The volume and temperature of the left chamber is V_1 and T_1, and that in the right chamber is V_2 and T_2. At equilibrium the pressure in both chambers is P. Giving the piston a small displacement Δx off balance to the right, find the maximum displacement to the left and the time needed to get there. The temperatures remain the same. (Hint: $\dfrac{1}{1-x} \cong 1+x$ for $x << 1$)



題 2（8 分）
如圖所示，兩封閉氣室間有一質量爲 m 面積爲 A 的活塞可無摩擦地左右滑動。左右氣室的體積和溫度分別爲 V_1 、T_1 和 V_2 、T_2。平衡時兩邊的氣壓均爲 P。現把活塞向右推一小距離 Δx，求活塞可向左運動的最大位移和所需時間。過程中兩邊溫度不變。(提示: $\dfrac{1}{1-x} \cong 1+x$ ，如果 $x << 1$)

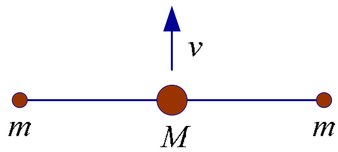
Q3 (8 points)
The shaded area in the figure is the side view of a disk shaped magnetic field region of radius R and thickness d. A parallel electron beam being accelerated by a voltage V is normally incident onto the region. Find the spatial distribution of the magnetic field (magnitude and direction) in the region such that the beam is focused to a point along the disk central axis at a distance L (>> R) from the disk. Your answer could also include the electron mass and charge.



題 3（8 分）
圖中陰影部分爲一半徑 R 厚度 d 的碟型磁場區的側面。一平行電子束被電壓 V 加速後垂直射入磁場區。如要求電子都被聚焦在碟軸線離磁場區距離 L (>> R)的點上，求磁場區裏磁場的空間分佈（方向和強度）。你的答案可包含電子的電荷和質量。

Q4 (10 points)

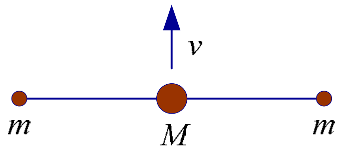
As shown, a large ball of mass M is connected on each end by a weightless thread of length l to a small ball of mass m. Initially the three balls are along the straight line on a smooth surface. The large ball is suddenly given an initial velocity v in the direction perpendicular to the line. Find



(a) The tension in the thread at the moment the large ball gets the impact;
(b) The tension in the thread at the moment the two small balls meet.

題 4（10 分）

長度為 2 l 的輕繩，兩端各系一個質量為 m 的小球，中央系一個質量為 M 的大球。同一直線上的三個球均靜止於光滑的水平桌面上。現給球 M 以一個衝量，使它獲得與繩垂直的水平速度 v。試求:

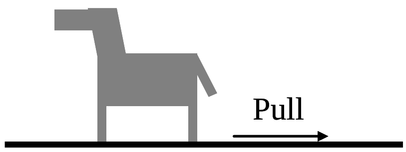


(a) 當 M 剛受到衝量時，繩中的張力。

(b) 在兩端小球發生碰撞的瞬間，繩中的張力。

Q5 (13 points)

A wooden toy horse rests on a tablecloth on a table, with its front legs 0.3 m from the cloth edge. It weighs 100 grams and its center of mass is 0.05 m from the front legs and 0.05 m above ground. The distance between the front and back legs is 0.15 m. The tablecloth is suddenly yanked horizontally with constant acceleration of 9.0 m/s^2 relative to the table. The friction coefficient between the cloth and the horse is μ = 0.75. Find



(a) the acceleration of the horse relative to the table;
(b) the force on each leg of the horse by the tablecloth;
(c) the velocity and the distance the horse has traveled relative to table when the edge of the tablecloth reaches the front legs.
(d) If the height of the center of mass could be adjusted, find the value above which the horse would tip off.

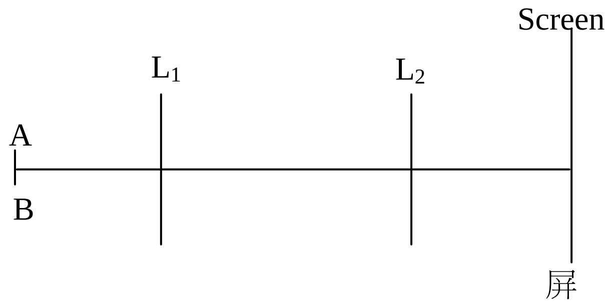
題 5（13 分）

一玩具木馬，質量為 100 克，重心在離前腿 0.05 m，離地 0.05 m 處，前後腿距離為 0.15 m，放在平桌面的桌布上，前腿離桌布邊 0.3 m。布與木馬間的磨擦係數為μ = 0.75。現突然將桌布以相對於桌面 9.0 m/s^2 的加速度拉走。求

(a) 木馬相對於桌面的加速度;
(b) 桌布對木馬前後腿的力;
(c) 當木馬到達桌布邊時相對於桌面的速度和位移。
(d) 如重心高度可變，求可保持木馬不翻轉的最大重心高度。

Q6 (13 points)

As shown, L_1 and L_2 are two thin lenses sharing the same optical axis and 30 cm apart. The focus length of L_1 is 10 cm, its radius 4.0 cm, and the focus length of L_2 is 5.0 cm, and its radius 2.0 cm. AB is a bright disk object of radius 2.0 cm and 20 cm from L_1. (a) Find the position of the screen so that a clear image of the disk is formed. (b) The edge of the image is found to be dimmer than the center. Why? (c) In order make the image uniformly bright on the screen, a third lens is added. Find the position, focus length, and radius of the lens.



題 6（13 分）

如圖所示，L_1 和 L_2 是兩個共軸透鏡。兩鏡距離爲 30 cm。L_1 的焦距爲 10 cm，半徑爲 4.0 cm。L_2 的焦距爲 5.0 cm，半徑爲 2.0 cm。AB 爲一半徑爲 2.0 cm 的明亮圓碟，與透鏡共軸，距離 L_1 爲 20 cm。屏上有一圓碟的像。（a）求屏的位置。(b)爲何像的邊緣不如中心明亮？(c)爲使邊緣和中心一樣亮，需加一透鏡 L_3。求 L_3 的位置，焦距，半徑。

Hong Kong Physics Olympiad 2005

Answers and suggested solutions

The Answer of Multiple-choice questions:

1. e	2. e	3. a	4. b	5. a	6. c	7. c	8. d	9. d	10. b
11. e	12. a	13. a	14. b	15. b	16. c	17. c	18. c	19. c	20. d

The Answer of Open Questions:

Q1 (8 points)

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t} \qquad ( )$$

$$\Rightarrow \oint E \bullet d\vec{l} = -\oint \frac{\partial B}{\partial t} ds \qquad ( )$$

$$\Rightarrow E.(2\pi R) = -\alpha \pi R^2 \qquad (2')$$

$$E = -\frac{\alpha R}{2}$$

$$\because v = \frac{ft}{m} = \frac{\alpha q R t}{2m} \qquad (2')$$

$$\Rightarrow F - qvB = \frac{mv^2}{R} \qquad (2')$$

$$F = \frac{\alpha q^2 R t}{4m}(2B_0 + \alpha t) \qquad (2')$$

Q2 (8 points)

$$PV_1 = P_1^{'}(V_1 + A\Delta x) \qquad (2')$$

$$P_1^{'} = \frac{P}{1 + (A\Delta x / V_1)} \approx P(1 - \frac{A\Delta x}{V_1})$$

$$PV_2 = P_2^{'}(V_2 - A\Delta x) \qquad (1')$$

$$P_2^{'} = \frac{P}{1 - (A\Delta x / V_2)} \approx P(1 + \frac{A\Delta x}{V_2})$$

$$F = A(P_1^{'} - P_2^{'}) \qquad (1')$$

$$= -PA^2\left(\frac{1}{V_1} + \frac{1}{V_2}\right)\Delta x$$

Which shows that the piston performs simple harmonic oscillation with $k = PA^2\left(\frac{1}{V_1} + \frac{1}{V_2}\right)$, the maximum displacement is Δx on the left side from equilibrium position. (2')

$$t = \pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{mV_1V_2}{PA^2(V_1+V_2)}} \qquad (2)$$

Q3 (8 points)

Solution 1:

$$t = \frac{d}{v} \quad \text{and} \quad v = \sqrt{\frac{2eV}{m}} \qquad (2')$$

When the electron enters the disk, the impulse is

$$m\Delta v = t(evB) \qquad (2')$$

$$\Rightarrow \Delta v = \frac{evBt}{m} = \frac{eBd}{m}$$

$$\Rightarrow \frac{\Delta v}{v} = \frac{eBd}{mv} = \frac{r}{L} \qquad \text{where } r \le R \qquad (3')$$

$$\Rightarrow B = \frac{r}{dL}\sqrt{\frac{2mV}{e}} \qquad (1')$$

Solution 2:

Let r' be the radius that electron changes it direction during inside the disk,

$$\frac{d}{r'} \approx \frac{R}{L} \quad \text{and} \quad v = \sqrt{\frac{2eV}{m}} \qquad (2')$$

$$evB = \frac{mv^2}{r} \quad \Rightarrow B = \frac{mv}{e}\frac{r}{dL} \quad \text{where} \quad r \le R \qquad (3')$$

$$\Rightarrow B = \frac{r}{dL}\sqrt{\frac{2mV}{e}} \qquad (2')$$

Q4: (10 points)

(a) Consider we observe the motion in the reference frame of mass M, the two small mass m will seen to be performing circular motion with initial velocity $-v$. The acceleration of M, by symmetry of the forces acting upon it, will be along $-v$ and perpendicular to the acceleration of the small masses. (1')

So, we have

$$T = \frac{mv^2}{l} \qquad (2')$$

(b) Ans:

$$2T_2 = Ma_M \quad \Rightarrow \quad a_M = \frac{2T_2}{M} \qquad (1) \qquad (1')$$

taking into account the initial force：

$$T_2 + ma_M = m\frac{{v_x}^2}{l}, \quad (2) \qquad (1')$$

From (1) and (2) one gets $T_2 = \dfrac{Mm{v_x}^2}{(M+2m)l}$. (1')

According to conservation of energy, kinetic energy of small balls in translational direction can be related as

$$2(\frac{1}{2}mv_x^2) = \frac{1}{2}Mv^2 - \frac{1}{2}(M+2m)v^2 \qquad (1')$$

$$\Rightarrow mv_x^2 = \frac{1}{2}Mv^2 - \frac{1}{2}(M+2m)\left(\frac{M}{M+2m}v\right)^2$$

$$\Rightarrow mv_x^2 = \frac{1}{2}Mv^2\left(\frac{2m}{M+2m}\right) \qquad (1')$$

Finally one gets

$$T_1 = \frac{M^2mv^2}{(M+2m)^2 l} \qquad (2')$$

Q5: (13 points)

(a) Let the acceleration of horse relative to the table be a_{ht},

$$ma_{ht} = \mu mg \qquad (1')$$

$$a_{ht} = \mu g = 0.75 \times 9.8ms^{-2} = 7.35ms^{-2} \qquad (1')$$

(b) Consider the net moment acting on the toy horse should be zero, we have

$$N_1 + N_2 = mg \qquad (1')$$

$$N_1 r_1 = N_2 r_2 + mg\mu h \qquad (2')$$

$$\Rightarrow (mg - N_2)r_1 = N_2 r_2 + mg\mu h$$

$$\Rightarrow N_2 = mg\frac{r_1 - \mu h}{r_1 + r_2} \qquad (1')$$

$$N_2 = (0.10kg)(9.8ms^{-2})\frac{0.05m - 0.75 \times 0.05m}{0.15m}$$

$$= 8.16 \times 10^{-2} N \qquad (1')$$

(c) Let the acceleration of horse relative to the tablecloth be a_{hc},

$$a_{hc} = a - a_{ht} = (9.0 - 7.35)ms^{-2} = 1.65ms^{-2} \qquad (1')$$

The time required for the horse reaches the edge of tablecloth is

$$t = \sqrt{\frac{2s}{a_{hc}}} = \sqrt{\frac{2(0.3m)}{1.65ms^{-2}}} = 0.603s \qquad (1')$$

The velocity of horse relative to table at time t is

$$v_{ht} = a_{ht}t = 4.432ms^{-1} \qquad (1')$$

The displacement on the table is

$$s' = \frac{1}{2}a_{ht}t^2 = \frac{1}{2}(7.35ms^{-1})(0.603s)^2 = 1.336m \qquad (1')$$

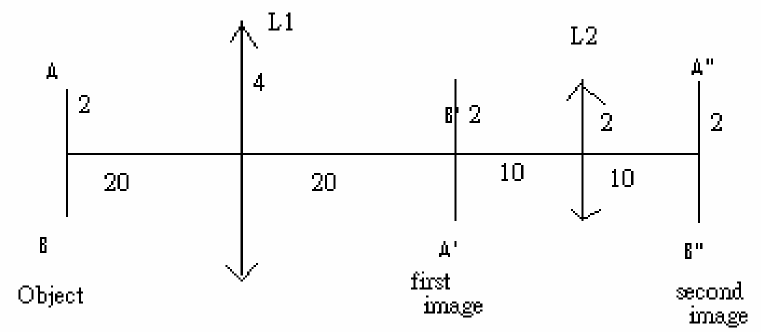
(d) When the horse is fallen, $N_2 = 0$. It implies that

$$r_1 - \mu h \geq 0 \qquad (1')$$

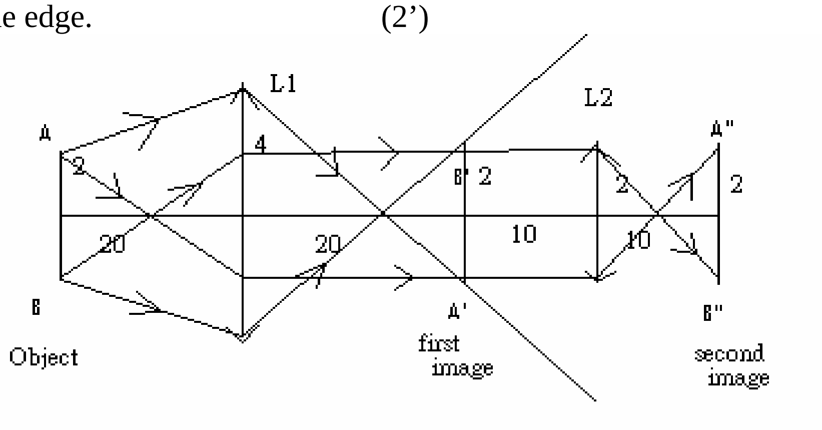
$$h \leq \frac{r_1}{\mu} = \frac{0.05m}{0.75} = 6.67 \times 10^{-2} m \qquad (1')$$

Q6 (13 points)

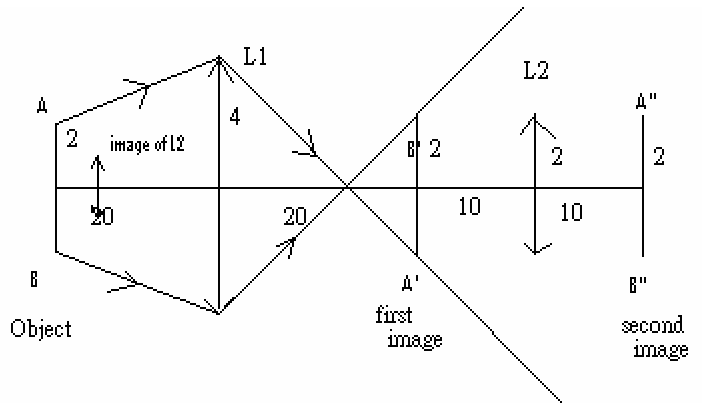
(a) Using the lens formula, (1')
the image of AB after L_1 is at 20 cm after L_1 and 10 cm from L_2. (1')
The image after L_2 is at 10 cm from it. That is where the screen should be. (1')



(b) All the light from the center point of AB that passes through L_1 will get through L_2. (1')
For the light from point-A which passes through the edge of L_1, using graphic method one can show that it will not pass through L_2 so it will not reach the image on the screen. Only the light through the central area of L_1 will pass L_2 and reach the screen. So the central image is brighter than the edge. (2')



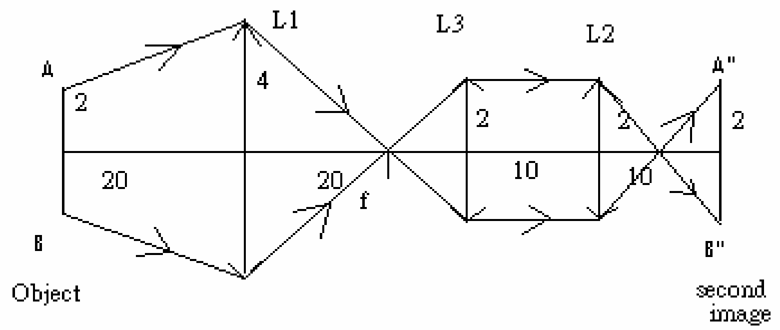
Alternatively, the image of L_2 by L_1 is at 15 cm from L_1 to the left. (1')
Its size is 2 x 15/30 = 1 cm (1')
Light from the edges of object AB can reach L_1 without going through the image of L_2, and the same conclusion as above is reached.



(c) To keep the image on the screen, L_3 must be placed at the image of AB after L_1. (2')

To allow all light pass, the size of L_3 must be at least that of the AB image, which is 2 cm in radius. (2')

To determine the focus length, one may use graphic method and geometry.



This is equivalent to require the image of L_2 formed by L_3 to coincide with L_1.
The focus length is then 6.67 cm. (3')

Hong Kong High School Physics Olympiad 2006
2006 年香港中學物理競賽
Written Examination
筆試

Jointly Organized by

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The Hong Kong Physical Society
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共同舉辦

May 28, 2006
2006 年 5 月 28 日

The following symbols and constants will be used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitation constant, 6.67 x 10^{-11} (N m^2/kg^2)
e – charge of an electron, –1.6 x 10^{-19} (A s)
ε_0 – electrostatic constant, 8.85 x 10^{-12} (A s)/(V m)
m_e – electron mass = 9.11 x 10^{-31} kg
c – speed of light in vacuum, 3.0 x 10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance = 1.5 x 10^{11} m
Earth-Moon distance = 3.84 x 10^8 m
Density of water = 1.0 x 10^3 kg/m^3
Density of iron = 7.7 x 10^3 kg/m^3
Density of mercury = 13.6 x 10^3 kg/m^3
Speed of sound in air = 340 m/s

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 重力常數, 6.67 x 10^{-11} (N m^2/kg^2)
e – 電子電荷, –1.6 x 10^{-19} (A s)
ε_0 – 靜電常數, 8.85 x 10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11 x 10^{-31} kg
c – 真空光速, 3.0 x 10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 = 1.5 x 10^{11} m
地球-月球距離 = 3.84 x 10^8 m
水的密度 = 1.0 x 10^3 kg/m^3
鐵的密度 = 7.7 x 10^3 kg/m^3
水銀的密度= 13.6 x 10^3 kg/m^3
空氣中聲速= 340 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

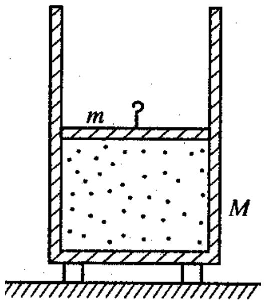
除非特別注明，否則下列條件將適用於本卷所有問題：

1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions (2 points each. Select one answer in each question.)
選擇題（每道題 2 分，每道題選擇一個答案）

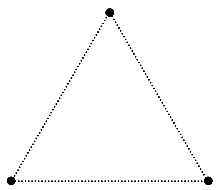
MC1
Given that the moon revolves around Earth at a period of about 27.3 days, use the parameters given in page-2 to find the mass of Earth.
利用**頁 2** 提供的資料，以及月亮大約每 27.3 天繞地球轉一次，估算地球的質量。
(a) 2.3×10^{24} kg (b) 4.6×10^{24} kg (c) 6.0×10^{24} kg (d) 7.8×10^{24} kg
(e) 9.8×10^{24} kg

MC2
As shown, a piston chamber of cross section area A is filled with ideal gas. A sealed piston of mass m is right at the middle height of the cylinder at equilibrium. The friction force between the chamber wall and the piston can be ignored. The mass of the rest of the chamber is M. The atmosphere pressure is P_0. Now slowly pull the piston upwards, find the maximum value of M such that the chamber can be lifted off the ground. The temperature remains unchanged.
如圖所示，在地面上放置一橫截面積為 A 的圓筒。筒內有一可上下無摩擦滑動且不漏氣的質量為 m 的活塞，活塞下方為理想氣體。平衡時活塞正好位於容器內的正中間位置。已知溫度不變，大氣壓強為 P_0。現在用力非常緩慢地上提活塞， 最後要能將氣缸提離地面，求氣缸的質量 M 的最大值。



(a) $M = \dfrac{P_0 A - 2mg}{2g}$ (b) $M = \dfrac{P_0 A - mg}{2g}$ (c) $M = \dfrac{P_0 A - mg}{g}$
(d) $M = \dfrac{P_0 A}{2g}$ (e) $M = \dfrac{P_0 A}{g}$

MC3
Find the electrostatic energy of an equilateral triangle of side a with a point charge q at each vertex, excluding the self-energy of the point charges.
邊長為 a 的等邊三角形的每個角上有一點電荷 q。求系統的靜電能量（不包括點電荷本身的能量）。



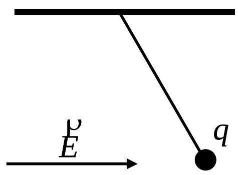
(a) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a}$ (b) $\dfrac{3}{4\pi\varepsilon_0}\dfrac{q^2}{a}$ (c) $\dfrac{3}{2\pi\varepsilon_0}\dfrac{q^2}{a}$
(d) $\dfrac{1}{8\pi\varepsilon_0}\dfrac{q^2}{a}$ (e) $-\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a}$

MC4
Two cars A and B are moving towards each other along the same line at 2/3 of the sound speed. Car-A sends out continuous sound waves at frequency f. The frequency of sound heard by the driver of car-B is _______.
兩車 A 和 B 各以 2/3 聲速的速度沿直線迎面駛近。A 車發出的聲波的頻率為 f。B 車聽到的聲波的頻率為_______。
(a) $5f$ (b) $9f$ (c) $f/5$ (d) $f/9$ (e) f

MC5

As shown, the tiny ball at the end of the thread of length 100 cm has a mass of 0.6 g and carries charge 5.88×10^{-6} C. It is in a horizontal electric field of intensity 1000 N/C (Newton per Coulomb). Find the vibration frequency of the ball near its equilibrium position.

如圖，一質量爲 0.6 g 的小球，帶電 5.88×10^{-6} C，繫在長爲 100 cm 的細繩一端，在強度爲 1000 N/C 的水平電場裏。求小球在平衡點附近的振動頻率。



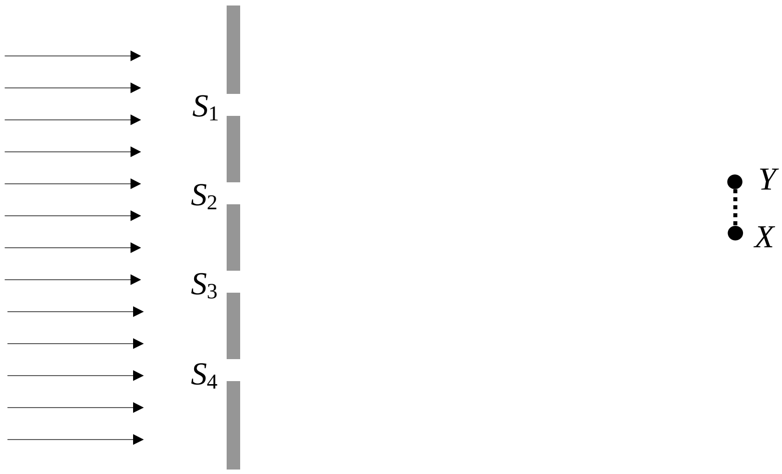
(a) 0.70 Hz (b) 0.59 Hz (c) 0.50 Hz (d) 3.2 Hz
(e) 6.4 Hz

MC6

As shown below, a plane microwave is normally incident on four evenly spaced identical narrow slits S_1, S_2, S_3 and S_4. XY is a line parallel to the plane of the four slits and far away from the slits. Point-X is at equal distance to slits S_2 and S_3. When only slits S_2 and S_3 are open, the wave intensity at point-X reaches the maximum value of A, and that at point-Y is zero. When all four slits are open, the intensity at point-X is _______.

如下圖，一平面微波正入射在四個等距窄縫 S_1、S_2、S_3、S_4 上。XY 連線離窄縫很遠，與窄縫平面平行。點 X 離 S_2、S_3 距離相等。當只有 S_2、S_3 開著時 X 點的波強度達到最大值 A，Y 點的波強度爲零。當四個窄縫都開時，點 X 處的波強度爲 _______ 。

(a) 0 (b) $2A$ (c) $4A$ (d) $8A$ (e) $16A$



MC7

Same condition as MC6. When all four slits are open, the intensity at point-Y is _______.

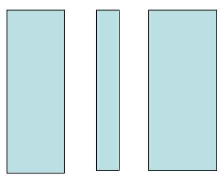
同上題。當四個窄縫都開時，點 Y 處的波強度爲 _______ 。

(a) 0 (b) $2A$ (c) $4A$ (d) $8A$ (e) $16A$

MC8

As shown, three large conductor plates of area A are placed parallel to one another at equal distance d. Find the capacitance between the left and the right plates.

如圖，三個大導電平板，面積爲 A，以等間距 d 排列。求左右板之間的電容。

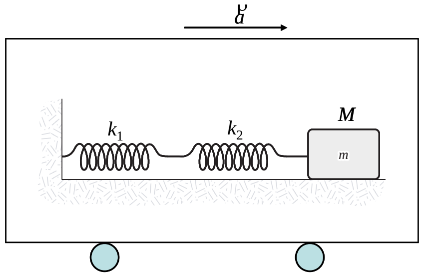


(a) $\frac{\varepsilon_0 A}{d}$ (b) $\frac{\varepsilon_0 A}{2d}$ (c) $\frac{2\varepsilon_0 A}{d}$ (d) $\frac{\varepsilon_0 A}{4d}$ (e) $\frac{4\varepsilon_0 A}{d}$

MC9

As shown, inside a cart that is accelerating horizontally at acceleration $\vec{a}$ there is a block of mass M connected to two light springs of force constants k_1 and k_2. The block can move without friction horizontally. Find the vibration frequency of the block.

如圖，在一以加速度 $\vec{a}$ 沿水平加速的車廂裏有一個質量爲 M 的物塊。物塊繫在兩根力常數分別爲 k_1、k_2 的輕彈簧上，可在水平面上無磨擦滑動。求物塊的振動頻率。



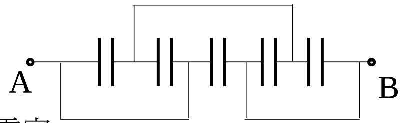
(a) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1+k_2}{M}}$ (b) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1+k_2}{M}+a}$ (c) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1+k_2}{M}-a}$

(d) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1k_2}{(k_1+k_2)M}}$ (e) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_1k_2}{(k_1+k_2)M}+a}$

MC10

Five identical 1 μF capacitors are connected as shown, find the capacitance between point-A and point-B.

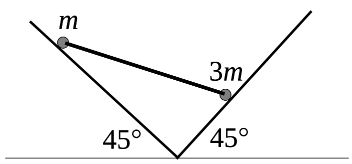
如圖。五個 1 μF 的電容器以圖示方式連接。求點 A 和 B 間的電容。



(a) 1 μF (b) 2 μF (c) 3 μF (d) 4 μF (e) 5 μF

MC11

Two small balls of mass m and $3m$, respectively, are connected by a thin and rigid bar with negligible mass, and are free to slide on the 45° inclines, as shown. Find the angle of the bar to the horizontal plane in equilibrium. The angle being negative means that the heavy ball is above the light ball.

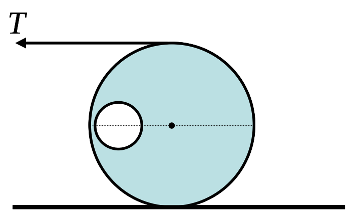


兩小球，質量分別爲 m 和 $3m$，由一輕細硬杆連接，可在兩相交的 45°平滑斜面上自由滑行。求當平衡時細杆與水平面的夾角。重球在上夾角爲負。

(a) 46.6° (b) 26.6° (c) 45° (d) – 26.6° (e) – 46.6°

MC12

A uniform cylinder of radius a originally had a weight of 80 N. After an off-axis cylinder hole at $2a/3$ was drilled through it as shown, it weighs 65 N. The axes of the two cylinders are parallel and their centers are at the same height. A force T is applied to the top of the cylinder horizontally. The value of the force should be _____ in order to keep the cylinder is at rest.



一均勻質量圓柱原重量爲 80 N，半徑爲 a。現在離軸心 $2a/3$ 處打一圓洞，洞軸與大圓柱軸心平行，並在同一高度。圓柱此時重量爲 65 N。在柱頂施一水平方向的力 T。如要柱體平衡，則力的大小爲____。

(a) 1 N (b) 3 N (c) 5 N (d) 8 N (e) 10 N

MC13

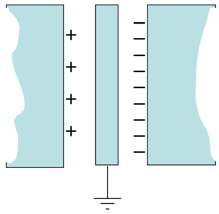
The electric field is $\vec{E} = (5\vec{x}_0 - 3\vec{y}_0)$ V/m. The potential at the point (0, 5 m, 5 m) is ___ if the potential at coordinate origin is taken as zero.

一電場爲 $\vec{E} = (5\vec{x}_0 - 3\vec{y}_0)$ V/m。如座標原點電勢爲零，則在 (0, 5 m, 5 m) 處的電勢爲 ____。

(a) – 25 V (b) – 15 V (c) 0 (d) 15 V (e) 25 V

MC14

As shown, an infinitely large surface on the left carries fixed surface charge density σ. The one on the right carries -2σ. A conductor slab is inserted between the two and its potential is fixed at zero. In equilibrium, the surface charge densities on the left and the right surfaces of the middle conductor are__________.

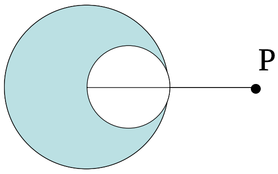


如圖，左邊的無窮大平面的固定表面電荷密度爲σ，右邊的無窮大平面的固定表面電荷密度爲 -2σ。現在兩面之間插入一導體板，其電勢保持爲零。平衡時導體板左、右表面的電荷密度分別爲_______。

(a) σ and -σ (b) -2σ and 2σ (c) -2σ and σ
(d) σ and -2σ (e) -σ and 2σ

MC15

A spherical cave of radius R/2 was carved out from a uniform sphere of radius R and original mass M. The center of the cave is at R/2 from the center of the large sphere. Point P is at a distance 2R from the center of the large sphere and on the joint line of the two centers. The gravitational field strength g at point P is ________.

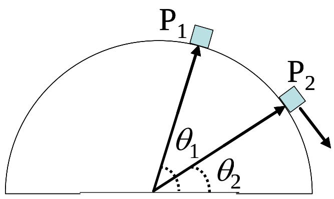


一均匀球，原來質量爲 M，半徑爲 R。現在其離球心 R/2 處挖一半徑爲 R/2 的空穴，求在球心與空穴中心連線上離球心 2R 處的引力場強度。

(a) $g=\frac{GM}{4R^2}$ (b) $g=\frac{5GM}{8R^2}$ (c) $g=\frac{3GM}{16R^2}$
(d) $g=\frac{GM}{2R^2}$ (e) $g=\frac{7GM}{36R^2}$

MC16

A small block slides down from rest at point P_1 on the surface of a smooth circular cylinder, as shown. At P_2 the particle falls off the cylinder. The equation relating the angles θ_1 and θ_2 is given by__________.



一小塊由靜止從光滑圓柱面上 P_1 點滑落，到 P_2 點離開柱面。兩角度θ_1 、θ_2 的關係須爲________。

(a) $\sin\theta_1=\frac{3}{2}\sin\theta_2$ (b) $\sin\theta_1=\frac{2}{3}\cos\theta_2$ (c) $\sin\theta_1=\cos\theta_2$

(d) $\cos\theta_1=\sin\theta_2$ (e) $\cos\theta_1=\frac{3}{2}\sin\theta_2$

MC17

50.0 g of ice at – 40 °C is mixed with 11.0 g of steam at 120 °C. Neglect any heat exchange with the surroundings. What is the final temperature of the mixture? (Specific heats of ice, water, and steam are 0.50, 1.00, and 0.481 cal (g °C)$^{-1}$, respectively. The heat of fusion of ice is 79.8 cal/g. The heat of vaporization of steam is 540 cal/g.)

將 50.0 g 在 – 40 °C 的冰與 11.0 g 在 120 °C 的水蒸汽混合。不計與外界的熱交換。求最終溫度。（冰、水、水蒸汽的比熱分別爲 0.50、1.00、0.481 cal (g °C)$^{-1}$。冰的熔化熱爲 79.8 cal/g。水的蒸發熱爲 540 cal/g。）

(a) 35.3°C (b) 30.3°C (c) 25.3°C (d) 20.3°C

(e) 15.3°C

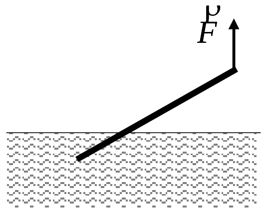
MC18

Electrons accelerated from rest by a voltage V enter a magnetic field of strength B which is perpendicular to the electron velocity. The trajectory of the electrons in the magnetic field is a circle of radius R. The electron charge to mass ratio e/m_e is then _________.

電子由靜止經過電壓 V 加速後進入強度爲 B 的磁場區，磁場方向與電子速度垂直。電子在磁場內的軌跡是半徑爲 R 的圓。由此得電子的電荷與質量之比 e/m_e 爲_____。

(a) $V/(BR)^2$ (b) $2V/(BR)^2$ (c) $4V/(BR)^2$ (d) $8V/(BR)^2$
(e) $16V/(BR)^2$

MC19

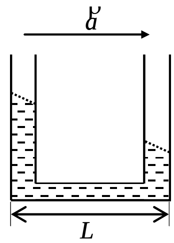


A thin uniform rod is partly immersed in water, while being lifted by a string fastened to one of its ends, as shown. If the density of the rod is 3/4 of that of water, what is the fraction of the length of the rod that is above the water when in equilibrium?

如圖。一均勻細杆，比重爲 3/4，一端吊在細繩上，部分在水裏。平衡時細杆有多少部分在水面上？

(a) 0.134 (b) 0.203 (c) 0.5 (d) 0.75 (e) 0.866

MC20



A U tube which is partially filled with liquid is in a vehicle under horizontal accelarating motion, as shown. Find the maximum difference in height of the liquid surface over the lateral distance of L.

如圖。一 U 型管，部分充有液體，在一作水平加速的車裏。求相距 L 的兩點液面的最大高度差。

(a) $\frac{g}{a}L$ (b) $\frac{a}{\sqrt{a^2+g^2}}L$ (c) $\frac{a}{g}L$ (d) $\sqrt{\frac{a}{g}}L$ (e) $\frac{g}{\sqrt{a^2+g^2}}L$

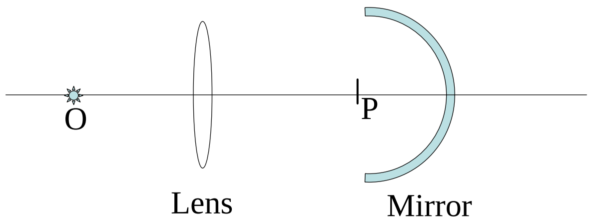
Open Problems 開放題
Total 6 problems 共 6 題

Q1 (5 points)
Describe, in less than one page, a way to determine the mass of a small object (~ kg) in weightless condition.

題 1（5 分）
以不多於一頁的篇幅，描述一種可在失重情況下確定一小物體質量（約 1kg）的方法。

Q2 (10 points)
As shown (not to actual scale), a thin convex lens with focus length 10 cm and a concave spherical mirror of radius 20 cm are placed on the same optical axis. The distance between the center of the sphere at point-P and the lens is 20 cm. A point light source O is placed on the optical axis and at 20 cm from the lens.

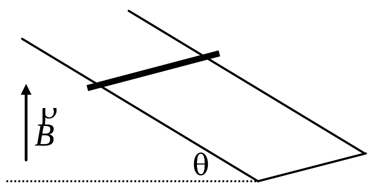
(a) Find the position of the final image of the light source.
(b) Assume the brightness of the final image in (a) is unity. The lens is then cut into two equal half disks, and the upper half is lifted by 1.0 mm from the original axis. Find the final image(s) and their brightness of the source.



題 2（10 分）
如上圖所示，一焦距爲 10 cm 的薄凸透鏡和一半徑爲 20 cm 的凹球面反射鏡共軸。P 點球心離透鏡 20 cm。一點光源 O 位於軸上離透鏡 20 cm 處。

(a) 求 O 的最終像位置。
(b) 設(a)中最終像的亮度爲 1。若將透鏡切成上下兩個半圓，上半部分提升到離原軸 1.0 mm 處，求 O 的最終像的位置和亮度。

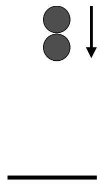
Q3 (10 points)
A conductor rod of length L, resistance R, and mass m is placed on an inclined rectangular frame made of perfect conductor in a uniform magnetic field B pointing upwards. The frame plane is at an angle θ to the horizon, as shown. Find the terminal velocity of the metal rod if
(a) there is no friction between the rod and the frame; and
(b) the friction coefficient between the rod and the frame is μ.



題 3（10 分）
如圖所示，一個理想導電金屬長方形框架以傾角 θ 放在方向向上的勻磁場 B 中，一長度爲 L, 質量爲 m, 電阻爲 R 的導體棒從框架上滑下。
(a) 求當棒和框架間無摩擦時棒的最終速度；
(b) 求當棒和框架間的摩擦係數爲 μ 時棒的最終速度。

Q4 (10 points)

Two small and hard spheres, one right on top of the other and almost in touch, are left to fall from a height H_0. The lower sphere of mass M collides with the ground, and almost instantaneously it collides with the upper sphere of mass $0.1M$. Both collisions are elastic. Find the maximum height the upper sphere can reach.



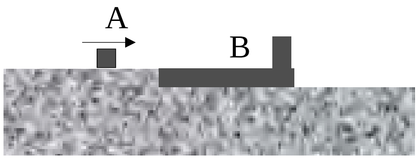
題 4（**10 分**）

兩個硬彈性小球，兩球連心線與地面垂直，兩球幾乎接觸，從高度 H_0 自由落下。設下球（質量爲 M）與地的碰撞以及緊接著它與上球（質量爲 $0.1M$）的碰撞都是瞬間的彈性碰撞，求上球回升的最大高度。

Q5 (12 points)

As shown, a block (Object-A) of mass m_A moves on a frictionless plane at initial speed v_0, and lands onto a cart (Object-B in the figure, mass m_B, length L, initially at rest) smoothly. Ignore the size of the block. The friction coefficient between A and B is μ and the cart is on a frictionless plane. The block collides elastically on the fixed wall at the end of the cart and eventually falls off the cart.

(a) Find the minimum value of v_0 such that the block can indeed fall off the cart.
(b) Suppose the initial speed is larger than the minimum value in (a), find the kinetic energy loss of the system (block + cart).
(c) Same condition as (b), find the time the block spends on the cart.
(d) If the initial speed is less than the minimum in (a), find the time the block spends on the cart.



題 5（**12 分**）

如圖，一質量爲 m_A 的小方塊以初始速度 v_0 在光滑平面上滑行，直到滑上一長度爲 L 質量爲 m_B 的靜止小車。方塊與小車之間的摩擦係數爲μ。小車可在光滑平面上滑行。方塊之後與車固定在末端的牆作彈性碰撞，最終從車上掉下。

(a) 如方塊確能從車上掉下，初始速度 v_0 最小是多少？
(b) 如方塊的初始速度大於(a)中的最小值，求方塊在小車上所花的時間。
(c) 在(b)的過程中系統損失的動能爲多少？
(d) 如初始速度 v_0 小於(a)中的最小值，方塊在小車上所花的時間是多少？

Q6 (13 points)

This is an experiment that demonstrates the ‘matter wave’, i. e., particles such as electrons move like light wave propagation. A beam of electrons (ignore interactions between the electrons) can be viewed as a plane wave with wavelength $\lambda = \frac{h}{p}$, where h is a fundamental constant called the Planck Constant, and p is the momentum of an electron. The intensity of the wave is proportional to the electrons density. The electrons ‘matter wave’ is used to replace the light waves in a typical setting of Young’s experiment (for light waves), with two narrow slits separated by a distance d, and a large screen at a distance D ($\gg d$) from the slits. A broad beam of electrons, accelerated by a voltage U from rest, is incident perpendicularly onto the slits plane. You can use the appropriate constants in page-2 without substituting the actual values in your answers.

(a) Find the electron density distribution on the screen, assuming that its maximum value is unity and the screen can somehow remain neutral.

(b) Add one more slit at the middle position between the two original ones. Find the electron density distribution on the screen, and its maximum value.

題 6（13 分）

這是一個演示‘物質波’的實驗。電子之類的微觀粒子的運動具有明顯的波動特性。一束電子如不考慮之間的相互作用可看作是一波長爲 $\lambda = \frac{h}{p}$ 的平面‘物質波’，其中 p 是單個電子的動量，h 是普朗克常數。‘物質波’的強度與電子數密度成正比。現將‘物質波’用來代替在典型的楊氏實驗裏所用的平面光波。實驗中兩窄縫的間距爲 d，到螢幕的距離爲 D ($\gg d$)。一寬電子束由靜止經過電壓 U 加速後，准直射向窄縫平面。你可利用頁 2 上提供的常數，但不用代入其數值。

(a) 求螢幕上電子密度分佈。令密度最大值爲 1，並假設螢幕可一直保持電中性。

(b) 在兩窄縫的中間位置再開一條和原來兩條一樣的窄縫。求螢幕上電子密度分佈和分佈的最大值。

END 完

Hong Kong High School Physics Olympiad 2006
Written Examination
Answers for M.C. Questions

(1) Apply $\frac{GM}{r^2} = r\omega^2$ then plug in numbers from the table

i.e. answer is c

(2) $\begin{cases} (P_{in} - P_0)A = mg \\ P_{in}V_0 = PV \le 2PV_0 \Rightarrow M \le \frac{P_0 A - mg}{2g} \\ Mg = (P_{in} - P)A \end{cases}$

where P_{in} is the pressure inside

P is the pressure after the lift

i.e. answer is b

(3) There are three combination of a pair.

i.e. answer is b

(4) $f' = \frac{c+v'}{c-v} f$, where *f'* and *v'* is the frequency received and the velocity of the observer

i.e. answer is a

(5) $\omega = \sqrt{\frac{F}{ml}}$, which reduce to $\sqrt{\frac{g}{l}}$ as usual, when *F=mg*, which cancel out with the other m because inertial mass is equal to gravitational mass

Where $F = \sqrt{(mg)^2 + (qE)^2}$ now

i.e. answer is b

(6) Note: Far Away, so that the rays are approximately parallel

E-field double, intensity 4 times

i.e. answer is c

(7) As S_2 and S_3 is destructive at Y, the total is destructive too. (the spacing are the same)

i.e. answer is a

(8) The existence of the middle conductor is useless. The system is equivalent to a gap of 2d

i.e. answer is b

(9) The acceleration only introduce a shift of equilibrium position

i.e. answer is d

(10) By symmetry: $q_1=q_5$, $q_2=q_4$

By Kirchhoff law: $q_1=-q_2$, $q_4=-q_5$, $q_2+q_3+q_4=0$, $q_3=C_0V$, where q1, 2, 3, 4, 5 is the charge on the corresponding capacitor form left to right, and C_0 is the capacitance of individual capacitor, and V is the arbitrary voltage across AB

Express the energy stored in two different ways:

$\frac{1}{2}CV^2 = \frac{1}{2}\frac{\sum_i q_i^2}{C_0}$. $C = 2C_0$

i.e. answer is b

(11) Horizontal component along the inclines must vanish:

$\begin{cases} T\ \cos(45^0 - \theta) = mg\ \cos 45^0 \\ T\ \cos(45^0 + \theta) = 3mg\ \cos 45^0 \end{cases} \Rightarrow 3 = \frac{\cos(45^0 + \theta)}{\cos(45^0 - \theta)}$

Where T is the tension of the bar, θ is the angle of the bar to the horizontal

Plug in the numbers from the choices to solve the equation i.e. answer is d

(12) If there is one more hole on the other side of the cylinder, the system is self equilibrium.
So, the system is equivalent to hold a mass filling the right hole
Considering the moment: 2*a T = 2a/3* (80-65)
i.e. answer is c

(13) In general, V=*-5x+3y*
i.e. answer is d

(14) By Gauss law and knowing that the inside of a conductor have no E-field
i.e. answer is e

(15) Calculate the g-field of the large sphere and then subtract the contribution of the small one. It works because of the superposition principle.
i.e. answer is e

(16) $$\begin{cases} \frac{1}{2}mv^2 = mgR(\sin\theta_1 - \sin\theta_2) \\ mg\ \ \sin\theta_2 - N = m\frac{v^2}{R} \\ N = 0 \end{cases}$$

i.e. answer is a

(17) From the choices, we know that the mixture is in its liquid state.
Including proper treatment on the heat of fusion, the answer can be found
i.e. answer is a

(18) $$\begin{cases} \frac{1}{2}mv^2 = eV \\ evB = m\frac{v^2}{R} \end{cases} \Rightarrow v^2 = 2V\frac{e}{m} = \left(\frac{e}{m}BR\right)^2$$

i.e. answer is b

(19) $$\begin{cases} \begin{cases} f = \rho_w Vg \\ \rho_r = \frac{3}{4}\rho_w \\ \rho_r V = (1-r)M \end{cases} \Rightarrow f = \frac{4}{3}(1-r)Mg \\ f + F\ \ =\ \ Mg \\ \left(\frac{1}{2} - \frac{1-r}{2}\right)Mg = \left(1 - \frac{1-r}{2}\right)F,\ moment \end{cases} \Rightarrow \frac{4}{3}(1-r) + \frac{r}{1+r} = 1 \Rightarrow r = \frac{1}{2}$$

Where f is the buoyancy force
ρ_w is the density of water, ρ_r is the density of the rod
r is the ratio of the rod above the water surface
and V is the volume occupied under the water
i.e. answer is c

(20) By principle of equivalence, the acceleration can be treated as gravity
The water surface should perpendicular to this gravity
i.e. $\tan\theta = \frac{a}{g} = \frac{h}{L}$, where h is the height required.
i.e. answer is c

Hong Kong High School Physics Olympiad 2006
Written Examination
Answers for Open Questions

Q1 (5 points)

State the principle, the setting, and the formula to obtain the result. For example, use a spring of known force constant k to attach to the mass, and measure the vibration frequency f. Then use $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ to determine the mass m.

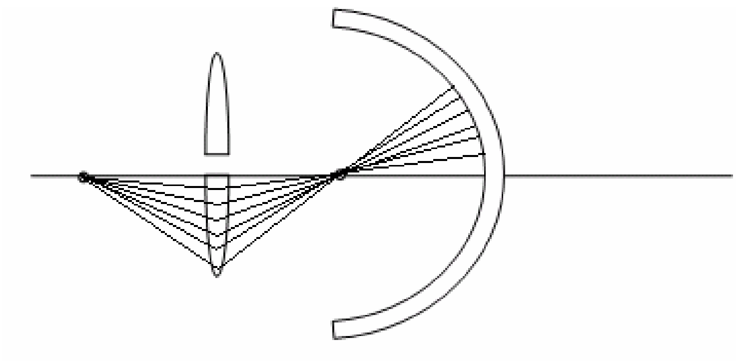
Q2 (10 points)

a) The first image due to the lens is at 20 cm from the lens, and at point-P. The second image due to the mirror is at point-P too. The final image of light source is back at the point O. (2 points)

b) Ignore the light passing through the gap since it is small comparing to the other dimension of the system. There are four possibilities:

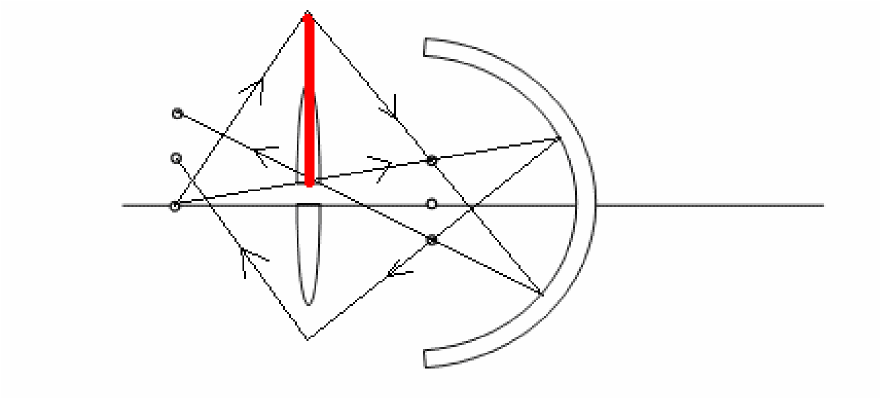
<u>Case 1:</u> The light pass through the lower half lens first. The intensity is reduced by 1/2.

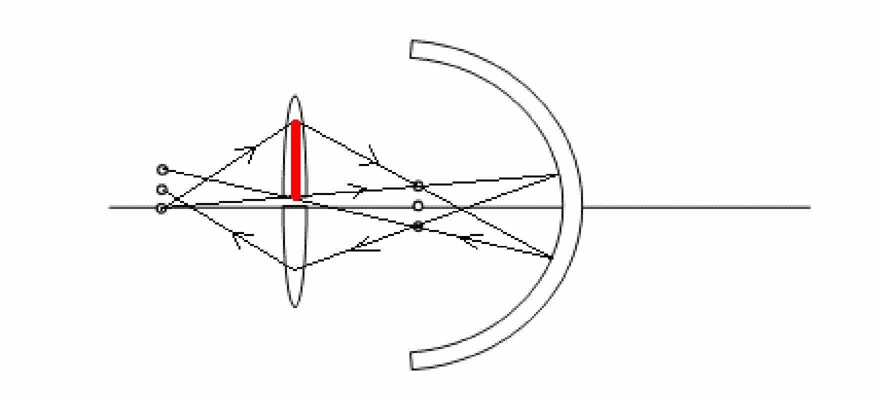
Case a: After that, by the reversibility of light, all of them will follow the same route going back to O. So, the final intensity of this spot is 1/2 of the original one. (2 points)



Case b: The light pass through the lower half lens, mirror, and then upper half lens is at 2.0mm above the source (concerning the axis of the upper lens when it travel back from the right hand side). But by the argument in (1a), after passing through the lower half lens, there will be NO light traveling trough the upper half lens. (2 points for identifying that there is no light traveling in this way)

<u>Case 2:</u> The light pass through the upper half lens first. The intensity is reduced by 1/2. After that, it can be easily found that the light can travel back via both the upper and lower half lens:





Case a: traveling back via upper half lens. By simple geometry and appropriate shifting of axis, one can find that the position of the image is 4.0mm above O.

Case b: traveling back via lower half lens. Again, one can find that the image is formed at 2.0mm above O.

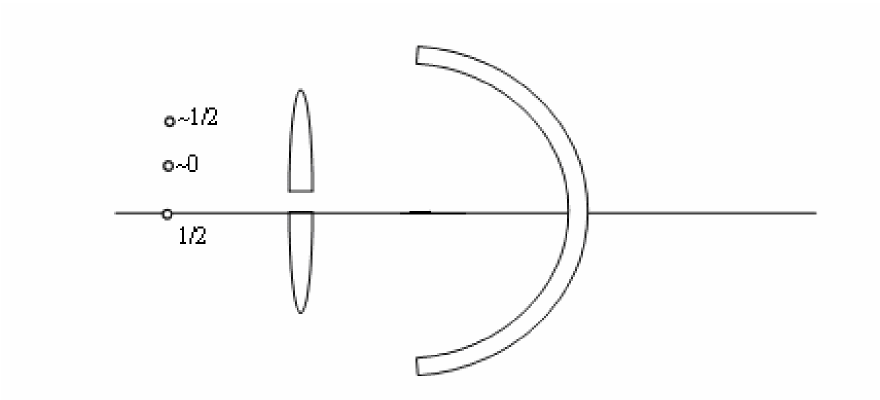
<u>To find out the intensity, neglect the effect of finite size of the lens.</u>

Method 1 (Qualitative argument):

From the above two figures, if the light coming out from the source passing through the red region, it will eventually go back via the lower half lens. As shown in the figures, the smaller the gap (which is 1.0mm in our case), the smaller the red region. In our case, the gap is very small, comparing to other physical dimension of the system. So, the red region contributes only a very small portion of the light. i.e. nearly all of them will going back via the upper half lens.

i.e. Intensity in case 2a is ~1/2.

And that in case 2b is ~0.

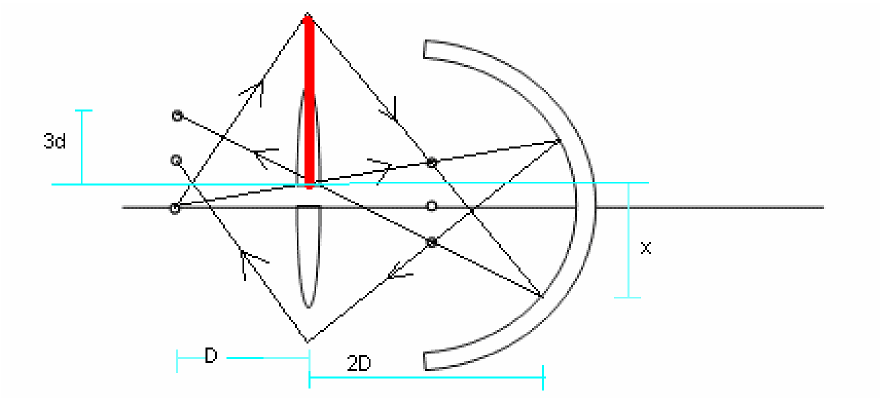


Method 2 (Quantitative argument)

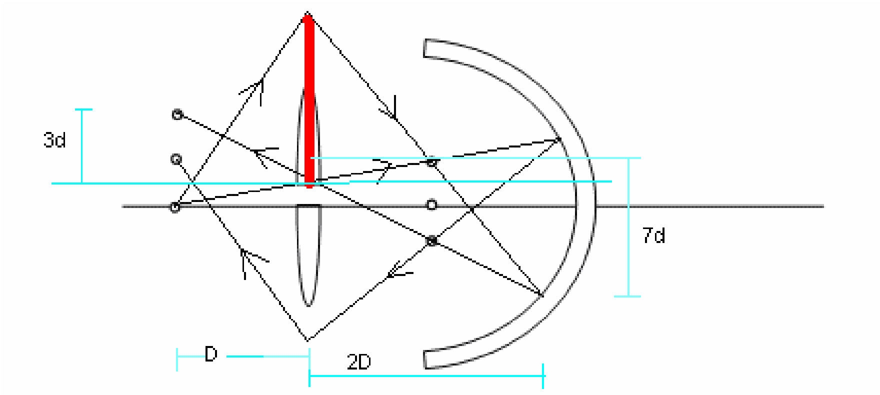
i) calculating the height of the red region first

ii) calculate the portion of light that will pass through the red region (i.e. going through the upper half lens and back via lower half lens)

answer to (i): (define d=1.0mm, D=20cm)



Small angle approximation (d<<D): x/2D = 3d/D => x = 6D



By the congruent triangle: height of red region: 7d + d = 8d

answer to (ii): the portion: $\int_0^{2\pi}\int_{\frac{\pi}{2}-\theta_1}^{\frac{\pi}{2}-\theta_2} \sin\theta d\theta d\phi$ divide by the total: $\int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin\theta d\theta d\phi$, where

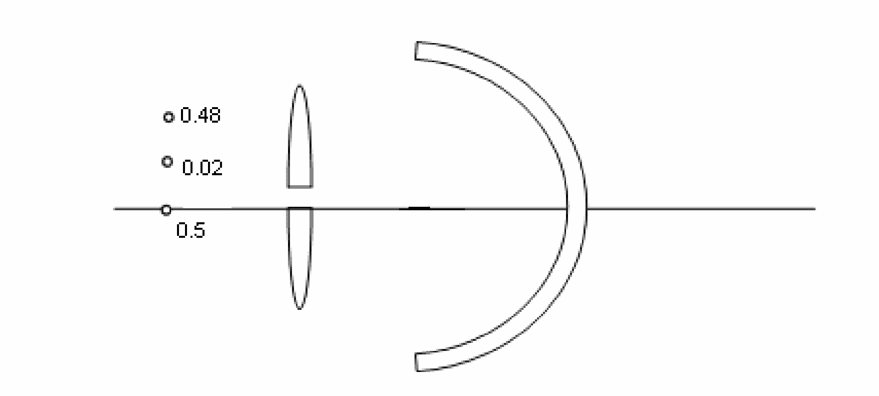
$\tan\theta_1 = \frac{9d}{D}$ and $\tan\theta_2 = \frac{d}{D}$

So, the portion of light is $\sin\theta_1 - \sin\theta_2 \approx \tan\theta_1 - \tan\theta_2 = \frac{8d}{D}$

Considering that the light passing through the upper half lens first is 1/2 of the original

Total intensity of the spot 2.0mm above O = 1/2 * 8d/D = 4*1.0mm/20cm = 0.02

The rest if light will focus on the spot 4.00 above O: 1-1/2-0.02 = 0.48



Q3 (10 points)

$$U = (\vec{V} \times \vec{B}) \cdot \vec{L} = VLB\cos\theta \Rightarrow I = \frac{U}{R} = \frac{VLB\cos\theta}{R} \quad \text{(2 points)}$$

$$a) F = I\vec{L} \times \vec{B} = ILB = \frac{VL^2B^2\cos\theta}{R} \text{.................................(1)} \quad \text{(1 point)}$$

Since the rod is in equipoise, we have $F\cos\theta = mg\sin\theta$

$$\therefore F = mg\tan\theta \text{...(2)} \quad \text{(1 point)}$$

combine (1) with (2), $V = \frac{mgR\sin\theta}{B^2L^2\cos^2\theta}$ (2 points)

b)

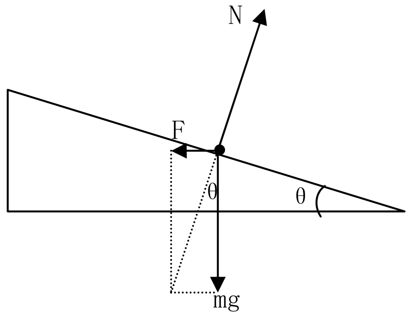
As the rod is in equipoise, we have

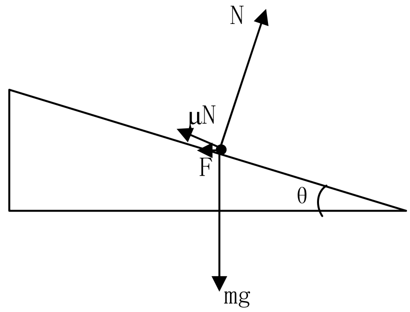
$$\mu N\sin\theta + N\cos\theta = mg \Rightarrow N = \frac{mg}{\cos\theta + \mu\sin\theta} \quad \text{(1 point)}$$

$$\mu N\cos\theta + F = N\sin\theta \quad \text{(1 point)}$$

$$\therefore F = \frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta} \times mg \text{...................(3)} \quad \text{(1 point)}$$

Combine(1) with(3), $V = \frac{mgR}{B^2L^2\cos\theta} \times \frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}$ (2 points)





Q4 (10 points)

When a ball with mass M1 and velocity V1 collides elastically with a ball with mass M2 and velocity V2, we have

$$M_1V_1 + M_2V_2 = M_1V_1' + M_2V_2'$$

$$\frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{1}{2}M_1V_1'^2 + \frac{1}{2}M_2V_2'^2$$ (2 points)

Then

$$V_1' = \frac{V_1(M_1 - M_2) + 2M_2V_2}{M_1 + M_2}$$

$$V_2' = \frac{V_2(M_2 - M_1) + 2M_1V_1}{M_1 + M_2}$$ (2 points)

In this question, regard the velocity upwards to be positive, V1 = -V, V2 =V , where $V = \sqrt{2gH_0}$ is the terminal speed of the balls falling from a height H0.

$$V_1' = \frac{-V(m_1 - m_2) + 2m_2V}{m_1 + m_2} = \frac{3m_2 - m_1}{m_1 + m_2}V = \frac{3-\gamma}{1+\gamma}V$$, where $\gamma = \frac{m_1}{m_2}$ (2 points)

In this problem, γ =1/10, so $V_1' = \frac{3-0.1}{1+0.1}V$ =2.64V (2 points)

So the maximum height the top sphere can reach is 2.642 H0 = 7H0.(2 points)

Q5 (12 points)

a) Considering the critical state that the block stops at the left edge of cart, according to the law of conservation of energy, we have

$$2\mu m_A gL + \frac{1}{2}(m_A + m_B)\frac{m_A^2 V_0^2}{(m_A + m_B)^2} = \frac{1}{2} m_A V_0^2 \quad \text{(2 points)}$$

$$\Rightarrow V_0 = 2\sqrt{(1+\frac{m_A}{m_B})\mu gL} \quad \text{(1 point)}$$

b) In the coordinate system moving with the cart,

the block is decelerating with initial velocity V_0 and acceleration $-(1+\frac{m_A}{m_B})\mu g$ (2 points)

$$V_0 t + \frac{a}{2}t^2 = 2L \Rightarrow t = \frac{V_0 - \sqrt{V_0^2 - 4(1+\frac{m_A}{m_B})\mu gL}}{(1+\frac{m_A}{m_B})\mu g} \quad \text{(3 points)}$$

c) The work done by the friction is $2\mu m_A gL$. This is the amount of kinetic energy loss. (3 points)

d) Infinite (1 points)

Q6 (13 points)

The momentum of the incident electron is

$$eU = \frac{p^2}{2m_e} \quad \text{(1 point)}$$

So the wavelength of the 'matter wave' is $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2eUm_e}}$ (1 point)

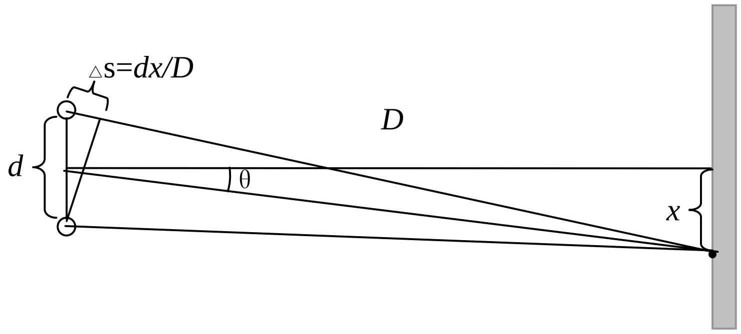
Consider a point-X which is at a distance x from the central line. The difference in distance from the two slits to point-X is given by

$$\Delta s = \frac{dx}{D} \quad \text{(2 points)}$$

$$W = W_0(1+e^{i\frac{\Delta s}{\lambda}}) \quad \text{(2 points)}$$

$$\Rightarrow I = |W|^2 = 2W_0^2(1+\cos\frac{\Delta s}{\lambda}) = 2W_0^2(1+\cos\frac{dx}{\lambda D}) \quad \text{(1 point)}$$

$$1 = I_{\max} = 4W_0^2, \text{ so } W_0^2 = 1/4 \quad \text{(1 point)}$$



b)the distance difference changes to $\frac{\Delta s}{2}$ (1 point)

$$W = W_0(1+e^{i\frac{\Delta s}{2\lambda}} + e^{-i\frac{\Delta s}{2\lambda}}) = W_0(1+2\cos\frac{\Delta s}{2\lambda})$$ (2 points)

$$\Rightarrow I(x) = |W|^2 = W_0{}^2(1+2\cos\frac{\Delta s}{2\lambda})^2 = W_0{}^2(1+2\cos\frac{dx}{2\lambda D})^2$$ (1 point)

$$I_{\max} = W_0{}^2(3)^2 = \frac{9}{4}$$ (1 point)

Hong Kong High School Physics Olympiad 2007
2007 年香港中學物理競賽

Written Examination
筆試

Jointly Organized by

Education and Manpower Bureau
教育統籌局
The Hong Kong Physical Society
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

May 27, 2007
2007 年 **5** 月 **27** 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions

   選擇題的答題紙將於比賽開始後一小時三十分收回。完成選擇題後，你即可開始作答開放題。

3. Your Participant Number is printed on the small green label. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant Number in the space of "I.D. No.", and fill out the appropriate circles fully. After that, write your English name in the space provided.

   你的參賽號碼印在已派發給你的綠色標簽上。請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 I.D. No.欄上寫上你的 8 位數參賽號碼，並把印有所填寫的數目字的圓圈完全塗黑，然後在適當的空格上填上你的英文姓名。

4. On the cover of the answer book, please write your Chinese and English name in the field of "Student Name" and your 8-dight Participant Number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Student Name 欄中填上你的中英文姓名；於 Student Number 填上你的 8 位數參賽號碼。 答題簿可雙面使用。

5. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet fully using a HB pencil.
   填寫選擇題答案時，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

6. The information provided in the text and in the figure of a question should be put to use together.
   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve it. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.
   開放題較長，最好將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

8. The questions with the '*' sign may require information on page-3.
   帶 * 的題可能需要用到第三頁上的資料。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitation constant, 6.67×10^{-11} (N m^2/kg^2)
e – charge of an electron, -1.6×10^{-19} (A s)
ε_0 – electrostatic constant, 8.85×10^{-12} (A s)/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Radius of Earth = 6378 km
Sun×-Earth distance (= 1 Astronomical Unit (AU)) = 1.5×10^{11} m
Earth-Moon distance = 3.84×10^8 m
Density of water = 1.0×10^3 kg/m^3
Density of iron = 7.7×10^3 kg/m^3
Density of mercury = 13.6×10^3 kg/m^3
Speed of sound in air = 340 m/s

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 萬有引力常數, 6.67×10^{-11} (N m^2/kg^2)
e – 電子電荷, -1.6×10^{-19} (A s)
ε_0 – 靜電常數, 8.85×10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 (= 1 天文單位) = 1.5×10^{11} m
地球-月球距離 = 3.84×10^8 m
水的密度 = 1.0×10^3 kg/m^3
鐵的密度 = 7.7×10^3 kg/m^3
水銀的密度= 13.6×10^3 kg/m^3
空氣中聲速= 340 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除非特別注明，否則下列條件將適用於本卷所有問題：

1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每道題 2 分，每道題選擇一個答案）

MC1

It takes 240 s for the escalator to bring a boy, who is standing still, from the bottom to the top. If the boy walks on the moving escalator, it takes 60 s for him to reach the top. If the escalator is not operating, how long does it take for the boy to walk from the bottom to the top?

一運行中的行人電梯從地下到頂樓需時 240 秒（s）。一男孩在運行中的電梯上走，從地下到頂樓需時 60 s。現在電梯停了。問男孩從地下走到頂樓需時多少？

(a) 80 s　(b) 140 s　(c) 150 s　(d) 60 s　(e) 120 s

MC2

Ship A moves due north at 40 km/h, while ship B moves due west at 30 km/h. Find the relative speed between the two ships.

船-A 以每小時 40 公里（40 km/h）的速度向北行駛，船-B 以 30 km/h 的速度向西行駛。求兩船之間的相對速度值。

(a) 10 km/h　(b) $\frac{30}{\sqrt{2}}$ km/h　(c) 50 km/h　(d) 40 km/h　(e) 60 km/h

MC3

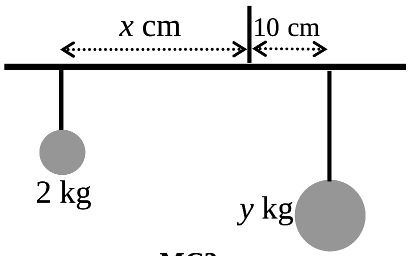
The balance with beam and strings of negligible masses is at equilibrium. Choose the correct values of x and y below.

秤的杆和細繩的質量可忽略。選出下列中正確的 x 和 y 答案。

(a) $x = 1, y = 2$　(b) $x = 10, y = 1$

(c) $x = 20, y = 4$　(d) $x = 40, y = 10$

(e) $x = 2, y = 2$

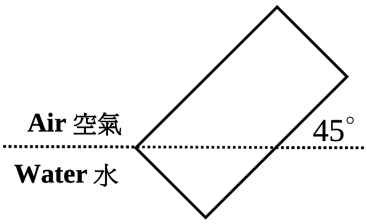


MC3

MC4*

An object of weight W has a uniform rectangular cross-section of $a \times 2a$ and density of 0.25 g/cm^3. Part of it is immersed in water and the rectangle is tilted by 45º, while one of its corners is just at the water surface. Find the torque of the buoyancy force to the center of mass of the object.

一均勻物體重量爲 W，比重爲 0.25 g/cm^3，截面爲 $a \times 2a$ 的長方形，部分浸在水裏，其中一角恰好在水面，長方形與水面成 45 º角。求浮力對物體重心的力矩。



MC4

(a) $2\sqrt{2}aW$　(b) $\frac{aW}{\sqrt{2}}$　(c) $\frac{aW}{2}$　(d) $\frac{aW}{2\sqrt{2}}$　(e) aW

MC5*

A dwarf planet, Eris, was discovered recently together with its satellite Dysnomia. The Eris-Dysnomia orbital period is about 14 (Earth) days. Assuming that the mass of Dysnomia is much smaller than Eris, and it performs circular motion with a radius of 33,000 km around Eris, find the mass of Eris.

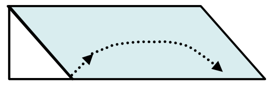
最近發現的矮行星 Eris，它的衛星 Dysnomia 繞它軌道運動的週期約爲 14 地球日，軌道半徑爲 33,000 km。Dysnomia 的質量比 Eris 小很多。求 Eris 的質量。

(a) 8.5×10^{21} kg (b) 1.45×10^{22} kg (c) 2.45×10^{22} kg (d) 2.85×10^{22} kg

(e) 3.5×10^{21} kg

MC6*

A small ball is projected up a smooth inclined plane with an initial speed of 9.8 m/s along the direction at 30° to the bottom edge of the slope. It returns to the edge after 2 s. The ball is in contact with the inclined plane throughout the process. What is the inclination angle of the plane?



MC6

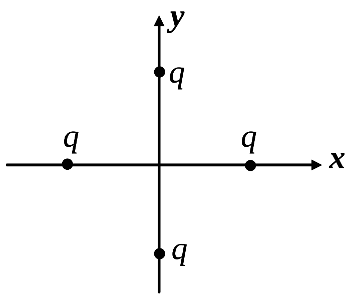
一小球以初速度 9.8 m/s 在一光滑斜面的底邊沿與底邊成 30°的方向貼著斜面射出。小球始終與斜面接觸。2 秒後小球回到底邊。求斜面的傾斜角。

(a) 15° (b) 60° (c) 37° (d) 45° (e) 30°

MC7

Four point charges, each carrying charge q, are at the positions with coordinates of $(a, 0)$, $(-a, 0)$, $(0, a)$, $(0, -a)$, respectively. Find the electric field strength at $(0, a/2)$.

四個電荷爲 q 的點電荷，分別放在坐標爲 $(a, 0)$, $(-a, 0)$, $(0, a)$, $(0, -a)$ 的位置。求 $(0, a/2)$ 處的電場值。



MC7

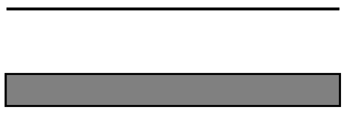
(a) $\dfrac{4}{4\pi\varepsilon_0}\dfrac{q}{a^2}$ (b) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{a^2}$ (c) $\dfrac{3.84}{4\pi\varepsilon_0}\dfrac{q}{a^2}$

(d) $\dfrac{1.84}{4\pi\varepsilon_0}\dfrac{q}{a^2}$ (e) $\dfrac{2.84}{4\pi\varepsilon_0}\dfrac{q}{a^2}$

MC8

A parallel-plate capacitor consists of two conductor plates of area A and separated by a distance d. A dielectric slab with dielectric constant ε, thickness $d/4$ and area A is inserted between the plates. What is the capacitance of the capacitor?

一面積爲 A 的平行板電容器，兩板間距離爲 d，板之間的空間有一厚爲 $d/4$、介電常數爲 ε 、面積同樣爲 A 的的電介質平板。求電容器的電容。



MC8

(a) $\dfrac{\varepsilon_0 A}{d}\left(\dfrac{4\varepsilon}{1+3\varepsilon}\right)$ (b) $\dfrac{\varepsilon_0 A}{d}\left(\dfrac{4\varepsilon}{1+\varepsilon}\right)$ (c) $\dfrac{\varepsilon_0 A}{d}\left(\dfrac{2\varepsilon}{1+\varepsilon}\right)$

(d) $\dfrac{\varepsilon_0 \varepsilon A}{4d}$ (e) $\dfrac{\varepsilon_0 \varepsilon A}{d}$

MC9*

An electron is in uniform circular motion in a uniform magnetic field perpendicular to the circular orbit. If the period of the circular motion is 1.0×10^{-6} s, what is the magnitude of the magnetic field?

一電子在均勻磁場內作勻速圓周運動，週期爲 1.0×10^{-6} 秒。磁場與圓平面垂直。求磁場大小。

(a) 3.0×10^{-4} T (b) 8.6×10^{-4} T (c) 1.6×10^{-5} T
(d) 2.6×10^{-5} T (e) 3.6×10^{-5} T

MC10*

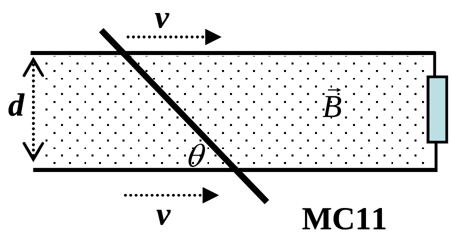
At a certain moment, the passengers on an airplane flying at a height of 8 km see sunrise. How long would it take for the people on the ground directly below the plane to see sunrise?

一架飛機在 8 千米上空飛行。機上乘客剛好見到日出。問飛機垂直下方的人們要過多久才能見到日出？（min. = 分钟）

(a) 5.8 min. (b) 7.3 min. (c) 9.1 min. (d) 11.5 min. (e) 13.2 min.

MC11

A conducting rod with resistance r per unit length is moving inside a vertical magnetic field $\vec{B}$ at speed v on two horizontal parallel ideal conductor rails. The ends of the rails are connected to a resistor R. The separation between the rails is d. The rod maintains a tilted angle θ to the rails. Find the external force required to keep the rod moving.



MC11

兩根平行的理想導電金屬導軌水平放在方向向上的均勻磁場 $\vec{B}$ 中，導軌間距爲 d。導軌端點連著電阻爲 R 的電阻器。一每單位長度電阻爲 r 的導體棒在導軌上以速度 v 滑動，導體棒與導軌保持角度 θ。求保持導體棒運動的外力大小。

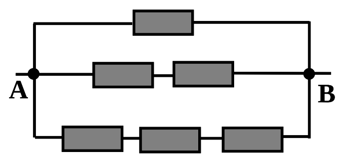
(a) $F = \dfrac{B^2d^2v}{(R+dr)}$ (b) $F = \dfrac{B^2d^2v}{(R+dr/\sin\theta)}$ (c) $F = \dfrac{B^2d^2v/\sin^2\theta}{(R+dr/\sin\theta)}$

(d) $F = \dfrac{B^2d^2v/\cos^2\theta}{(R+dr/\cos\theta)}$ (e) $F = \dfrac{B^2d^2v}{(R+dr/\cos\theta)}$

MC12

The resistance of each resistor is 1Ω. Find the equivalent resistance between points A and B.

每個電阻爲 1Ω。求 A、B 間的等效電阻。



MC12

(a) 6/11Ω (b) 11/6Ω (c) 2/3Ω
(d) 3/4Ω (e) 2Ω

MC13*

A rocket is launched vertically upward from ground and moves at a constant acceleration of 19.6 m/s^2. By accident, the engine is suddenly shut off 10 seconds after launch. To escape, the astronauts must eject at least 3 seconds before the rocket hits the ground. Neglect air resistance. How long will the astronauts have before ejection?

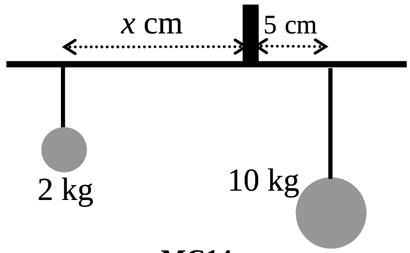
一火箭由地面以 19.6 m/s^2 的加速度垂直發射。由於故障，火箭在 10 秒鐘後突然熄滅。若宇航員必須在火箭撞地前至少 3 秒鐘彈射跳傘，問宇航員還有多少時間準備跳傘？

(a) 45.5 s (b) 34.5 s (c) 6.5 s (d) 41.5 s (e) 13.5 s

MC14

An object of about 10 kg is measured by a not-so-ideal balance. The pivot is provided by a 4.0 mm wide ribbon, so the uncertainty of the position of the pivot is 4 mm. All distances on the scale are measured from the midpoint of the ribbon. Estimate the percentage error if the weight of the object is measured using this balance.

一個不太理想的秤被用來量度一個重量約 10 kg 的物體。秤的支點爲一寬 4.0 mm 的布帶，因此支點的位置有 4.0 mm 的不確定性。秤杆上各點的位置均以布帶的中點爲參照點。估計所量物體重量的百分誤差。



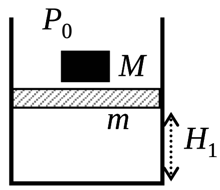
MC14

(a) 0 ~ 1.0 % (b) 1.6 % ~ 2.0 % (c) 8 % ~ 10 % (d) 20 % ~ 30 %
(e) 50 % ~ 60 %

MC15

A piston chamber of cross section area A is filled with ideal gas. At equilibrium a sealed piston of mass m is at the height H_1 from the bottom of the cylinder. The friction force between the chamber wall and the piston can be ignored. The atmosphere pressure is P_0. Now a weight of mass M is added onto the piston. Find the height of the piston. The temperature remains unchanged.

一橫截面積爲 A 的圓筒，筒內有一可上下無摩擦滑動且不漏氣的質量爲 m 的活塞，活塞下方爲理想氣體。平衡時活塞位於離圓筒底 H_1 的位置。已知溫度不變，大氣壓強爲 P_0。現在活塞上加一質量爲 M 的重物，求氣缸的位置。



MC15

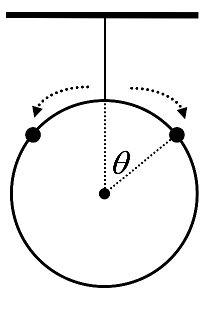
(a) $\dfrac{P_0A+mg}{(M+m)g+P_0A}H_1$ (b) $\dfrac{P_0A}{Mg+P_0A}H_1$ (c) $\dfrac{P_0A-mg}{(M+m)g-P_0A}H_1$

(d) $\dfrac{P_0A+mg}{(M-m)g+P_0A}H_1$ (e) H_1

MC16

A smooth circular track of mass M is vertically hung by a string down the ceiling. Two small rings, each of mass m, are initially at rest at the top of the track. They then slide down simultaneously along the track in opposite directions. Find the position of the rings when the tension in the string is zero.

一質量爲 M 的光滑圓圈用細繩垂直掛在天花板上。兩個質量爲 m 的小圓環從圈頂由靜止開始同時向兩邊下滑。求當細繩張力爲零時小圓環的位置。



MC16

(a) $\theta=\cos^{-1}\left[\dfrac{5}{3}\left(1+\sqrt{m-\dfrac{3M}{2m}}\right)\right]$

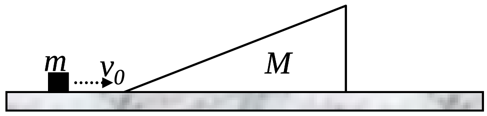
(b) $\theta=\cos^{-1}\left[\dfrac{5}{3}\left(1+\sqrt{1-\dfrac{3m}{2M}}\right)\right]$ (c) $\theta=\cos^{-1}\left[\dfrac{1}{3}\left(1+\sqrt{1-\dfrac{3M}{2m}}\right)\right]$

(d) $\theta=\cos^{-1}\left[\dfrac{1}{3}\left(1+\sqrt{1-\dfrac{5M}{3m}}\right)\right]$ (e) $\theta=\sin^{-1}\left[\dfrac{1}{3}\left(1+\sqrt{1-\dfrac{5m}{3M}}\right)\right]$

MC17

A small block of mass m is moving on a smooth horizontal table surface at initial speed v_0. It then moves smoothly onto a sloped big block of mass M. The big block can also move freely on the table surface. After the small block reaches the height h on the slope, it slides down. Find the height h.

一質量爲 m 的小物塊以初速度 v_0 在光滑水平桌面上滑行。之後它平滑地滑上一質量爲 M，有平滑斜面的大物塊。大物塊可在桌面上自由滑行。到達斜面最高點後小物塊滑下大物塊。求最高點的高度 h。



MC17 & 18

(a) $h=\dfrac{v_0^2}{2g}$, (b) $h=\dfrac{1}{g}\dfrac{Mv_0^2}{m+M}$, (c) $h=\dfrac{1}{2g}\dfrac{mv_0^2}{m+M}$,

(d) $h=\dfrac{1}{2g}\dfrac{Mv_0^2}{m+M}$, (e) $h=\dfrac{v_0^2}{g}$

MC18

Following MC17, find the speed v of the small block after it leaves the slope.

接著 **MC17**，求小物塊離開大物塊後的速率 v。

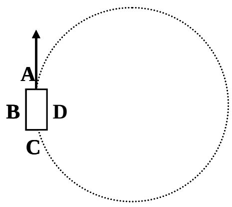
(a) $v=-v_0$ (b) $v=\dfrac{m}{m+M}v_0$ (c) $v=\dfrac{M-m}{m+M}v_0$

(d) $v=\dfrac{m-M}{m+M}v_0$ (e) $v=\dfrac{M}{m+M}v_0$

MC19

A balloon filled with helium gas is tied by a light string to the floor of a car. With all the windows shut, the string remains vertical when the car is moving at a constant velocity. If the car is traveling with constant speed along a circular path, what direction will the string tilt towards?

一充滿氦氣的氣球繫在細繩的上端，細繩的下端固定在密封的車的地板上。當車作勻速運動時，細繩是豎直的。當車作勻速圓周運動時，細繩應向哪方向傾斜？



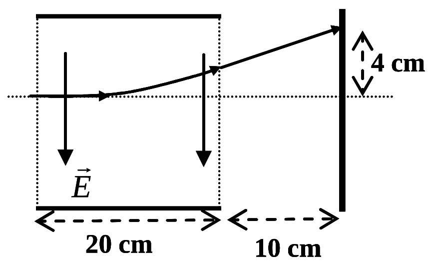
MC19

(a) A (b) B (c) C (d) D (e) Remains vertical 保持豎直

MC20*

An electron is moving at a horizontal velocity of 5×10^6 m/s before it enters a region of uniform downward electric field. It leaves the region after traveling a horizontal distance of 20cm. If the electron is deflected by 4 cm when it reaches a screen 10 cm from the electric field region, find the field magnitude.

一電子以 5×10^6 m/s 的水平初速度進入長爲 20cm 的均勻電場區，區內電場垂直向下。電場區與屏幕之間是一長爲 10 cm 的無場區。若電子的偏離爲 4 cm，求電場強度。



MC20

(a) 142 N/C (b) 102 N/C (c) 213 N/C (d) 42 N/C

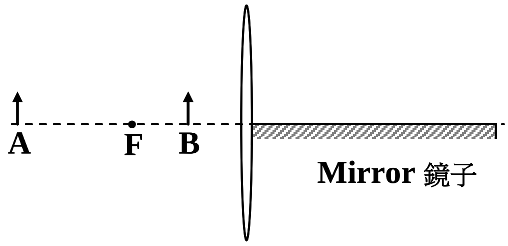
(e) 355 N/C

《**End of MC's** 選擇題完》

Open Problems 開放題
Total 5 problems 共 5 題

Q1 (8 points)

Object-A is at a distance $2f$ from a positive lens of focal length f, and Object-B is at $f/2$ from the lens. On the other side of the lens there is a flat mirror laying parallel to and coincide with the optical axis of the lens. Find the positions of the final images and the magnifications of the two objects. Also, determine whether the images are real or virtual.

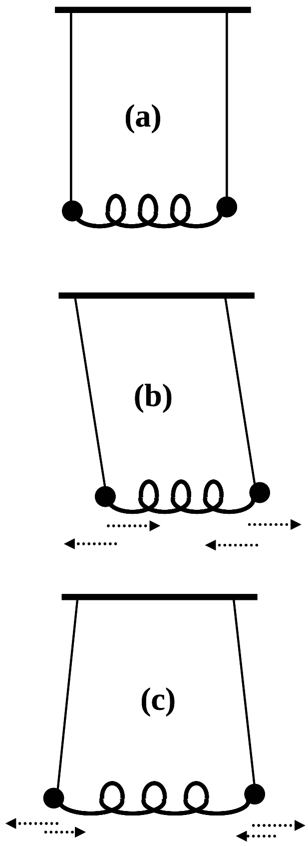


題 1（8 分）

一焦距爲 f 的正透鏡左邊有兩個物。物-A 離透鏡的距離爲 $2f$，物-B 離透鏡的距離爲 $f/2$。透鏡右邊有一鏡子，鏡面與透鏡光軸平行並與之重合。求兩個物最後的成像位置和放大率，並確定像的虛實。

Q2 (8 points)

Shown in Figure-a are two identical simple pendulums joint by an ideal spring of force constant k and natural length d, which is equal to the distance between the fixed points of the two pendulums. At equilibrium the two strings (length L) attached to the two weights (both having mass M) are straight upright. For such a system even the small amplitude simple harmonic motions are quite complicated, but there are two 'normal mode' motions that are relatively simple. One is shown in Figure-b, in which the two weights move in unison, i. e., in the first quarter of the cycle, both weights move to the right by the same amount, and in the third quarter of the cycle, both move to the left. The spring is neither stretched nor compressed in the entire cycle. In the second mode shown in Figure-c, the two weights are moving in opposite directions while keeping their combined center of mass fixed. In the first quarter of the cycle, both move outwards by the same amount, and in the third quarter of the cycle both move inward by the same amount. Find the periods of the two modes.

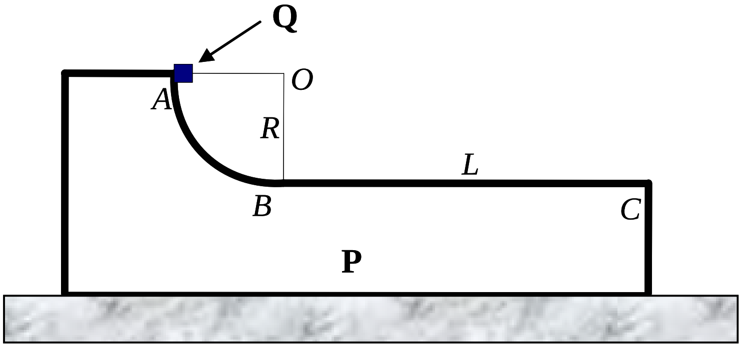


題 2（8 分）

如圖-a 所示，兩個相同的單擺，細線長爲 L，重物質量爲 M，細線上端固定點間的距離爲 d。重物間以一理想彈簧相連，彈簧的彈性係數爲 k，自然長度與細線上端固定點間的距離相同。平衡時細線與地面垂直。這樣的系統的小幅簡諧振蕩運動也相當複雜，但有兩個相對簡單的 '正則' 振盪模式。模式-1 如圖-b 所示，兩個重物同步運動，在第一個 1/4 週期兩個重物一起向右運動且幅度相同，在第三個 1/4 週期兩個重物一起向左運動且幅度相同。彈簧在整個週期無伸長或壓縮。模式-2 如圖-c 所示，兩個重物運動正好相反，而它們共同的質心保持靜止。在第一個 1/4 週期兩個重物一起向外運動且幅度相同，在第三個 1/4 週期兩個重物一起向裏運動且幅度相同。求這兩個振盪模式的振盪週期。

Q3 (12 points)
On a smooth horizontal ground surface there is a big block P of mass M. Its AB section is 1/4 of a circle of radius R, while its BC section is a horizontal surface of length L. A small cube Q of mass m is released from the top of the arc from rest and slide down. When it reaches point-B at the bottom of the arc its speed relative to P is v. It then continues to move forward and finally stops at point-C.

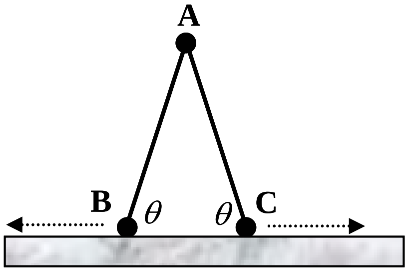
(a) Find the maximum value of v when Q reaches point-B in terms of R, M, m, and g.
(b) Given speed v, find the kinetic friction coefficient between P and Q.
(c) Find the displacement of P relative to the ground when Q reaches point-C.



題 3（12 分）
光滑的水平地面上有一質量爲 M 的滑塊 P，其 AB 段爲 l/4 圓弧，半徑爲 R，BC 段是水平面，長爲 L。一質量爲 m 的物塊 Q，從 A 點由靜止釋放，沿 ABC 滑行，到 B 點時 Q 相對與 P 的速度爲 v，最後停在 C 點。

(a) 求物塊 Q 到 B 點時速度 v 的最大可能值。(以 R, M, m, g 表達)
(b) 如果已知速度 v，求物塊 Q 與 BC 面之間的滑動摩擦係數。
(c) 求物塊 Q 到 C 點時滑塊 P 相對與桌面的位移。

Q4 (10 points)
As shown, two weightless and rigid thin rods are connected by a spherical joint A of mass M. The rods can swing freely around the joint. On the other ends of the rods are two small hard balls (B and C) of masses M and $2M$, respectively. Originally both rods are upright on a smooth table surface with joint A on the top, and B and C are on the surface. After releasing, balls B and C remain on the surface and move sideways, while the rods remain in the plane perpendicular to the table surface. (a) Find the velocity of A right before it hits the table surface. (b) Find the velocity of A when the rods are at angle θ to the table surface after releasing from the upright position. (c) Using the results in (b) to verify your answer in (a).

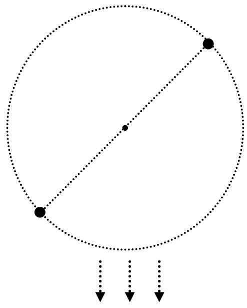


題 4（10 分）
兩根長度均爲 L 的剛性輕細杆，一端用質量爲 M 的球型鉸鏈 A 相連，另端分別安裝質量爲 M 和 $2M$ 的小球 B、C。細杆可繞鉸鏈自由轉動。開始時兩杆併攏，鉸鏈 A 在上，豎直放置在光滑桌面上，從靜止釋放，小球 B、C 始終在桌面上向兩邊滑動，兩杆始終在與桌面垂直的平面裏。細杆的質量和各種磨擦均可忽略。(a) 求鉸鏈 A 碰到桌面前瞬間的速度。(b) 求從靜止釋放後，當杆與桌面成角度 θ 時鉸鏈 A 的速度。(c) 用(b)的結果驗證(a)的答案。

Q5* (22 points) 題 5*（**22** 分）

About half of the stars in the sky are actually binaries, i. e., two stars revolving around a fixed point bound by their mutual gravity. Because the two stars in a binary system are quite close to each other and they are at far distance from us, they look like a single star in the sky. Consider a binary system consisting of two stars of the same mass as our Sun, and the distance between them is 1 Astronomical Unit. Both revolve around a fixed center in a circular orbit, as shown.

星空中大約有一半的星其實是雙星，也就是兩個恒星在相互重力作用下繞一固定點轉動。由於雙星間的距離不大，但離我們很遠，所以看起來像是一顆星。現有兩個質量與太陽相同的恒星組成雙星，雙星間的距離爲 1 天文單位，兩恒星均繞一固定點在圓軌道上轉動。



Earth 地球

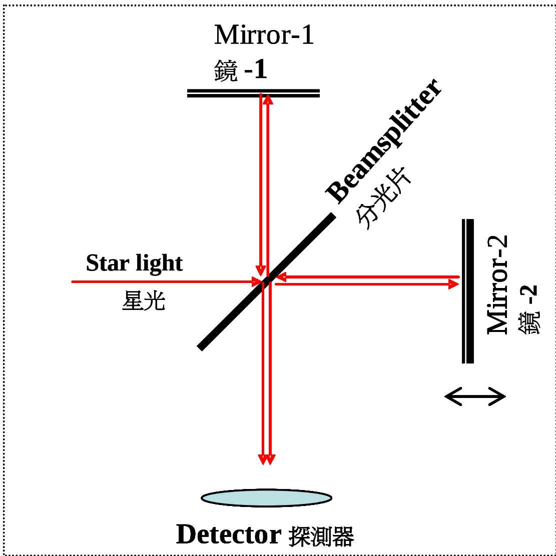
(a) Where is the fixed point? (2 points)
固定點在哪裏？（2 分）

(b) Find the revolving period in the unit of year. (3 points)
求軌道運動的週期（以年爲單位）。（3 分）

(c) Our Earth happens to be in such a position that our line-of-sight is parallel to the orbital plane of the binary. For the light of wavelength λ = 500 nm emitted from the stars, find the maximum Doppler shift $\delta\lambda$, and draw a sketch of the Doppler shift as a function of time. (4 points)

從地球上看，我們的視線與雙星軌道平面恰好平行。雙星發射光的波長爲 λ = 500 nm，求光的最大多普勒頻移，並用簡圖畫出頻移與時間的關係。（4 分）

(d) A Michelson interferometer is employed to detect the Doppler shifts when the separation between the two stars appears largest. The light from the stars entering the instrument is a narrow parallel beam containing two waves of equal intensity I_0 with wavelengths $\lambda - \delta\lambda$ and $\lambda + \delta\lambda$, where $\lambda \gg \delta\lambda$. The beamsplitter reflects half (in intensity) of the incident waves, and let the other half through, without introducing any phase shifts. The optical path distance between Mirror-1 and the beamsplitter is fixed at L while the distance between the beamsplitter and Mirror-2 is $L + x$, where x can be varied. The waves reflected from Mirror-1 and Mirror-2 finally meet at the detector and interfere. The total light intensity received by the detector can be expressed as $I(x) \cong I_0(1 + f(x)\cos(4\pi x/\lambda))$. Find the function $f(x)$, and the value of x_0 when $f(x_0) = f(0)/2$.
(Hint: The total intensity due to the interference of two waves of equal intensity I_0 and equal wavelength is $I = 2I_0(1 + \cos\Delta)$, where Δ is the phase difference between the two waves.) (5 points)



Michelson Interferometer 干涉儀

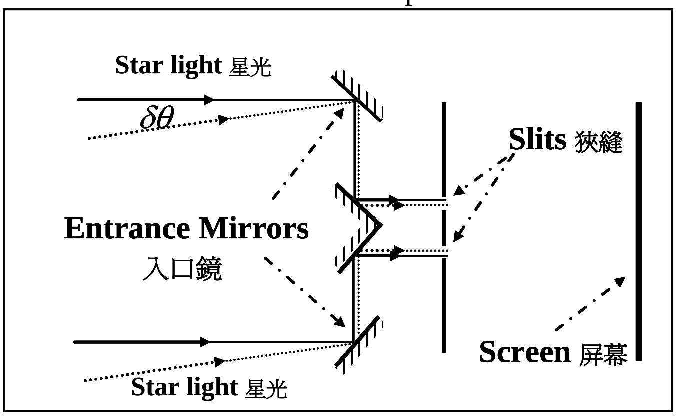
邁克爾遜干涉儀被用來測量當雙星看上去分開最遠時的多普勒頻移。此時進入干涉儀的星光可當作一束包含兩個強度均爲 I_0，波長分別爲 $\lambda-\delta\lambda$ 和 $\lambda+\delta\lambda$ 的平行光波，其中 $\lambda \gg \delta\lambda$。分光片將一半強度的入射光反射，另一半讓其透射，但不引入位相變化。鏡-1 到分光片的光程爲固定的 L，鏡-2 到分光片的光程爲 $L + x$，其中 x 可調。從兩鏡上反射的光波在探測器相干。探測器測到的總光強可表達成 $I(x) \cong I_0(1+f(x)\cos(4\pi x/\lambda))$，求涵數 $f(x)$，以及當 $f(x_0) = f(0)/2$ 時 x_0 的值。
（提示：兩個波長相同、強度均爲 I_0 的波干涉所產生的總光強爲 $I = 2I_0(1+\cos\Delta)$，其中 Δ 爲兩波之間的位相差。）（5 分）

The Doppler Effect can be ignored for the rest of the problem.
題目餘下的部分可忽略多普勒效應。

(e) The binary is at 1000 light-years distance from Earth. Find the maximum angular separation $\delta\theta$ between the two stars in the unit of arc degree. (1 point)
雙星離地球 1000 光年。求雙星最大角距離 $\delta\theta$（用弧度爲單位）。（1 分）

(f) A Stella Interferometer is used to accurately measure the angular separation between the two stars. As shown, the light from the stars can be approximately treated as two broad parallel light waves of 500 nm in wavelength; one (wave-1) is at normal incidence and the other (wave-2) is off by a small angle $\delta\theta$. Each wave is then split into two by the two entrance mirrors. The distance between the mirrors is D. There is no phase difference between the two waves split from wave-1 at the entrances. Find the phase difference between the two waves split from wave-2 at the entrance mirrors. (3 points)

星光干涉儀可用來精確測量雙星的角距離。從兩恒星來的波長爲 500 nm 的寬大的平行光波，一束（波-1）正入射，另一束（波-2）與正入射方向成一小偏角 $\delta\theta$。每束波在兩入口鏡被分成兩束，入口鏡之間離距爲 D。從波-1 分出的兩束波在入口鏡處的位相差爲零。求從波-2 分出的兩束波在入口鏡處的位相差。（3 分）



Stella Interferometer 星光干涉儀

(g) The waves at the entrances are then brought to the two narrow slits of a Young's experiment, without introducing further path differences between the waves through the upper entrance and the ones through the lower entrance. If on the screen the bright interference fringes of one star exactly overlap the dark fringes of the other, what should be the minimum distance between the two entrance mirrors? (In actual operation, the entrance mirrors are moved slowly until the fringes on the screen disappear.) (4 points)

從入口鏡進入干涉儀的光波被引導到 Young's 干涉實驗的狹縫上，從上入口到上狹縫的光程與從下入口到下狹縫的光程相等。若屏幕上一顆星所形成的干涉亮條紋與另一顆星所形成的干涉暗條紋剛好重疊，求兩入口鏡之間的最小距離。（實際操作時，入口鏡被慢慢拉開，直到屏幕上的干涉條紋剛好消失。）（4 分）

《**END** 完》

Answers for M.C. Questions

1) 80s, a; **2)** 50 km/h, c; **3)** x = 20, y = 4, c; **4)** , $\dfrac{aW}{2\sqrt{2}}$ d; **5)** 1.45 $\times 10^{22}$ kg, b; **6)** 30°, e;

7) $\dfrac{2.84}{4\pi\varepsilon_0}\dfrac{q}{a^2}$, e; **8)** $\dfrac{\varepsilon_0 A}{d}\left(\dfrac{4\varepsilon}{1+3\varepsilon}\right)$, a; **9)** 3.6 × 10^{-5} T, e; **10)** 11.5 min., d;

11) $F = \dfrac{B^2 d^2 v}{(R + dr/\sin\theta)}$, b; **12)** 6/11Ω , a; **13)** 41.5 s, d; **14)** 8 % ~ 10 % , c;

15) $\dfrac{P_0 A + mg}{(M+m)g + P_0 A} H_1$, a; **16)** $\theta = \cos^{-1}\left[\dfrac{1}{3}\left(1+\sqrt{1-\dfrac{3M}{2m}}\right)\right]$, c;

17) $h = \dfrac{1}{2g}\dfrac{Mv_0^2}{m+M}$, d; **18)** $v_1 = \dfrac{m-M}{m+M} v_0$, d; **19)** d; **20)** 142 N/C, a

Details

MC 01.

Escalator velocity$=u$, boy velocity$=v$ and L be the length of path

$t_1 = 240s$ and $t_2 = 60s$

$L = ut_1$ and $L = (u+v)t_2$

=> $L = \left(\dfrac{L}{t_1} + v\right)t_2$

=> $\dfrac{L}{v} = \dfrac{t_1 t_2}{t_1 + t_2} = \dfrac{(240s)(60s)}{(240s - 60s)} = 80s$

MC02.

Simply velocity addition,

$v_{rel} = \sqrt{v_x^2 + v_y^2} = \sqrt{(30kmh^{-1})^2 + (40kmh^{-1})^2} = 50kmh^{-1}$

MC03.

Consider the balance of moment,

$m_1 L_1 = m_2 L_2$

=> $2x = 10y$

=> $x : y = 5:1$

=> The choice is (c). $x = 20$ and $y = 4$

MC04.

$W =$ buoyancy force

Consider the center of mass and symmetry of immersed part of the object,

$r = \dfrac{1}{4}\sqrt{a^2 + a^2} = \dfrac{a}{2\sqrt{2}}$

=> $\Gamma = Wr = \dfrac{aW}{2\sqrt{2}}$

MC05.

$$\frac{GMm}{R^2} = m\omega^2 R \qquad => \qquad GM = \omega^2 R^3 \quad => \qquad M = \frac{\omega^2 R^3}{G}$$

$$\omega = \frac{2\pi}{14\times 24\times 3600s} = 5.194\times 10^{-6} s^{-1}$$

=> $$M = \frac{\omega^2 R^3}{G} = \frac{\left(5.194\times 10^{-6} s^{-1}\right)^2 \left(3.3\times 10^7 m\right)^3}{6.67\times 10^{-11} Nm^{-2} kg^{-2}} = 1.45\times 10^{22} kg$$

MC06.

Let v_y be the velocity along the plane in the y-direction,

$$v_y = v\sin\theta \qquad \theta = 30^0$$

$v = u + at$ => $v\sin\theta = -v\sin\theta + \left(g\sin\phi\right)t$ $\quad\phi$ is angle of incline

=> $$\sin\phi = \frac{2v\sin\theta}{gt} = \frac{2\left(9.8ms^{-1}\right)\left(\sin 30^0\right)}{\left(9.8ms^{-2}\right)\left(2s\right)} = 0.5$$

=> $\phi = 30^0$

MC07.

From the symmetry of charges, we only have to count y-componets,

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\left(2\times\frac{1}{1^2+0.5^2}\frac{1}{\sqrt{5}} + \frac{1}{1.5^2} - \frac{1}{0.5^2}\right)$$

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\left(2.84\right)$$

MC08.

Consider the capacitor as two capacitors with different dielectric and thickness in series.

$C = \frac{\varepsilon_0 A}{d}$ (parallel plate capacitor)

=> $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ where $C_1 = \frac{\varepsilon_0 A}{\left(3/4\right)d}$ and $C_1 = \frac{\varepsilon\varepsilon_0 A}{\left(1/4\right)d}$

=> $$C = \frac{\varepsilon_0 A}{d}\left(\frac{4\varepsilon}{1+3\varepsilon}\right)$$

MC09.

$qvB = m\omega^2 r$ and $\qquad v = \omega r$

=> $B = \frac{\omega m}{q}$ and $\omega = \frac{2\pi}{T} = 6.28\times 10^6 s^{-1}$, $m_e = 9.11\times 10^{-31} kg$, $e = 1.6\times 10^{-19} C$

=> $B = 3.6\times 10^{-5} T$

MC10.

Radius of earth $R_e = 6378km$

$$\cos\theta = \frac{R_e}{R_e + h} = \frac{6378km}{6378km + 8km} = 0.9987$$

=> $\theta = 2.87^0$

$$T_{delay} = \left(24\times 60\min\right)\frac{2.87^0}{360} = 11.45\min$$

MC11.

Resistance of rod $= r\dfrac{d}{\sin\theta}$

$\xi = \left|\dfrac{d\phi}{dt}\right| = Bdv$

$P = \dfrac{\xi^2}{R_{total}} = \dfrac{B^2d^2v^2}{r(d/\sin\theta)+R} = Fv$

=> $F = \dfrac{B^2d^2v}{r(d/\sin\theta)+R}$

MC12.

Simply consider the resistances in series and parallel,

$\dfrac{1}{R} = \dfrac{1}{1\Omega} + \dfrac{1}{2\Omega} + \dfrac{1}{3\Omega}$ => $R = \dfrac{6}{11}\Omega$

MC13.

At the point the engine just shutting down,

$s = \dfrac{1}{2}at^2 = \dfrac{1}{2}(19.6ms^{-2})(10s)^2 = 980m$

$v = at = (19.6ms^{-2})(10s) = 196ms^{-1}$

Then, consider the free falling,

$s = ut + \dfrac{1}{2}at^2$

$-980 = (196)t + \dfrac{1}{2}(-9.8)t^2$ => $4.9t^2 - 196t - 980 = 0$

=> $t = 44.5s$ or $-171.5s$ (rejected)

=> The time left is 41.5 s.

MC14.

Find $x = 25cm$

Consider the max uncertainty at extreme case as

$m_{\max} = \dfrac{25.4cm}{4.6cm}(2kg) = 11.04kg$ =>

$\%error = \dfrac{11.04-10}{11.04}\times 100\% = 9.4\%$

$m_{\min} = \dfrac{24.6cm}{5.4cm}(2kg) = 9.11kg$ =>

$\%error = \dfrac{10-9.11}{9.11}\times 100\% = 9.8\%$

The % error is about 10%.

MC15.

According to Boyle's law,

$P_1V_1 = P_2V_2 \quad => \quad P_1H_1 = P_2H_2$

$$\left(P_0 + \frac{mg}{A}\right)H_1 = \left(P_0 + \frac{(m+M)g}{A}\right)H_2$$

=> $$H_2 = \frac{P_0A + mg}{(m+M)g + P_0A}H_1$$

MC16.

Balance of force in vertical direction,

$Mg + 2mg\cos^2\theta = 2N\cos\theta$

Normal force component,

$$\frac{mv^2}{R} = N$$

Conservation of energy,

$$mgR(1-\cos\theta) = \frac{1}{2}mv^2$$

=> $Mg + 2mg\cos^2\theta = 4mg\cos\theta(1-\cos\theta)$

$6m\cos^2\theta - 4m\cos\theta + Mg = 0$

$$\cos\theta = \frac{4m \pm \sqrt{16m^2 - 24mM}}{12m}$$

=> $$\cos\theta = \frac{1}{3} \pm \frac{1}{3}\sqrt{1 - \frac{3}{2}\frac{M}{m}}\,.$$

MC17.

To find h, consider at the highest point, mass m and M moving with the same velocity. Then, by conservation of momentum,

$$mv_0 = (m+M)V \quad => \quad V = \frac{m}{m+M}v_0\,.$$

By conservation of energy,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}(m+M)V^2 + mgh$$

=> $$\frac{1}{2}mv_0^2 = \frac{1}{2}(m+M)\left(\frac{m}{m+M}v_0\right)^2 + mgh$$

=> $$h = \frac{v_0^2}{2g}\left(\frac{M}{m+M}\right).$$

MC18.

Similarly for finding v,

$$mv_0 = mv + MV \quad => \quad V = \frac{m}{M}(v_0 - v) \text{ and } \frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

=> $$v = v_0\left(\frac{m-M}{M+m}\right).$$

MC19. (D) It is not towards 'B' as would be 'obvious'. The balloon experiences a buoyancy force by the air which is opposite to 'gravity'. Inside the car, the 'gravity' component due to centrifugal force points outwards to 'B'.

MC20.

Consider the cases as inside E-field and outside E-field,

Inside the E-field $t_1 = \frac{d}{v} = \frac{20cm}{5\times10^6 ms^{-1}} = 4\times10^{-8} s$

Outside the E-filed $t_2 = 2\times10^{-8} s$

Given $a = \frac{Eq}{m}$, $y_1 = \frac{1}{2}\frac{Eq}{m}t_1^2$, $y_2 = \left(\frac{Eq}{m}t_1\right)t_2$

=> $y = y_1 + y_2 = \frac{Eq}{m}t_1\left(\frac{t_1}{2} + t_2\right)$

Given $y = 4cm$, $q = 1.6\times10^{-19} C$, $m_e = 9.11\times10^{-31} kg$

=> $E = 142 NC^{-1}$

Answers for Open Questions

Q1. (8 points)

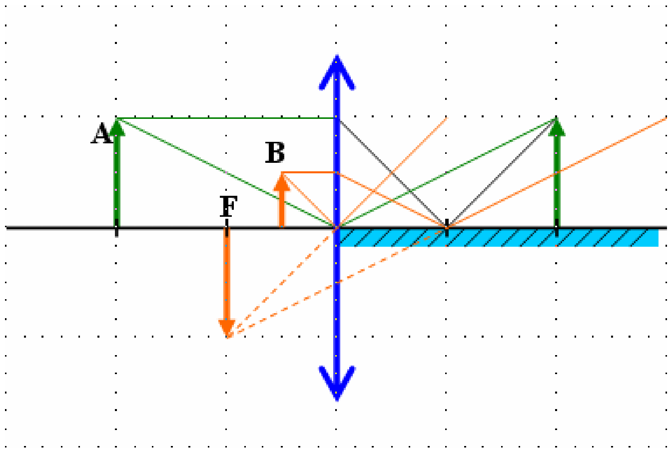
For Object A,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad => \quad \frac{1}{2f}+\frac{1}{v}=\frac{1}{f} \qquad \text{(1 point)}$$

=> $v=2f$ (1 point)

and $m=\left|\frac{v}{u}\right|=1$ (1 point)

=> real, up (1 point)



For Object B,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad =>$$

$$\frac{1}{f/2}+\frac{1}{v}=\frac{1}{f} \qquad \text{(1 point)}$$

=> $v=-f$ (1 point)

and

$m=\left|\frac{v}{u}\right|=2$ (1 point)

=> virtual, inverted (1 point)

Q2. (8 points)

For the first mode (Figure b), it is exactly the same as the simple pendulum. (1 point)

$$\omega=\sqrt{\frac{g}{L}} \quad => \quad T=2\pi\sqrt{\frac{L}{g}} \qquad \text{(2 points)}$$

For the second mode (Figure c),

Consider the spring constants contributed from different parts,

From the pendulum,

$k_1=\frac{mg}{L}$ (1 point)

From the spring,

$k_2=2k$ (1 point)

$$k_{eff}=k_1+k_2=\frac{mg}{L}+2k=\frac{mg+2kL}{L} \qquad \text{(1 point)}$$

=> $T=2\pi\sqrt{\frac{m}{k_{eff}}}=2\pi\sqrt{\frac{mL}{mg+2kL}}$ (2 points)

Q3. (12 points)

(a) Consider the arc AB is smooth,

By conservation of momentum,

$$mv_1 = MV_1 \quad => \quad V_1 = \frac{m}{M}v_1 \text{ (1 point)}$$

By conservation of energy,

$$\frac{1}{2}mv_1^2 + \frac{1}{2}MV_1^2 = mgR \text{ (1 point)}$$

$$=> \quad \frac{1}{2}mv_1^2 + \frac{1}{2}M\left(\frac{m}{M}v_1\right)^2 = mgR$$

$$=> \quad v_1 = \sqrt{2gR}\sqrt{\frac{M}{m+M}} \quad \text{and} \quad V_1 = \frac{m}{M}\left(\sqrt{2gR}\sqrt{\frac{M}{m+M}}\right)$$

$$=> \quad v = v_1 + V_1 = \sqrt{2gR}\sqrt{\frac{M}{m+M}}\left(1+\frac{m}{M}\right) = \sqrt{2gR}\sqrt{1+\frac{m}{M}} \text{ (2 points)}$$

(b) At the end point, both m and M stopped simultaneously,

By conservation of energy,

$$\frac{1}{2}m\left(\frac{M}{m+M}v\right)^2 + \frac{1}{2}M\left(\frac{m}{m+M}v\right)^2 = \mu mgL \text{ (3 points)}$$

$$=> \quad \mu = \left(\frac{M}{m+M}\right)\frac{v^2}{2gL} \text{ (1 point)}$$

(c) The displacement of P is

$$-u^2 = 2aL' \quad \text{(1 point)} => \quad -2gR\frac{m}{M}\left(\frac{M}{m+M}\right) = 2\left(-\frac{R}{L}g\right)L' \text{ (1 point)}$$

(Or because the center of mass of the system remains fixed) (2 points)

$$L' = \left(\frac{m}{m+M}\right)L \text{ (2 points)}$$

Q4. (10 points)

(a) Since the velocity of B and C are zero at the moment when A hits the table, by conservation of energy,

$$MgL = \frac{1}{2}Mv^2 \quad \text{(1 point)} \Rightarrow \quad v^2 = 2gL \quad \Rightarrow \quad v = \sqrt{2gL} \quad \text{(1 point)}$$

Method-1 for part-b

According to the condition, B will move to the left at v_B, C will move to the right at v_C, while A will move at $\vec{v}_A = v_{Ax}\vec{x}_0 + v_{Ay}\vec{y}_0$, where x-direction is horizontal and y-direction is vertical. (1 point)

Energy conservation: $mgL(1-\sin\theta) = \frac{1}{2}m(v_{Ax}^2 + v_{Ay}^2) + \frac{1}{2}mv_B^2 + mv_C^2$ (1) (1 point)

Horizontal momentum conservation: $v_{Ax} + v_B = 2v_C$ (2) (1 point)

The rods are rigid, so the relative velocity between A and B and between A and C along the rod direction must be zero. This leads to two equations below.

$-v_{Ax}\cos\theta + v_{Ay}\sin\theta = v_C\cos\theta$ (3) (1 point)

$v_{Ax}\cos\theta + v_{Ay}\sin\theta = v_B\cos\theta$ (4) (1 point)

(4) – (3) leads to $-2v_{Ax} = v_B - v_C$ (5).

From Eq. (2) and (5) we get $3v_{Ax} = v_C$, and $5v_{Ax} = v_B$. (1 point)

Put in Eq. (4) we get $4v_{Ax}\cos\theta = v_{Ay}\sin\theta$. Put v_{Ax}, v_B and v_C into Eq. (1), we get

$v_{Ax}^2 = \frac{(1-\sin\theta)\sin^2\theta}{44\sin^2\theta + 16\cos^2\theta}2gL$, and $v_{Ay}^2 = \frac{(1-\sin\theta)\cos^2\theta}{11\sin^2\theta + 4\cos^2\theta}2gL$.

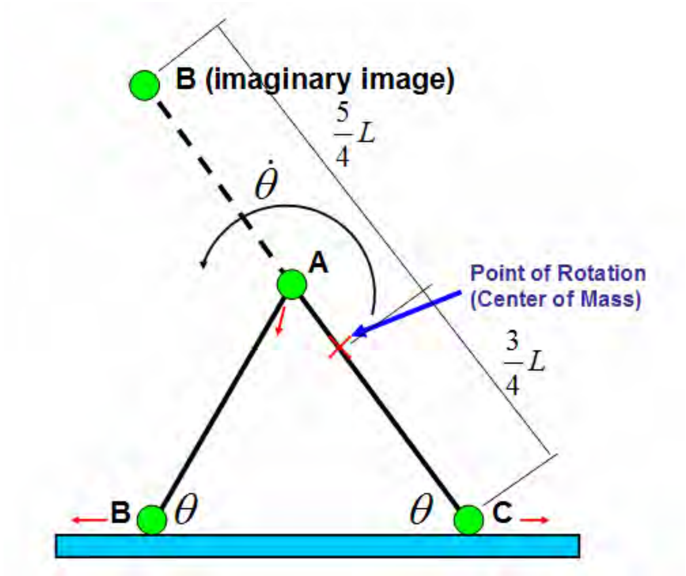
Finally, $v_A^2 = \frac{(1-\sin\theta)(\sin^2\theta + 16\cos^2\theta)}{44\sin^2\theta + 16\cos^2\theta}2gL$ (1 point)

Method-2 for part-b

(b) Find the lateral shifting of the position of A first relative to the center of mass which remains fixed in the horizontal direction,

$$M_A(x-L) + M_B(x-2L) + M_C(x) = 0$$

$$\Rightarrow \quad M(x-L) + M(x-2L) + 2M(x) = 0 \quad \Rightarrow \quad x = \frac{3}{4}L$$



To find the x-component of velocity of each ball, consider the rotation axis at Center of Mass, while the y-component of velocity of ball A requires to consider the rotation axis at C. Then, we can express the velocity of each mass in term of angular velocity,

$$v_{Ax} = \frac{1}{4}L\dot{\theta}\sin\theta, \qquad v_{Ay} = L\dot{\theta}\cos\theta,\ v_B = \frac{5}{4}L\dot{\theta}\sin\theta,\ v_C = \frac{3}{4}L\dot{\theta}\sin\theta$$

By conservation of energy,

$$mgL(1-\sin\theta) = \frac{1}{2}M_A v_A^2 + \frac{1}{2}M_B v_B^2 + \frac{1}{2}M_C v_C^2$$

=> $$mgL(1-\sin\theta) = \frac{1}{2}m\left(v_{Ax}^2 + v_{Ay}^2\right) + \frac{1}{2}mv_B^2 + \frac{1}{2}(2m)v_C^2$$

=> $$2gL(1-\sin\theta) = L^2\dot{\theta}^2\left[\frac{1}{16}\sin^2\theta + \cos^2\theta + \frac{25}{16}\sin^2\theta + \frac{18}{16}\sin^2\theta\right]$$

$$= L^2\dot{\theta}^2\left[\frac{11}{4}\sin^2\theta + \cos^2\theta\right] = L^2\dot{\theta}^2\left[\frac{7}{4}\sin^2\theta + 1\right]$$

=> $$\dot{\theta}^2 = \frac{8g}{L}\left(\frac{1-\sin\theta}{4\cos^2\theta + 11\sin^2\theta}\right)$$

=> $$v_A^2 = v_{Ax}^2 + v_{Ay}^2 = L^2\dot{\theta}^2\left[\frac{\sin^2\theta}{16} + \cos^2\theta\right] = \frac{gL}{2}(1-\sin\theta)\left(\frac{\sin^2\theta + 16\cos^2\theta}{11\sin^2\theta + 4\cos^2\theta}\right)$$

=> $$v_A = \sqrt{\frac{gL}{2}(1-\sin\theta)\left(\frac{\sin^2\theta + 16\cos^2\theta}{11\sin^2\theta + 4\cos^2\theta}\right)}$$

(c) Put $\theta = 0^0$ =>` $v_A = \sqrt{2gL}$ (1 point)

Q5. (22 points)

(a) Center of mass

(b) The force on one of the star is $\frac{GM_{Sun}^2}{R^2}$, where R is 1 AU. The acceleration is $\omega^2 R/2$

Therefore $M_{sun}\omega^2 \frac{R}{2} = \frac{GM_{Sun}^2}{R^2}$

=> $\omega = \sqrt{\frac{2GM_{Sun}}{R^3}}$ => $\omega = \sqrt{2}\omega_0$, where ω_0 is the orbital frequency of Earth around Sun, and $T_0 \equiv \frac{2\pi}{\omega_0} = 1$ year.

=> $T = 0.71$ years

(c) $v = (7.5\times10^{10}\,m)(2.8\times10^{-7}\,s) = 2.1\times10^4\,ms^{-1}$

$$\delta\lambda = \lambda\left(\frac{v}{c}\right) = (500nm)\left(\frac{2.1\times10^4\,ms^{-1}}{3\times10^8\,ms^{-1}}\right) = 0.035nm$$

The graph of Doppler shifts versus time is sinusoidal.

(d)
$$I(x) = \frac{I_0}{2}\left(1+\cos\left(\frac{4\pi x}{\lambda-\delta\lambda}\right)\right) + \frac{I_0}{2}\left(1+\cos\left(\frac{4\pi x}{\lambda+\delta\lambda}\right)\right)$$
$$= I_0 + \frac{1}{2}I_0\left[\cos\left(\frac{4\pi x}{\lambda-\delta\lambda}\right)+\cos\left(\frac{4\pi x}{\lambda+\delta\lambda}\right)\right]$$
$$= I_0 + I_0\cos\left[2\pi x\left(\frac{1}{\lambda-\delta\lambda}+\frac{1}{\lambda+\delta\lambda}\right)\right]\cos\left[2\pi x\left(\frac{1}{\lambda-\delta\lambda}-\frac{1}{\lambda+\delta\lambda}\right)\right]$$
$$= I_0\left[1+\cos\left[2\pi x\left(\frac{2\lambda}{\lambda^2-\delta\lambda^2}\right)\right]\cos\left[2\pi x\left(\frac{2\delta\lambda}{\lambda^2-\delta\lambda^2}\right)\right]\right]$$
$$\approx I_0\left[1+\cos\left(\frac{4\pi x}{\lambda}\right)\cos\left(\frac{4\pi x}{\lambda}\frac{\delta\lambda}{\lambda}\right)\right]$$

=> $f(x) = \cos\left(\frac{4\pi x}{\lambda}\frac{\delta\lambda}{\lambda}\right)$ $\quad f(0) = 1$

=> $f(x_0) = \frac{f(0)}{2}$ => $x_0 = \cos^{-1}\left(\frac{1}{2}\right)\left(\frac{\lambda^2}{4\pi\delta\lambda}\right) = \frac{1}{12}\frac{\lambda^2}{\delta\lambda} = 0.59$ mm

(e) 1 light year $= 365\times24\times3600s\times(3\times10^8\,ms^{-1}) = 9.46\times10^{15}\,m$

$$\theta = \frac{1.5\times10^{11}\,m}{1000\times9.46\times10^{15}\,m} = 1.6\times10^{-8}\,rad$$

(f) $\Delta x = D\sin\delta\theta$ => $\Delta\theta = \frac{2\pi D\sin\delta\theta}{\lambda} \approx \frac{2\pi D\delta\theta}{\lambda}$

(g) $\Delta\theta = \frac{2\pi D\delta\theta}{\lambda} = \pi$ => $D = \frac{\lambda}{2}\frac{1}{\delta\theta} = 1.56m$

Hong Kong Physics Olympiad 2008
2008 年香港物理奧林匹克競賽

Written Examination
筆試

Jointly Organized by

Education Bureau
教育局
The Hong Kong Physical Society
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

May 25, 2008
2008 年 5 月 25 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for further announcement.

   選擇題的答題紙將於比賽開始後一小時三十分收回。完成選擇題後，你即可開始作答開放題。同學若果提早完成了選擇題後，即可爭取時間開始作答開放式問答題，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID number in the field of “I.D. No., and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of “Course number & Section No.”

   請依照選擇題答題紙的指示，用HB鉛筆在選擇題答題紙的I.D. No.欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格上填上你的英文姓名，最後於“Course & Section No. ”欄內填上你的身分證號碼。

4. On the cover of the answer book, please write your Hong Kong ID number in the field of “Course Title”, and write your English name in the field of “Student Name” and your 8-dight Participant ID number in the field of “Student Number”. You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

5. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.
   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用HB鉛筆完全塗黑。

6. The information provided in the text and in the figure of a question should be put to use together.
   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.
   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitational constant, 6.67 × 10^{-11} (N m^2/kg^2)
e – charge of an electron, –1.6 × 10^{-19} (A s)
ε_0 – electrostatic constant, 8.85 × 10^{-12} (A s)/(V m)
m_e – electron mass = 9.11 × 10^{-31} kg
c – speed of light in vacuum, 3.0 × 10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance (= 1 Astronomical Unit (AU)) = 1.5 × 10^{11} m
Earth-Moon distance = 3.84 × 10^8 m
Mass of the sun = 1.99 × 10^{30} kg
Density of water = 1.0 × 10^3 kg/m^3
Density of iron = 7.7 × 10^3 kg/m^3
Density of mercury = 13.6 × 10^3 kg/m^3
Speed of sound in air = 340 m/s

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 萬有引力常數, 6.67 × 10^{-11} (N m^2/kg^2)
e – 電子電荷, –1.6 × 10^{-19} (A s)
ε_0 – 靜電常數, 8.85 × 10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11 × 10^{-31} kg
c – 真空光速, 3.0 × 10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 (= 1 天文單位) = 1.5 × 10^{11} m
地球-月球距離 = 3.84 × 10^8 m
太陽質量 = 1.99 × 10^{30} kg
水的密度 = 1.0 × 10^3 kg/m^3
鐵的密度 = 7.7 × 10^3 kg/m^3
水銀的密度= 13.6 × 10^3 kg/m^3
空氣中聲速= 340 m/s

The following conditions will be applied unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除非特別注明，否則下列條件將適用於本卷所有問題：

1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每道題 2 分，每道題選擇一個答案）

The MC questions with the '*' sign may require information on page-3.
帶 * 的選擇題可能需要用到第三頁上的資料。

MC1

An object in constant acceleration motion with zero initial velocity covers 1 meter within the 1st second. How many meters will it cover within the 4th second?

一物體作初速度爲零的勻加速運動，在第一秒鐘內走過的距離爲 1 米。問物體在第四秒鐘內走過的距離爲多少米？

(a) 7 (b) 1 (c) 9 (d) 3 (e) 16

MC2

Spaceship A moves at velocity 40($\vec{x}_0 + \vec{y}_0$) km/s, while spaceship B moves at 30 ($\vec{x}_0 - \vec{y}_0$) km/s. Find the relative velocity in the unit of km/s between the two ships.

飛船-A 以 40($\vec{x}_0 + \vec{y}_0$)每秒公里的速度行駛，飛船-B 以 30 ($\vec{x}_0 - \vec{y}_0$)每秒公里的速度行駛。求兩飛船之間的相對速度（以每秒公里爲單位）。

(a) $10\vec{x}_0 + 10\vec{y}_0$ (b) $10\vec{x}_0 + 70\vec{y}_0$ (c) $70\vec{x}_0 + 10\vec{y}_0$ (d) $10\vec{x}_0 + 10\vec{z}_0$
(e) $10\vec{x}_0 - 10\vec{y}_0$

MC3*

Jupiter revolves around the sun at 11 years and 315 days per revolution. Find the distance between Jupiter and Sun **in terms of astronomical unit (A. U.).**

木星每 11 年 315 天繞太陽運行一圈。求木星與太陽間的距離（**以天文單位表達**）。

(a) 5.2 (b) 1.0 (c) 11.9 (d) 15 (e) 3.0

MC4

An object is in motion with constant acceleration. Which of the following statement is correct?

(a) The object must move in a straight line.
(b) The distance traveled must be proportional to the square of the time.
(c) The speed of the object must increase uniformly with time.
(d) The velocity must be constant.
(e) The total external force must do work to the object.

一物體作勻加速運動。以下哪句論述是對的？

(a) 該物體必須作直線運動。
(b) 該物體運動的距離必須與時間平方成正比。
(c) 該物體的速率必須是隨時間均勻增加的。
(d) 該物體的速度必須是常數。
(e) 合外力必須對物體作功。

MC5

An object is placed on the left side of a positive lens and the real image is formed on the right side of the lens. If the object is moved slightly to the right, the image will _______.

(a) not move (b) move to the left (c) move to the right

(d) move up (e) move down

一物體放在一正透鏡的左邊，它的實像在透鏡的右邊。若將物體向右移動一點，則像會_______。

(a) 不動 (b) 向左移動 (c) 向右移動

(d) 向上移動 (e) 向下移動

MC6

Same initial condition as MC5 above. If the upper half of the lens is covered with black cloth, the image will _____ .

(a) totally disappear (b) move up (c) move down
(d) become brighter (e) become dimmer

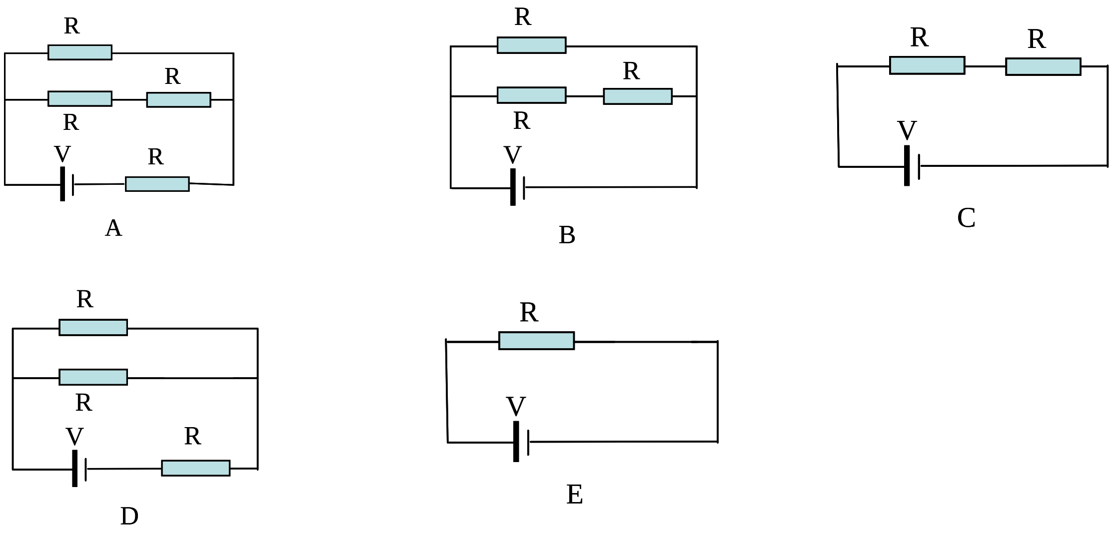
初始情況與 MC5 相同。若將透鏡上半部用黑布遮住，則像會_______。

(a) 完全消失 (b) 向上移動 (c) 向下移動 (d) 亮一點
(e) 暗一點

MC7

Five circuits are shown below. The batteries all have the same voltage *V* and all resistors have the same resistance *R*. In which circuit does the battery output the most power?

在下列電路圖中，所有電池有相同的電壓 *V*，所有電阻器有相同的電阻值 *R*。問哪個電路中的電池輸出功率最大？



(a) A (b) B (c) C (d) D (e) E

MC8

A 20 μF capacitor charged to 2.0 kV and a 40 μF capacitor charged to 4.0 kV are connected to each other, with the positive plate connected to the positive plate, and the negative plate to the negative plate. What is the final charge on the 20 μF capacitor?

一個 20 μF 的電容充電至 2.0 kV 的電壓，一個 40 μF 的電容充電至 4.0 kV 的電壓。現將兩電容的正極與正極相連，負極與負極相連，問在 20 μF 的電容上有多少電荷?

(a) 50 mC (b) 200 mC (c) 40 mC (d) 67 mC (e) 120 mC

MC9

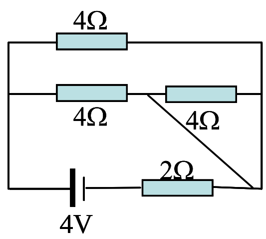
A cylinder is filled with ideal gas with pressure *P*. The gas is then heated up by an external source and has doubled its volume and temperature, while half the gas has leaked out. Find the pressure of the remaining gas.

一汽缸充滿理想氣體，初始壓強爲 *P*。一外部熱源對氣體加熱，氣體的體積和溫度都增加到原來的兩倍，但有一半的氣體漏掉了。求剩下氣體的壓強。

(a) 2*P* (b) 4*P* (c) *P* (d) 0.5*P* (e) 0.25*P*

MC10

In the circuit, the electric current through the battery is _____ Amperes.



在電路中，流經電池的電流爲_____ 安培。

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

MC11

Which of the process below DOES NOT involve energy transfer?

(a) Lifting a cup (b) Turning on a lamp (c) Burning some wood
d) Motion of a simple pendulum (e) Moon revolving around Earth in circular motion

以下哪個過程沒有能量轉換？

(a) 拿起個杯子 (b) 開亮盞燈 (c) 燒木柴
d) 單擺運動 (e) 月亮繞地球作圓周運動

MC12*

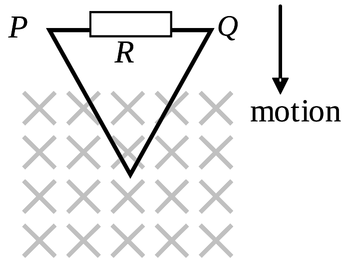
Ignore friction between water and pipe wall. Water is pumped from a reservoir up to a height of 10 m at the rate of 30 kg per second through a pipe of 2.0×10^{-3} m^2 in cross section area. Find the pumping power.

不考慮水與管壁的摩擦力。抽水機經一橫截面面積爲 2.0×10^{-3}平方米的水管將水庫的水以每秒 30 公斤的速率抽上 10 米的高處。求抽水機的功率。

(a) 6315 (b) 2940 (c) 3573 (d) 1305 (e) 4960

MC13

As shown, a triangular coil is moving into a uniform magnetic field region (field pointing into the paper) at uniform velocity. Which of the following statements is/are correct?



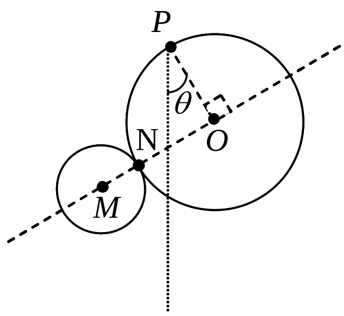
(1) An induced current flows from P to Q through resistor R.
(2) The magnetic flux passing through the coil is increasing at a uniform rate.
(3) The induced emf is increasing at a uniform rate

如圖，一個三角型線圈以勻速進入一均勻磁場區域（磁場指向紙面）。問下列哪些論述是正確的？
(1) 感應電流經電阻 R 由 P 流向 Q 。
(2) 穿過線圈的磁通量以勻速增加。
(3) 感應電動勢以勻速增加。

(a) (1)   (b) (3)   (c) (1) and (2)   (d) (2) and (3)
(e) (1) and (3)

MC14

Two uniform circular discs of the same material and thickness are adhered together at point-N. Both discs lie in the same vertical plane. The rigid body is hinged at point-P and it can rotate freely about P in the vertical plane. $PO \perp OM$ and ON = 2 NM, where point-O and point-M are the centres of the discs. When static equilibrium is reached, find the angle θ between PO and the vertical direction.

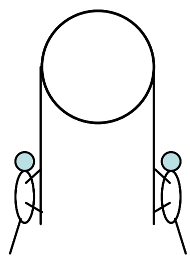


兩個同樣厚度的均勻圓盤，由同樣物質組成，在 N 點粘在一起，組成一剛體。盤面處於垂直面，大圓盤吊在 P 點，整個剛體可繞 P 點自由轉動。O 和 M 分別是兩圓盤的中心，$PO \perp OM$ ，ON = 2 NM。求當平衡時 PO 與垂直方向的夾角θ 。

(a) 16.7^o (b) 26.6^o (c) 30^o (d) 36.9^o (e) 11.7^o

MC15

Two monkeys of the same weight are holding tightly the two ends of a rope with negligible mass. The rope passes through a smooth pulley, as shown in the figure. The two monkeys are initially at rest. Now the left monkey starts to climbs up the rope at an average speed v relative to the pulley. Find its average speed relative to the right monkey.

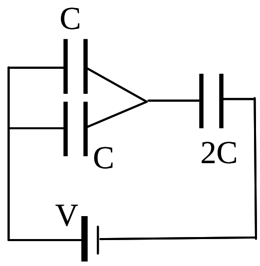


如圖所示，兩隻重量同樣的猴子緊緊抓住無重量細繩的兩端，細繩掛在光滑的滑輪上。現在左邊的猴子開始以相對於滑輪為 v 的平均速度向上爬，求左邊猴子相對於右邊猴子的平均速度。

(a) 0   (b) v upwards   (c) 2v upwards   (d) v downwards
(e) 2v downwards

MC16

As shown, find the charge on one of the capacitors with capacitance C.



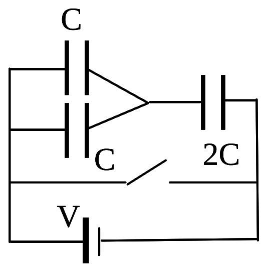
如圖，求其中一個電容值爲 C 的電容上的電荷。

(a) $2CV$ (b) CV (c) $\frac{1}{2}CV$ (d) $\frac{1}{4}CV$

(e) $\frac{1}{8}CV$

MC17

Same initial condition as MC16. If the battery is suddenly taken away and the switch shown in the circuit diagram is closed, find the total charge that will pass through the switch.



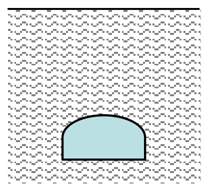
初始情況與 MC16 相同。若突然拿走電池，併且合上電路的開關，求將流過開關的總電荷。

(a) $2CV$ (b) CV (c) $\frac{1}{2}CV$ (d) $\frac{1}{4}CV$

(e) $\frac{1}{8}CV$

MC18*

An object of volume V is submerged in a liquid of density ρ. It has a flat bottom of area A which is at a depth of H in the liquid, and a dome shaped top. Find the total force of liquid acting on the top of the object.



(a) $|\rho HAg - \rho Vg|$ and upward (b) $|\rho HAg - \rho Vg|$ and downward

(c) ρVg and downward (d) ρVg and upward

(e) ρHAg and upward

一體積爲 V 的物體完全浸沒在密度爲ρ的液體裏。物體有一圓的拱頂和平的底部，底部處於的深度爲 H，底部面積爲 A。求液體對拱頂的合力。

(a) $|\rho HAg - \rho Vg|$ 向上 (b) $|\rho HAg - \rho Vg|$ 向下 (c) ρVg 向下

(d) ρVg 向上 (e) ρHAg 向上

MC19

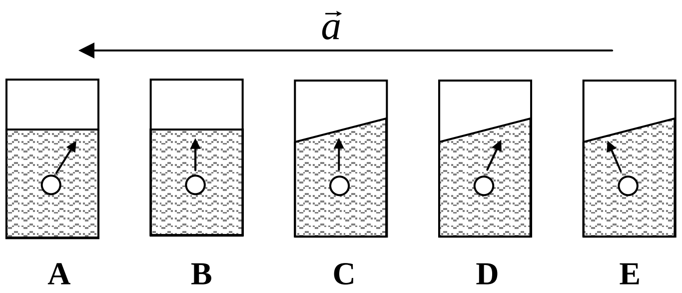
An AC generator consists of 6 turns of rectangular wire coil. Each turn has an area of 0.040 m^2. The coil rotates in a uniform field (B = 0.20 T) at a constant frequency of 50 Hz. Find the maximum induced electro-motive force.

一交流發電機有 6 圈的長方型線圈，每圈面積爲 0.040 平方米。線圈在 0.20 T 的均勻磁場中以每秒 50 周的頻率轉動。 求最大感應電動勢。

(a) 4.8 V (b) 2.5 V (c) 3.0 V (d) 15 V (e) 7.5 V

MC20
A cup of water is placed in a car under constant acceleration to the left, as shown. Inside the water is a small air bubble. The following figures show five situations of the shape of the water surface and the direction of motion of the bubble as indicated by the arrow on the bubble. Which one is correct?

在一輛作勻加速運動的車裏有一杯水，水裏有一個小氣泡。下圖展現了五個水面形狀和以小箭頭表示的氣泡運動方向的情形。哪個是正確的？



(a) A (b) B (c) C (d) D (e) E

《End of MC's 選擇題完》

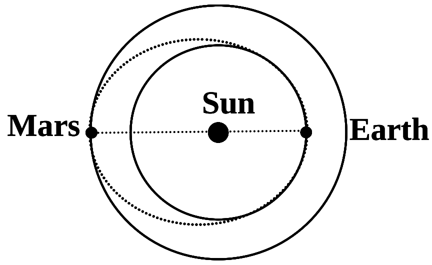
Open Problems 開放題
Total 5 problems 共 5 題

The Open Problem(s) with the '*' sign may require information on page-3.
帶 * 的開放題可能需要用到第三頁上的資料。

Q1* (10 points)
A particle of mass m carrying charge q is placed mid-way between two fixed point charges, each carrying charge Q. All three particles are on the x-axis. The distance between the fixed charges is $2d$. The particle is confined to move along the x-axis only.
(a) Determine whether the equilibrium position of the particle is stable when q and Q are of the same sign or of opposite sign. (3 points)
(b) In the case of stable equilibrium, find the vibration frequency of small amplitude oscillation of the particle. (7 points)
(Hint: for $x << 1$, $(1+ax+bx^2)^n \approx (1+ax)^n \approx 1+nax$, where n can be a fraction number or an integer, and a and b are constants.)
題 1*（10 分）
一質量爲 m 帶電爲 q 的粒子放在兩個帶電爲 Q 的固定點電荷的正中間。這三個點電荷都在 x-軸上。兩固定點電荷之間的距離爲 $2d$。 粒子只能在 x-軸上作一維運動。
(a) q 和 Q 可爲同號或反號，問哪種情況下粒子的平衡是穩定的？（3 分）
(b) 在粒子的平衡是穩定的情況下，求粒子作小幅振動的頻率。（7 分）
(提示: 當 $x << 1$, $(1+ax+bx^2)^n \approx (1+ax)^n \approx 1+nax$, 其中 n 可以是分數或整數， a 和 b 爲常數。)

Q2* (12 points)

(a) The distance between Mars and Sun is 2.28×10^{11} meters. Find the period of revolution of Mars (in the unit of days) around the sun. (2 points)

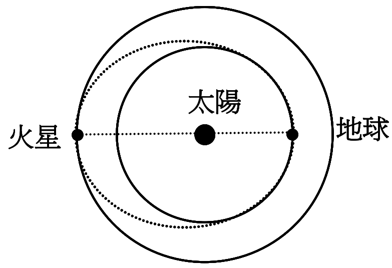


(b) To launch a spaceship from Earth to Mars, the spaceship should follow an elliptical orbit with Earth on one end of the long axis and Mars on the other end. The sun is also on the long axis, as shown. The time it takes to fly half the elliptical orbit is 259 days. Where should Mars be relative to Earth when the spaceship is launched? Express the position of Mars in terms of the angle between the line joining Earth and Sun and the line joining Mars and Sun. (5 points)

(c) To come back from Mars the spaceship will follow the same elliptical orbit and take the same amount of days as the first journey. How many days should the spaceship stay on Mars before it is launched again on its way back to Earth? (5 points)

題 2*（**12** 分）

(a) 火星離太陽的距離爲 2.28×10^{11}米。求火星繞太陽公轉一周的時間（以天爲單位）。（2 分）

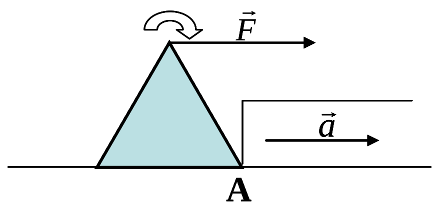


(b) 從地球向火星發射的飛船須沿如圖所示的橢圓軌道運行。地球和火星必須分別處於橢圓長軸的兩端。太陽也在長軸上。飛船運行半個橢圓軌道需時 259 天。若以地球與太陽和火星與太陽連線的夾角來描述地球與火星的相對位置，問應該在什麼相對位置時發射飛船？（5 分）

(c) 回程的飛船軌道也是同樣的橢圓，需時也相同。爲了回到地球，飛船要在火星等多久？（5 分）

Q3* (8 points)

As shown, a uniform block of mass m and with equal lateral triangle cross section is placed at the edge of a step on a car floor which is under constant acceleration a ($< g$) to the right. A horizontal force on the apex of the triangle $\vec{F}$ is large enough to just tilt the block.



(a) Find the magnitude of $\vec{F}$. (3 points)

(b) Find the force at point-A where the block is in contact with the step edge. (5 points)

題 3*（**8** 分）

如圖所示，一質量爲 m 橫截面爲等邊三角型的均勻物塊放在一汽車地板上的台階邊，汽車以勻加速 a ($< g$)向右行駛。一水平力 $\vec{F}$ 作用於三角型物塊的頂上，剛好可以使物塊翻轉。

(a) 求 $\vec{F}$ 的大小。（3 分）

(b) 求台階邊與物塊接觸的 A 點處的作用力。（5 分）

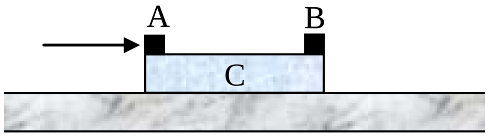
Q4* (14 points)

As shown, a block-C of mass M_C and length L is placed on a smooth horizontal floor. On its two ends are two small blocks A and B with masses of M_A and M_B, respectively. The friction coefficient between block-A and block-C is μ_A, that between block-B and block-C is μ_B. The three objects are originally at rest. Block-A is then given a short impulse I, and starts to move to the right.

(a) Find the condition that block-B will remain rest relative to block-C. (4 points)

(b) Assuming that block-B remains rest relative to block-C, find the minimum impulse I_{min} such that block-A can reach block-B. (5 points)

(c) Assume $I > I_{min}$. Block-A and block-B then collide elastically. Find the condition that block-A's velocity relative to block-C is reversed after the collision. (5 points)



題 4*（**14** 分）

如圖所示，一質量爲M_C 長度爲L的物塊-C放在光滑的水平面上。在它的兩端有兩個小物塊A和B。物塊-A的質量爲M_A，與物塊-C的磨擦係數爲 μ_A 。物塊-B的質量爲M_B，與物塊-C的磨擦係數爲 μ_B 。三物體原處於靜止狀態。物塊-A突然受到一短促衝量I，於是開始向右運動。

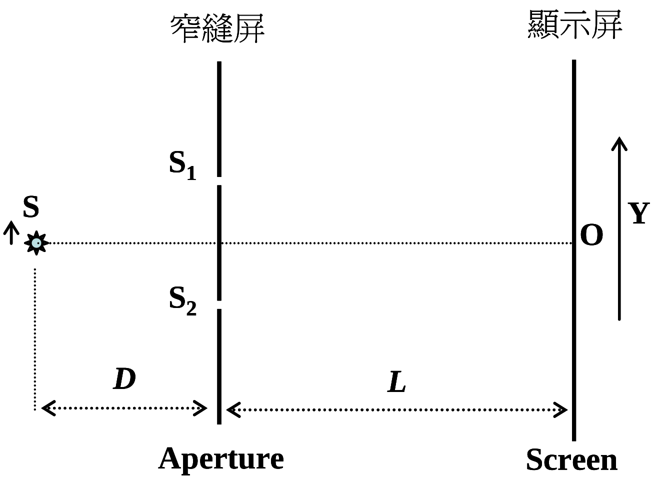
(a) 求物塊-B 與物塊-C 保持相對靜止的條件。（4 分）

(b) 設物塊-B與物塊-C可以保持相對靜止，求能使物塊-A到達物塊-C另一端的最小衝量I_{min}。（5 分）

(c) 設$I > I_{min}$。物塊-A與物塊-B作完全彈性碰撞，求碰撞後能使物塊-A相對於物塊-C的速度反向的條件。（5 分）

Q5 (16 points)

Shown in the drawing is a typical Young's interference experiment setup (not drawn to the true scale, though). A point light source S emitting monochromatic light waves of wavelength λ is placed at equal distance to two narrow slits S_1 and S_2 of width a on an aperture plane. The distance between S and the aperture plane is D. A display screen is parallel to the aperture plane and is at a distance L from it. D and $L >> \lambda$. A Y-coordinate is imprinted upright on the screen, its origin (Y = 0) being at point-O. The line joining S and O is the central axis of the setup. It is perpendicular to the aperture plane, and passes through the aperture plane at the middle point between the two slits, which are separated by a distance d (<< D and L). The light intensity on the screen can be expressed as $I(Y)/I_{max} = f(Y)$, where the maximum value of $f(Y)$ is 1. (Hint: The total intensity due to the interference of two waves of equal intensity I_0 is given by $I = 2I_0(1+\cos\Delta)$, where Δ is the phase difference between the two waves.)



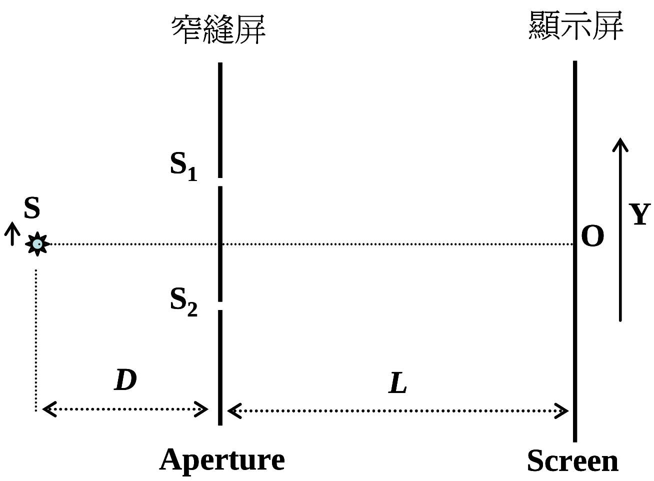
(a) When $a << \lambda$, the amplitude E of the light wave emitted from the slits is constant in all propagation directions. Find $f(Y)$. (4 points)

(b) Now the light source S is moved upwards by a small distance b (<< D) as the small arrow indicates, find $f(Y)$. (6 points)

(c) The light source is moved back to the central axis. When a is comparable to λ, the amplitude E of the light wave emitted from the slits is no longer constant. It is instead given by $E(\theta)=\dfrac{\sin(\beta)}{\beta}$, where $\beta\equiv\dfrac{\pi a}{\lambda}\theta$, and θ (<< 1) is the angle between the emitting (propagating) direction and the central axis. Find $f(Y)$ and the position on the screen where the light intensity is maximum. (6 points) (Hint: $\dfrac{\sin(x)}{x}\to 1$ when $x\to 0$.)

題 5（**16** 分）

圖中不按比例所畫的是楊氏干涉實驗示意圖。一個點光源 S 發出波長爲λ的單色光，放在離兩個寬度爲a的在窄縫屏上的S_1和S_2 窄縫相等距離的位置，光源離窄縫屏的距離爲D。距離窄縫屏L 處有一個與窄縫屏平行的顯示屏。D和L >> λ。顯示屏上印有豎直的Y-坐標，其原點(Y = 0) 在O點。S和O的連線是整個實驗裝置的中心軸，與窄縫屏垂直，並經過S_1 和 S_2的中點。S_1 和 S_2之間的距離爲 d (<< D 和 L)。顯示屏上的光強可表達爲 $I(Y)/I_{\max}=f(Y)$，其中 $f(Y)$ 的最大值爲 1。



（提示：兩個強度均爲I_0的波干涉所產生的總光強爲 $I=2I_0(1+\cos\Delta)$，其中Δ爲兩波之間的位相差。）

(a) 當 a << λ 時，由窄縫發出的光波的振幅 E 在各個傳播方向都是一樣的，求 $f(Y)$。（4 分）

(b) 現將 S 如箭頭所示向上移一小距離 b (<< D) ，求 $f(Y)$。（6 分）

(c) 將 S 移回到中心軸。當 a 和λ 差不多大小時，由窄縫發出的光波的振幅 E 在各個傳播方向是不同的。取而代之的是$E(\theta)=\dfrac{\sin(\beta)}{\beta}$, 其中 $\beta\equiv\dfrac{\pi a}{\lambda}\theta$, θ (<< 1) 是光傳播方向與中心軸的夾角。 求 $f(Y)$ ，以及顯示屏上的光強最大的位置。（6 分）(提示: 當 $x\to 0$ ，$\dfrac{\sin(x)}{x}\to 1$。)

《**END** 完》

Solutions for Multiple Choice Questions

1a 2b 3a 4e 5c 6e 7b 8d 9d 10a 11e 12a 13b 14a 15a 16c 17b 18b 19d 20e

Solutions for Open Questions

Q1

(a) Suppose the charge *q*'s position is *r* to the midpoint of two *Q*s.

$$F_1 = \frac{kqQ}{(d+r)^2}, \qquad F_2 = \frac{kqQ}{(d-r)^2}$$

The total force on *q* is $F_{total} = F_1 - F_2 = -kqQ[\frac{1}{(d-r)^2} - \frac{1}{(d+r)^2}]$. Obviously, the factor $\frac{1}{(d-r)^2} - \frac{1}{(d+r)^2}$ is positive. When *q* and *Q* are of the same sign, *F* is negative, so its effect is to push the charge *q* back to the midpoint. Therefore *q* is stable at the midpoint.

On the contrary, when *q* and *Q* are of opposite sign, a small deviation from the midpoint will increase the in-balance of the two forces, so *q* is not stable at the midpoint.

(b) $F = -kqQ[\frac{1}{(d-r)^2} - \frac{1}{(d+r)^2}] = -kqQ\frac{4dr}{(d^2-r^2)^2} = -\frac{4kqQr}{d^3}$

so $\omega^2 = \frac{4kQq}{md^3}$

Q2

(a) The gravity force of the sun provide the acceleration for Mars' circular motion

$$\frac{Gm_M M_S}{r^2} = m_M\omega^2 r = m_M r\left(\frac{2\pi}{T}\right)^2$$

$$\Rightarrow T = \sqrt{\frac{4\pi^2 r^3}{GM_S}} = \sqrt{\frac{4\pi^2(2.28\times10^{11})^3}{6.67\times10^{-11}\times1.99\times10^{30}}} = 5.93 \times 10^7 \text{ s} = 686 \text{ days}$$

(b) Within 259 days, the angular displacement of Mars is $\frac{259}{686}\times360^\circ = 136^\circ$, so Mars should be ahead of Earth by 180º - 136º = 44º.

(c) When the spaceship arrives in Mars, Earth has covered angular distance $\frac{259}{365}\times360^\circ = 255^\circ$. Earth is now AHEAD of Mars by 255º -180º = 75º. When the spaceship is launched again in Mars, Earth will cover the same angular distance. Therefore, Earth should be BEHIND Mars by 255º -180º = 75º, or ahead of Mars by 360º - 75º = 285º. To wait for that to happen, and noted that the angular speeds of Earth is $\omega_E = \frac{360}{365}$ degree/day and that of Mars is $\omega_M = \frac{360}{686}$ degree/day, the spaceship should stay on Mars for

$$\frac{(360-75)-75}{\frac{360}{365}-\frac{360}{686}} = \frac{365\times 210\times 686}{360\times(686-365)} = 455 \text{ days.}$$

Q3

Using point-A as the pivotal point, the total torque is

$$\tau = mg\cdot\frac{\sqrt{3}}{4}d + ma\cdot\frac{\sqrt{3}}{4}d - F\cdot\frac{\sqrt{3}}{2}d = 0$$

$$\Rightarrow F = \frac{m}{2}(g+a)$$

Use the force equilibrium equation then.

In the vertical direction, $f_v = mg$

In the horizontal plane, $f_h + ma = F = \frac{m}{2}(g+a) \Rightarrow f_h = \frac{m}{2}(g-a)$

The total force at point A includes f_v and f_h.

Q4

(a) The friction force of block-A on Block-C is $\mu_A m_A g$. The acceleration of C and B is then $\frac{\mu_A m_A g}{(m_B+m_C)}$

The condition for B to stay with C is then $\mu_B \geq \frac{\mu_A m_A}{(m_B+m_C)}$.

(b) When block-A reaches the end of Block-C while its velocity becomes the same as C&B, the speed of the system A&B&C is $v_f = \frac{I}{(m_A+m_B+m_C)}$. By energy conservation,

$\frac{I^2}{2m_A} - \mu_A m_A gL \geq \frac{I^2}{2(m_A+m_B+m_C)}$, or $I \geq \sqrt{\frac{2\mu_A m_A^2 gL}{(m_B+m_C)}(m_A+m_B+m_C)}$

(c) Suppose before the collision the speed of A is V_A, and that of B and C is V_B.

During the collision process

$$m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$$

$$m_A V_A^2 + m_B V_B^2 = m_A V_A'^2 + m_B V_B'^2$$

From them we have

$$V_A' = \frac{V_A(m_A-m_B)+2m_B V_B}{m_A+m_B}$$

$$V_B' = \frac{V_B(m_A-m_B)+2m_A V_A}{m_A+m_B}$$

The condition for the relative speed of A to C is reversed is $V_A{}' < V_B$, that is, $V_A{}' = \dfrac{V_A(m_A - m_B) + 2m_B V_B}{m_A + m_B} < V_B \Rightarrow V_A(m_A - m_B) < V_B(m_A - m_B)$, since A collides with B means that $V_A > V_B$, the condition is $m_A < m_B$.

Alternatively, in the instantaneous reference where C is at rest, B is initially at rest. So
$$\begin{aligned} m_A V_A &= m_A V_A{}' + m_B V_B{}' \\ m_A V_A{}^2 &= m_A V_A{}'^2 + m_B V_B{}'^2 \end{aligned}.$$
Then
$$\begin{aligned} V_A{}' &= \frac{(m_A - m_B)}{m_A + m_B} V_A \\ V_B{}' &= \frac{2m_A}{m_A + m_B} V_A \end{aligned}$$
. So $m_A < m_B$ will make $V_A{}'$ negative.

Q5

(a) The path difference is $\delta = \sqrt{L^2 + (Y + d/2)^2} - \sqrt{L^2 + (Y - d/2)^2}$. As $d << L$, only the linear terms of d should be kept.

$$\delta = L\sqrt{1 + (\frac{Y}{L} + \frac{d}{2L})^2} - L\sqrt{1 + (\frac{Y}{L} - \frac{d}{2L})^2} = L\sqrt{1 + \left(\frac{Y}{L}\right)^2 + \frac{dY}{L}} - L\sqrt{1 + \left(\frac{Y}{L}\right)^2 - \frac{dY}{L}}$$
$$= L\approx(1 + \left(\frac{Y}{L}\right)^2 + \frac{dY}{2L^2}) - L\approx(1 + \left(\frac{Y}{L}\right)^2 - \frac{dY}{2L^2}) = \frac{dY}{L}.$$

The answer indicates that the pass difference is given by $\delta = d\theta$, where $\theta = \dfrac{Y}{L}$ is the angle between the emission direction and the central axis.

Using the hint given, we get $f(Y) = \dfrac{1}{2}\left(1 + \cos\left(\dfrac{2\pi dY}{\lambda L}\right)\right)$.

(b) Now there is also a path difference for light from the source to the two slits. Using the same calculation procedure as in (a), such path difference is $\delta_1 = \dfrac{bd}{D}$, and the total path difference is $\delta = \dfrac{bd}{D} + \dfrac{dY}{L}$. Finally $f(Y) = \dfrac{1}{2}\left(1 + \cos\left(\dfrac{2\pi dY}{\lambda L} + \dfrac{2\pi db}{\lambda D}\right)\right)$, and the interference fringes shift by a distance $Y_0 = \dfrac{Lb}{D}$.

(c) Following (a), the emission direction angle is given by $\theta = \dfrac{Y}{L}$, the amplitude is now direction dependent, so $f(Y) = \dfrac{1}{2}\left(1 + \cos\left(\dfrac{2\pi dY}{\lambda L}\right)\right)\dfrac{\sin(\beta)}{\beta}$, where $\beta \equiv \dfrac{\pi a}{\lambda}\dfrac{Y}{L}$. The maximum occurs at $Y = 0$ where $f(Y) = 1$.

Hong Kong Physics Olympiad 2009
2009 年香港物理奧林匹克競賽

Written Examination
筆試

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
香港特區政府教育局
The Hong Kong Physical Society
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

May 24, 2009
2009 年 5 月 24 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for further announcement.

   選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID number in the field of “I.D. No., and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of “Course number & Section No.”

   請依照選擇題答題紙的指示，用HB鉛筆在選擇題答題紙的I.D. No.欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格上填上你的英文姓名，最後於“Course & Section No. ”欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.
   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用HB鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of “Course Title”, and write your English name in the field of “Student Name” and your 8-dight Participant ID number in the field of “Student Number”. You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.
   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.
   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitational constant, 6.67×10^{-11} (N m^2/kg^2)
e – charge of an electron, -1.6×10^{-19} (A s)
ε_0 – electrostatic constant, 8.85×10^{-12} (A s)/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance (= 1 Astronomical Unit (AU)) = 1.5×10^{11} m
Earth-Moon distance = 3.84×10^8 m
Mass of the sun = 1.99×10^{30} kg
Density of water = 1.0×10^3 kg/m^3
Standard atmosphere pressure $p_0 = 1.0 \times 10^5$ N/m^2

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 萬有引力常數, 6.67×10^{-11} (N m^2/kg^2)
e – 電子電荷, -1.6×10^{-19} (A s)
ε_0 – 靜電常數, 8.85×10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 (= 1 天文單位) = 1.5×10^{11} m
地球-月球距離 = 3.84×10^8 m
太陽質量 = 1.99×10^{30} kg
水的密度 = 1.0×10^3 kg/m^3
標準大氣壓 $p_0 = 1.0 \times 10^5$ N/m^2

The following conditions will be applied to all questions unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除非特別注明，否則下列條件將適用于本卷所有問題：

1) 所有物體都處于地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每道題 2 分，每道題選擇一個答案）

The MC questions with the '*' sign may require information on page-3.
帶 * 的選擇題可能需要用到第三頁上的資料。

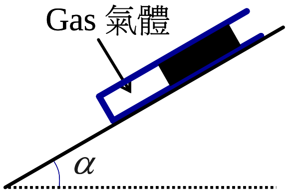
MC1

Spaceship-A moves at velocity $40(\vec{x}_0+\vec{y}_0)$ km/s relative to a fixed coordinate system. Spaceship-B moves at 30 $(\vec{x}_0-\vec{y}_0)$ km/s as seen by Spaceship-A. Find the velocity of Spaceship-B in the fixed coordinate system in the unit of km/s.
飛船-A 以 $40(\vec{x}_0+\vec{y}_0)$公里/秒的速度相對于一固定坐標行駛，飛船-A 看到飛船-B 以 30 $(\vec{x}_0-\vec{y}_0)$公里/秒的速度行駛。求飛船-B 相對于固定坐標的速度（以公里/秒爲單位）。

(a) $10\vec{x}_0+10\vec{y}_0$ (b) $10\vec{x}_0+70\vec{y}_0$ (c) $10\vec{x}_0+10\vec{z}_0$ (d) $70\vec{x}_0+10\vec{y}_0$
(e) $10\vec{x}_0-10\vec{y}_0$

MC2*

A light glass tube with a sealed lower end and cross section area S = 2.5 cm^2 contains a column of mercury of mass m = 2 kg. Between the mercury and the lower end is some trapped gas. The glass tube is sliding down a slope with an inclining angle α = 30°. The dynamic friction coefficient between the tube and the slope is $\mu=\sqrt{3}/6$. Find the pressure of the trapped gas in terms of the standard atmosphere pressure p_0.



下端封閉、橫截面積 S = 2.5 cm^2 的輕玻璃管內裝有質量 m = 2 kg 的水銀柱。一些氣體被封閉在水銀柱與下端之間。玻璃管沿傾角α = 30°的斜面下滑，玻璃管與斜面間的滑動摩擦系數爲 $\mu=\sqrt{3}/6$。求被封閉氣體的壓強（以標準大氣壓 p_0 爲單位）。

(a) 1.7 (b) 1.2 (c) 1.9 (d) 2.3 (e) 16

MC3*

Saturn revolves around the sun at 29.5 years per revolution. Find the distance between Saturn and Sun **in terms of astronomical unit (A. U.).**
土星每 29.5 年繞太陽運行一圈。求土星與太陽間的距離（**以天文單位表達**）。

(a) 5.2 (b) 9.5 (c) 11.9 (d) 15 (e) 3.0

MC4

The pixel size of a digital camera is 5.0 × 10^{-3} mm. The focal length of the lens is 10 mm. The image plate from the lens is 15 mm. Find the minimum size of an object the camera can resolve.
一數碼相機的像素尺寸爲 5.0 × 10^{-3} mm，相機鏡頭的焦距爲 10 mm，像面離鏡頭 15 mm。求相機能分辨的最小物體。

(a) 5.0 × 10^{-3} mm (b) 1.5 × 10^{-3} mm (c) 1.5 × 10^{-2} mm
(d) 2.5 × 10^{-3} mm (e) 1.0 × 10^{-2} mm

MC5

Following MC4, if the diameter of the camera lens is doubled, what will be the increase of light intensity on the image plate?

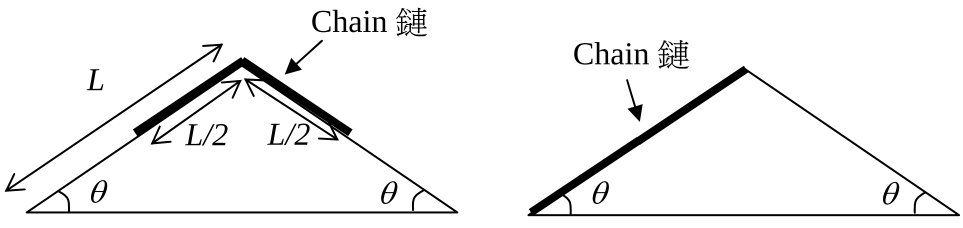
(a) 4 times (b) 2 times (c) 1.5 times (d) ½ times (e) ¼ times

接 MC4。若相機鏡頭的直徑加倍，則像面上的光強爲原來的_______。

(a) 4 倍 (b) 2 倍 (c) 1.5 倍 (d) ½ (e) ¼

MC6

A uniform chain of mass m and length L is originally placed mid-way on the top of a fixed smooth double-sided wedge (Figure-A). The length of each side of the wedge is L. It is then given a slight push. Find the kinetic energy of the chain when the whole chain has just slid to the left side of the wedge (Figure-B).



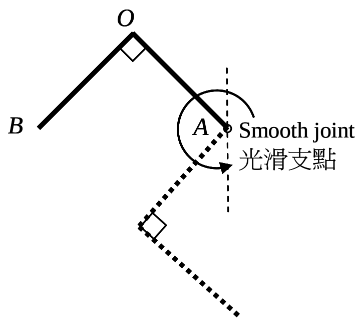
(A) (B)

一質量爲 m 長度爲 L 的均勻鏈條， 原來放在一邊長爲 L 的固定的光滑等邊雙斜面上（圖-A）。現將鏈條輕輕推一下，求當鏈條剛好全部滑到左斜面時（圖-B）的動能。

(a) $mgL\sin\theta$ (b) $\dfrac{mgL\sin\theta}{2}$ (c) $\dfrac{mgL\sin\theta}{4}$ (d) $\dfrac{mgL\sin\theta}{8}$

(e) $2mgL\sin\theta$

MC7



A uniform "L"-shaped rigid body AOB of mass m, where $AO = OB = l$ and $\angle AOB$ is a right angle, is hinged to a smooth joint at Point A of the body and can swing freely in a vertical plane. Initially, the body is released from rest with AB horizontal. Find the ***maximum kinetic energy*** of the body.

一質量爲 m 的均勻"L"-形剛體 AOB，$AO = OB = l$，角 $\angle AOB$ 爲直角。剛體的一端（點-A）挂在光滑支點上，剛體可在垂直面內自由擺動。剛體處于初始位置時 AB 連綫是水平的。求將剛體從該位置放開後剛體可得的***最大動能***。

(a) $\left(\dfrac{\sqrt{2}+\sqrt{10}}{4}\right)mgl$ (b) $\left(\dfrac{\sqrt{2}+\sqrt{5}}{2}\right)mgl$ (c) $\left(\dfrac{\sqrt{3}+\sqrt{10}}{4}\right)mgl$

(d) $\left(\dfrac{\sqrt{3}+\sqrt{5}}{2}\right)mgl$ (e) $\left(\dfrac{\sqrt{2}}{2}\right)mgl$

MC8

A 20 μF capacitor charged to 2.0 kV and a 40 μF capacitor charged to 4.0 kV are connected to each other, with the positive plate connected to the negative plate, and the negative plate to the positive plate. What is the final charge on the 20 μF capacitor?

一個 20 μF 的電容充電至 2.0 kV 的電壓，一個 40 μF 的電容充電至 4.0 kV 的電壓。現將兩電容的正極與負極相連，負極與正極相連，問在 20 μF 的電容上有多少電荷?

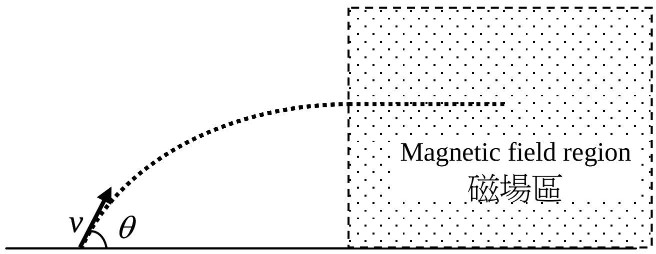
(a) 50 mC (b) 200 mC (c) 40 mC (d) 67 mC (e) 120 mC

MC9

A particle of mass M and carrying charge Q is launched with initial speed v and at an angle of θ relative to the horizontal direction. When it reaches the maximum height it enters a region of uniform magnetic field. In the region it moves at constant velocity in the horizontal direction. Determine the direction and strength of the magnetic field.

(a) outward, $\frac{Mg}{Qv\cos\theta}$ (b) inward, $\frac{Mg}{Qv\cos\theta}$ (c) outward, $\frac{Mg}{Qv\sin\theta}$

(d) inward, $\frac{Mg}{Qv\sin\theta}$ (e) inward, $\frac{Mg}{Qv\tan\theta}$



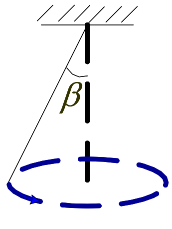
一質量爲 M 帶電量爲 Q 的粒子以速度 v 相對于水平綫角度θ射出。粒子到達最高點後進入一均勻磁場區，并以恒定速度沿水平綫運動。求磁場的方向和大小。

(a) 向外, $\frac{Mg}{Qv\cos\theta}$ (b) 向內, $\frac{Mg}{Qv\cos\theta}$ (c) 向外, $\frac{Mg}{Qv\sin\theta}$

(d) 向內, $\frac{Mg}{Qv\sin\theta}$ (e) 向內, $\frac{Mg}{Qv\tan\theta}$

MC10

As shown, one end of a light thread is fixed on the ceiling and the other end tied to a small sphere. The angle between the thread and the vertical direction is β. When β = α and α is a small angle, the sphere is in simple harmonic motion like a pendulum with period T. When β = α_1 or α_2 ($\alpha < \alpha_1 < \alpha_2$), the sphere is in a uniform circular motion in a horizontal plane with period T_1 or T_2, respectively. Then the correct relation is________.



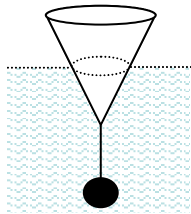
一根輕細綫，上端固定在天花板上，下端連接一小球，小球在水平面上做勻速圓周運動。設偏斜角可取值α_1和α_2，與之對應的小球圓周運動的周期爲 T_1和 T_2。若$\alpha < \alpha_1 < \alpha_2$，其中$\alpha$是小球做單擺簡諧振動的擺角，$T$是簡諧振動的周期，則正確的關係式是________ 。

(a) $T < T_1 < T_2$ (b) $T = T_1 = T_2$ (c) $T > T_1 > T_2$ (d) $T_1 < T < T_2$
(e) $T_1 > T > T_2$

MC11

A cone of height H with a mass attached is floating upside down in water, as shown. The water reaches $H/2$ when in equilibrium. Ignore friction. Find the vibration frequency after the cone is slightly pushed downwards.

一圓錐體高度爲 H，頂端系一重物，倒浮在水裏。平衡時水浸到 $H/2$ 處。不計摩擦，求將錐體輕輕往下一按後的振動頻率。



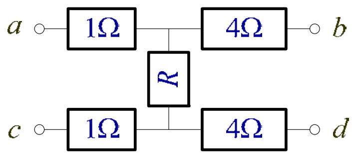
(a) $\frac{1}{2\pi}\sqrt{\frac{6g}{H}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{g}{H}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{3g}{H}}$

(d) $\frac{1}{2\pi}\sqrt{\frac{8g}{H}}$ (e) $\frac{1}{2\pi}\sqrt{\frac{2g}{H}}$

MC12

In the circuit, when the voltage between a and c is 20 V, the voltage between b and d is 10 V. When the voltage between b and d is 20 V, what is the voltage between a and c?

在如圖所示的電路中，當 a、c 兩端加上 20V 的電壓時，測得 b、d 兩端的電壓爲 10V；若改在 b、d 兩端加上 20V 的電壓，測得 a、c 兩端的電壓爲多少?

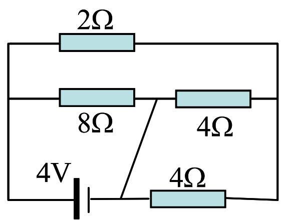


(a) 4V (b) 5V (c) 8V (d) 10V (e) 20V

MC13

In the circuit, the electric current through the battery is _____ Ampere(s).

在電路中，流經電池的電流爲_____ 安培。



(a) 1 (b) 2 (c) 2.5 (d) 4 (e) 1.5

MC14

Object-A is dropped from a height h. At the same instant object-B is thrown vertically upward from the ground. Right before they collide in mid-air, the speed of A is twice the speed of B. Determine the height where the collision occurs.

物體-A 從高度爲 h 的高處釋放，物體-B 同時從地上垂直往上拋。兩物體在空中相撞前瞬間，物體-A 的速率是物體-B 速率的二倍。求它們相撞的高度。

(a) $\frac{2h}{3}$ (b) $\frac{h}{\sqrt{3}}$ (c) $\frac{3h}{4}$ (d) $\frac{h}{2}$ (e) $\frac{h}{4}$

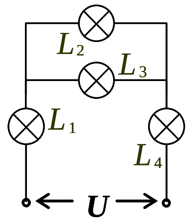
MC15

An object of mass m is placed on a horizontal floor. The static friction coefficient between the object and the floor is $\mu = 1$. Find the minimum force that can move the object.

質量爲 m 的物體置于水平地面上，物體和地面間的靜摩擦係數$\mu = 1$，求能使該物體移動的最小外力。

(a) $\frac{mg}{2}$ (b) $\frac{mg}{\sqrt{2}}$ (c) mg (d) $\sqrt{2}\,mg$ (e) $2mg$

MC16



In the circuit L_1, L_2, L_3, and L_4 are four light bulbs. L_1 and L_2 are labeled "220V, 25W". L_3 and L_4 are labeled "220V, 60W". When voltage U is applied, the sequence of the brightness of the four bulbs is, from the brightest to the dimmest, ________.

四盞電燈接入如圖所示的電路中，其中 L_1 和 L_2 標稱爲"220V, 25W", L_3 和 L_4 的標稱爲"220V, 60W"。加上電壓 U 後時電燈由亮至暗的排列次序是________。

(a) 1,2,3,4 (b) 2,3,4,1 (c) 4,2,1,3 (d) 3,4,1,2
(e) 1,4,3,2

MC17

A charged object is launched inside a time varying electric field. Its motion is recorded by a video camera on a video tape. When it is at a certain moment *A*, its position vector ***r***, velocity ***v***, and acceleration ***a*** are measured. A student watches the video at a later time but mistakenly plays the tape in the reverse direction. What are the position, velocity, and acceleration of the object at moment *A* observed by the student?

一帶電粒子在一隨時間變化的電場中的運動被錄影機記錄在錄影帶上。在某一時刻 A 它的位置是 ***r***、速度是 ***v***、加速度是 ***a***。一學生後來在播放錄影帶時把播放方向弄反了。問到了 A 時刻他看到粒子的位置、速度、加速度是什麽？

(a) ***r***, ***v***, ***a*** (b) ***r***, -***v***, -***a*** (c) ***r***, -***v***, ***a*** (d) ***r***, ***v***, -***a*** (e) -***r***, -***v***, -***a***

MC18

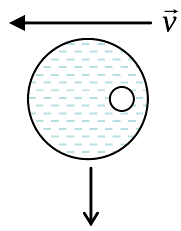
An object of mass m is attached to a spring. The restoring force of the spring is $F = -\lambda x^3$, where x is the displacement. The oscillation period now depends on the oscillation amplitude. Suppose the object is initially at rest. If the initial displacement is D then its period is τ. If the initial displacement is $2D$, find the period. (Hint: use dimension analysis.)

一質量爲 m 的物體系在一彈簧上，彈簧的回復力與位移 x 的三次方成正比，即 $F = -\lambda x^3$ ，因此物體的振動頻率與它的振幅有關。若物體都由靜止釋放，振幅爲 D 時的振動周期爲 τ，問振幅爲 $2D$ 時的振動周期爲多少?（提示：利用量綱分析。）

(a) 8τ (b) 2τ (c) τ (d) $\tau/2$ (e) $\tau/8$

MC19

A small air bubble is inside a drop of water residing in a space station on an orbit around Earth. The direction to Earth is downwards and the space station is moving to the left relative to Earth, as shown. The air bubble will __________ relative to the water drop.



Earth 地球

(a) move to the left (b) move to the right (c) move up
(d) move down (e) not move

在一個繞地球運轉的空間站裏有一相對于空間站静止的水珠，水珠裏有一小氣泡。空間站相對地球往左運動，空間站下方是地球。則氣泡相對與水珠的運動是__________。

(a) 向左 (b) 向右 (c) 向上 (d) 向下 (e) 不動

MC20

A satellite of mass *m* is at a distance *a* from a star of mass *M*. The speed of the satellite is *u*. Suppose the law of universal gravity is $F=-G\dfrac{Mm}{r^{2.1}}$ instead of $F=-G\dfrac{Mm}{r^{2}}$, find the speed of the satellite when it is at a distance *b* from the star.

一質量爲 m 速率爲 u 的衛星位于離一質量爲 M 的星球距離爲 a 的位置。若萬有引力爲 $F=-G\dfrac{Mm}{r^{2.1}}$ 而不是 $F=-G\dfrac{Mm}{r^{2}}$，求衛星離星球距離爲 b 時的速率。

(a) $\sqrt{u^2+2GM\left(\dfrac{1}{b^{1.1}}-\dfrac{1}{a^{1.1}}\right)}$

(b) $\sqrt{u^2+GM\left(\dfrac{1}{a^{1.1}}-\dfrac{1}{b^{1.1}}\right)}$

(c) $\sqrt{u^2+\dfrac{2}{1.1}GM\left(\dfrac{1}{b^{1.1}}-\dfrac{1}{a^{1.1}}\right)}$

(d) $\sqrt{u^2+\dfrac{2}{2.1}GM\left(\dfrac{1}{b^{1.1}}-\dfrac{1}{a^{1.1}}\right)}$

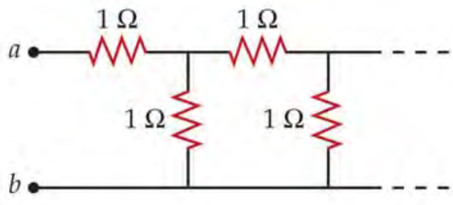
(e) $\sqrt{u^2+\dfrac{2}{1.1}GM\left(\dfrac{1}{b}-\dfrac{1}{a}\right)}$

《End of MC's 選擇題完》

Open Problems 開放題
Total 5 problems 共 5 題

The Open Problem(s) with the '*' sign may require information on page-3.
帶 * 的開放題可能需要用到第三頁上的資料。

Q1 (8 points)



Consider the figure of an infinite ladder of resistors as shown in the figure. Calculate the equivalent resistance between point *a* and point *b*.

題 1（8 分）

右圖爲一無限長電阻綫路。求 *a*、*b* 點之間的等效電阻。

Q2 (12 points)

A firework exploded into *N* fragments and converted chemical energy *E* into the kinetic energies of the fragments. The masses of the fragments are $\{m_i\}$, $i = 1, \dots N$. The spinning motions of the fragments are ignored.

(a) Consider the 2nd to the Nth fragments as a whole system, find the velocity of the center-of-mass of the system $\vec{v}_c$ in terms of the fragment masses and the velocity of the 1st fragment $\vec{v}_1$. (4 points)

(b) Let the velocities of the 2nd to the Nth fragments relative to the center-of-mass of the system be $\{\vec{v}_i\}$, $i = 2, \dots N$. Find their total kinetic energy in terms of their masses, $\vec{v}_c$, and $\{\vec{v}_i\}$, $i = 2, \dots N$. (4 points)

(c) Find the maximum kinetic energy the 1st fragment can have in terms of *E* and the masses of the fragments. (4 points)

題 2（12 分）

一烟花爆炸後形成 *N* 塊碎片，并將化學能 *E* 轉化成碎片的動能。碎片的質量爲$\{m_i\}$, $i = 1, \dots N$，它們的轉動可以忽略。

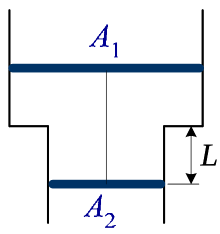
(a) 將第二到第 N 塊碎片看做是一個系統，用第一塊碎片的速度 $\vec{v}_1$ 和各碎片的質量來表達該系統的質心的速度 $\vec{v}_c$。 (4 分)

(b) 若第二到第 N 塊碎片相對于系統質心的速度爲 $\{\vec{v}_i\}$, $i = 2, \dots N$，求它們的總動能，并以 $\vec{v}_c$, $\{\vec{v}_i\}$, $i = 2, \dots N$，和碎片的質量來表達。 (4 分)

(c) 求第一塊碎片可得的最大動能，并以 *E* 和碎片的質量來表達。 (4 分)

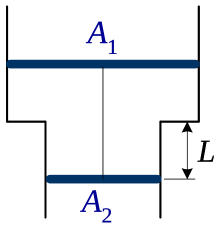
Q3 (15 points)

Two vertical pistons of total mass *M* and different diameters are connected by a rigid light rod of length *H*. Trapped in between is *N* mol of ideal gas. The area of the upper piston is A_1 and that of the lower piston is A_2. The air pressure outside is p_0. The wall of the cylinder is smooth. The gas constant is *R*. (a) Find the distance between the bottom of the large cylinder and the lower piston *L* when the system is in equilibrium. (b) Find the vibration frequency of the pistons near the equilibrium position.



(Hint: for $x \ll 1$, $(1+x)^n \approx 1+nx$, where *n* can be a fraction number or an integer.)

題 3（**15 分**）



直立的氣缸由半徑不同的兩個薄圓筒連接而成，由剛性的長爲 H 的輕杆連接的兩個活塞把 N mol 的理想氣體封閉在氣缸內。兩個活塞的總質量爲 M，上活塞截面積爲 A_1、下活塞爲 A_2，氣缸外部大氣的壓強爲 p_0。活塞與氣缸壁的接觸是光滑的，普適氣體恆量爲 R。(a)求平衡時下活塞到大圓筒底部的距離 L。(b)求在平衡點附近的振蕩頻率。

(提示: 當 $x << 1$, $(1+x)^n \approx 1+nx$, 其中 n 可以是分數或整數。)

Q4* (15 points)

(a) Use suitable parameters on Page-3, calculate the mass of Earth. (3 points)

(b) Geostationary satellites seem stationary in the sky all the time. One such satellite is right above Singapore on the Equator. Find the orbit radius of the satellite. (5 points)

(c) Hong Kong is 22° N in latitude. To maintain a geostationary satellite of mass m right above Hong Kong, the orbit of the satellite is no longer around the Earth center and a constant force f from the small rocket engines on the satellite must be provided. Find the height of the satellite when the force f is minimum. (For A, B, $x > 0$, the function $y = \dfrac{\sqrt{1+A\left(1-Bx^3\right)^2}}{x^2}$ is minimum when $x = \left(\dfrac{\sqrt{1+8\left(1+1/A\right)}-1}{2B}\right)^{1/3}$) (7 points)

.

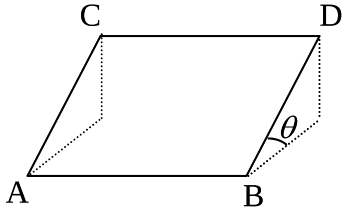
題 4*（**15 分**）

(a) 利用第三頁上的參數，求地球的質量。 (3 分)

(b) 同步衛星看上去就像固定在天上一樣。有一顆同步衛星正好在位于赤道的新加坡上空。求該衛星的軌道半徑。 (5 分)

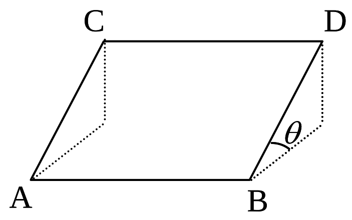
(c) 香港位于北緯 22°。要在香港上空維持一顆質量爲 m 的同步衛星，則該衛星的軌道不再以地球中心爲圓心，并要由衛星上的小火箭提供一作用力 f。求使 f 爲最小值時衛星的高度。 (若 A, B, $x > 0$, 函數 $y = \dfrac{\sqrt{1+A\left(1-Bx^3\right)^2}}{x^2}$ 在 $x = \left(\dfrac{\sqrt{1+8\left(1+1/A\right)}-1}{2B}\right)^{1/3}$ 時爲最小值。) (7 分)

Q5 (10 points)



According to quantum mechanics, a particle of momentum p can be regarded as a plane matter wave with wavelength $\lambda = h/p$, where h is the Planck Constant. As shown in the figure, ABDC is a flat square of side length L at an inclined angle θ to the horizontal plane. A neutron beam of initial kinetic energy E_0 is divided into two beams at point-A. One beam moves along the path ACD and the other beam along the path ABD. When the two beams meet at point-D they interfere. The mass of a neutron is m. How many times can one get maximum neutron number readings at point-D when θ changes from 0° to 90°?

題 5（**10** 分）



根據量子力學，一動量爲 p 的粒子可當作是一平面物質波，其波長爲 $\lambda = h / p$ ，h 爲普朗克常數。如圖所示，ABCD 爲一邊長爲 L 的平面正方形，與水平面的夾角爲 θ。一束初始動能爲 E_0 的中子在 A 點被分成兩束，一束沿路徑 ACD 傳播，一束沿路徑 ABD 傳播。到達 D 點的兩束波相互干涉。一個中子的質量爲 m。把 θ 從 0° 轉到 90° 在 D 點可測到幾次中子數的最大值？

《END 完》

HKPhO 2009 Solutions

MC1 (d) $70\vec{x}_0 + 10\vec{y}_0$ MC2 (b) 1.2 MC3 (b) 9.5

MC4 (e) 1.0×10^{-2} mm MC5 (a) 4 times MC6 (c) $\frac{mgL\sin\theta}{4}$

MC7 (a) $\left(\frac{\sqrt{2}+\sqrt{10}}{4}\right)mgl$ MC8 (c) 40 mC MC9 (b) inward, $\frac{Mg}{Qv\cos\theta}$

MC10 (c) $T > T_1 > T_2$ MC11 (a) $\frac{1}{2\pi}\sqrt{\frac{6g}{H}}$ MC12 (a) 4V

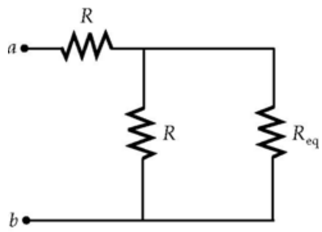
MC13 (e) 1.5 MC14 (a) $\frac{2h}{3}$ MC15 (b) $\frac{mg}{\sqrt{2}}$ MC16 (e) 1,4,3,2

MC17 (c) ***r***, -***v***, ***a*** MC18 (d) $\tau/2$ MC19 (e) not move

MC20 (c) $\sqrt{u^2 + \frac{2}{1.1}GM\left(\frac{1}{b^{1.1}} - \frac{1}{a^{1.1}}\right)}$

Q1

Let R be the resistance of each resistor in the ladder and let R_{eq} be the equivalent resistance of the infinite ladder. If the resistance is finite and non-zero, then adding one or more stages to the ladder will not change the resistance of the network. We can apply the rules for resistance combination to the diagram shown to the right to obtain a quadratic equation in R_{eq} that we can solve for the equivalent resistance between points a and b.



The equivalent resistance of the series combination of R and ($R \parallel R_{eq}$) is R_{eq}, so:

$$R_{eq} = R + R\|R_{eq} = R + \frac{RR_{eq}}{R + R_{eq}}$$

Simplify to obtain:

$$R_{eq}^2 - RR_{eq} - R^2 = 0$$

Solve for R_{eq} to obtain:

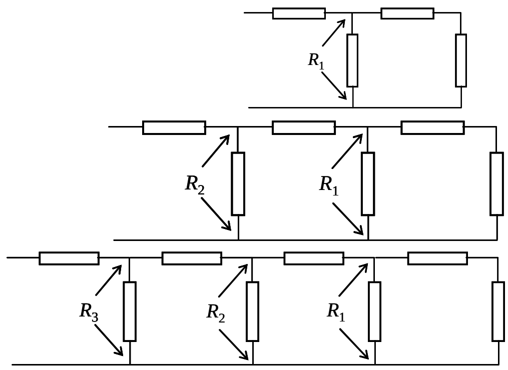
$$R_{eq} = \left(\frac{1+\sqrt{5}}{2}\right)R$$

For $R = 1\Omega$:

$$R_{eq} = \left(\frac{1+\sqrt{5}}{2}\right)(1\Omega) = \boxed{1.62\,\Omega}$$

<u>**Alternative**</u>:

Do it the brute force way.

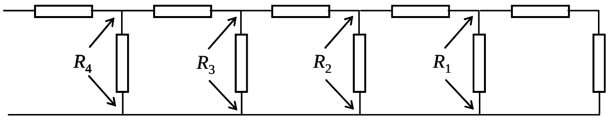


$$R_1 = \frac{1\times 2}{1+1} = 0.667\Omega.$$

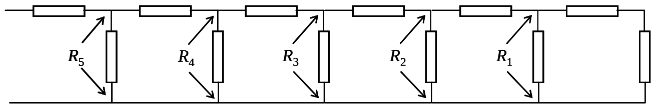
$$R_2 = \frac{1\times(1+R_1)}{1+(1+R_1)} = \frac{1\times(1+0.667)}{1+(1+0.667)} = 0.625\Omega$$

$$R_3 = \frac{1\times(1+R_2)}{1+(1+R_2)} = \frac{1\times(1+0.625)}{1+(1+0.625)} = 0.619\Omega$$

$$R_4 = \frac{1\times(1+R_3)}{1+(1+R_3)} = \frac{1\times(1+0.619)}{1+(1+0.619)} = 0.618\Omega$$



$$R_5 = \frac{1\times(1+R_4)}{1+(1+R_4)} = \frac{1\times(1+0.618)}{1+(1+0.618)} = 0.618\Omega$$



The total resistance is $R = (1 + 0.618) = 1.618\ \Omega$

Q2

(a) The total momentum of the system is $\sum_{i=2}^{N} m_i \vec{v}_c$. By momentum conservation, $\sum_{i=2}^{N} m_i \vec{v}_c + m_1\vec{v}_1 = 0$. So $\vec{v}_c = -\dfrac{m_1\vec{v}_1}{\sum_{i=2}^{N} m_i} = -\dfrac{m_1\vec{v}_1}{m_c}$, where $m_c \equiv \sum_{i=2}^{N} m_i$.

(b) The kinetic energy $KE = \frac{1}{2}\sum_{i=2}^{N} m_i(\vec{v}_c+\vec{v}_i)^2 = \frac{1}{2}\sum_{i=2}^{N} m_i v_c^2 + \frac{1}{2}\sum_{i=2}^{N} m_i v_i^2 + \sum_{i=2}^{N} m_i \vec{v}_c\cdot\vec{v}_i$. Note that $\sum_{i=2}^{N} m_i\vec{v}_i = 0$. So $\sum_{i=2}^{N} m_i\vec{v}_c\cdot\vec{v}_i = \vec{v}_c\cdot\left(\sum_{i=2}^{N} m_i\vec{v}_i\right) = 0$. So $KE = \frac{1}{2}m_c v_c^2 + \frac{1}{2}\sum_{i=2}^{N} m_i v_i^2$.

(c) Since v_c is dictated by momentum conservation, for minimum kinetic energy of the 2nd to the Nth fragments, the condition should be $v_i = 0$, $i = 2, 3, \ldots, N$. Then, using the result in (a) and energy conservation $E = \frac{1}{2}m_c v_c^2 + \frac{1}{2}m_1 v_1^2$, we get

$$Max.\ KE = \frac{1}{2}m_1 v_1^2 = \frac{m_c}{m_1+m_c}E\ .$$

Q3

(a) Force balance $Mg + p_0\left(A_1 - A_2\right) = p\left(A_1 - A_2\right)$, so $p = p_0 + \dfrac{Mg}{A_1 - A_2}$

Ideal gas law: $p\left(LA_2 + \left(H-L\right)A_1\right) = NRT$

Combine the two equations, we get

$$\frac{NRT}{LA_2 + \left(H-L\right)A_1} = p_0 + \frac{Mg}{A_1 - A_2}$$

$$\frac{LA_2 + \left(H-L\right)A_1}{NRT} = \frac{A_1 - A_2}{p_0\left(A_1 - A_2\right) + Mg}$$

$$L\left(A_1 - A_2\right) = HA_1 - \frac{A_1 - A_2}{p_0\left(A_1 - A_2\right) + Mg}NRT$$

$$L = \frac{HA_1}{A_1 - A_2} - \frac{NRT}{p_0\left(A_1 - A_2\right) + Mg}\ .$$

(b) Take downward as positive in displacement, we have

$$F = Mg + p_0\left(A_1 - A_2\right) - p\left(A_1 - A_2\right)$$

And $p\left(\left(L+x\right)A_2 + \left(H-L-x\right)A_1\right) = NRT$.

So $F = Mg + p_0\left(A_1 - A_2\right) - \dfrac{NRT\left(A_1 - A_2\right)}{\left(L+x\right)A_2 + \left(H-L-x\right)A_1}$

Tidy up the equation,

$$F = Mg + p_0\left(A_1 - A_2\right) - \frac{NRT\left(A_1 - A_2\right)}{LA_2 + \left(H-L\right)A_1 - x\left(A_1 - A_2\right)}$$

$$F = Mg + p_0\left(A_1 - A_2\right) - \frac{NRT\left(A_1 - A_2\right)}{LA_2 + \left(H-L\right)A_1}\frac{1}{1 - \dfrac{A_1 - A_2}{LA_2 + \left(H-L\right)A_1}x}$$

$$F = Mg + p_0\left(A_1 - A_2\right) - \frac{NRT\left(A_1 - A_2\right)}{LA_2 + \left(H-L\right)A_1}\left(1 + \frac{A_1 - A_2}{LA_2 + \left(H-L\right)A_1}x\right)$$

Using the answer in (a), we get

$$F = -\frac{NRT\left(A_1 - A_2\right)}{LA_2 + \left(H-L\right)A_1}\frac{A_1 - A_2}{LA_2 + \left(H-L\right)A_1}x = -NRT\left(\frac{A_1 - A_2}{LA_2 + \left(H-L\right)A_1}\right)^2 x = -kx$$

$$k = NRT\left(\frac{A_1 - A_2}{LA_2 + \left(H-L\right)A_1}\right)^2, \quad \omega = \sqrt{\frac{NRT}{M}}\frac{A_1 - A_2}{LA_2 + \left(H-L\right)A_1} = \frac{p_0\left(A_1 - A_2\right) + Mg}{\sqrt{MNRT}}$$

Q4

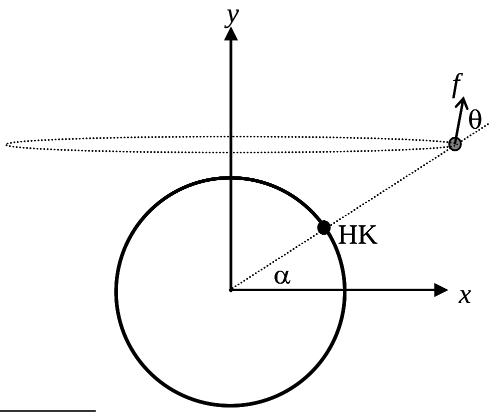
(a) $g = G\dfrac{M}{R^2}$, so $M = gR^2 / G = 9.8\times(6378\times10^3)^2 / (6.67\times10^{-11}) = 5.98\times10^{24}$ kg

(b) $G\dfrac{M}{r^2} = \omega^2 r = \left(\dfrac{2\pi}{T}\right)^2 r$, so

$$r = \left(G\frac{MT^2}{4\pi^2}\right)^{1/3} = \left(\frac{gR^2T^2}{4\pi^2}\right)^{1/3} = \left(\frac{9.8\times(6378\times10^3)^2\times(24\times3600)^2}{39.48}\right)^{1/3} = 4.23\times10^7 \text{ m}$$

(c)

Let the latitude be α.



$$\begin{cases} \dfrac{GM}{r^2}\sin\alpha = f\sin\left(\theta+\alpha\right) \\ \omega^2 r\cos\alpha = \dfrac{GM}{r^2}\cos\alpha - f\cos\left(\theta+\alpha\right) \end{cases}$$

Eliminate f from the above equations:

$$\cot\left(\theta+\alpha\right) = \left(1 - \frac{\omega^2 r^3}{GM}\right)\cot\alpha .$$

From the first equation,

$$f = \frac{GM}{r^2}\sin\alpha\csc\left(\theta+\alpha\right) = \frac{GM}{r^2}\sin\alpha\sqrt{1 + \left(1 - \frac{\omega^2 r^3}{GM}\right)^2\cot^2\alpha}$$

Hence f is minimum when

$$r=\left(\frac{\sqrt{1+8\left(1+\tan^2\alpha\right)}-1}{2\omega^2/GM}\right)^{1/3}=\left(\frac{\sqrt{1+8\sec^2\alpha}-1}{2}\right)^{1/3}\left(\frac{GM}{\omega^2}\right)^{1/3}\approx 1.034\left(\frac{GM}{\omega^2}\right)^{1/3}.$$

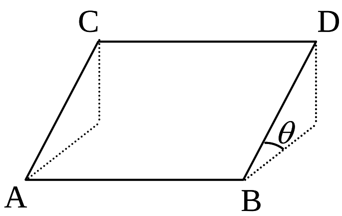
So the orbit radius of the satellite is about 3.4 % larger than a truly geostationary satellite above the Equator.

Q5

The phase difference for the matter waves to take the two paths is $\Delta\Phi=\Phi_{ABD}-\Phi_{ACD}$,

$\Phi_{ACD}=\Phi_{AC}+\Phi_{CD}$, and $\Phi_{ABD}=\Phi_{AB}+\Phi_{BD}$

Note that $\Phi_{AC}=\Phi_{BD}$,



The momentum on path AB is $p_{AB}=\sqrt{2mE_0}$, and the momentum on path CD is $p_{CD}=\sqrt{2m(E_0-mgL\sin\theta)}$.

So $\Delta\Phi=\Phi_{AB}-\Phi_{CD}=\frac{2\pi L}{h}\left(\sqrt{2mE_0}-\sqrt{2m(E_0-mgL\sin\theta}\right)$

Maximum reading of neutron number occurs when

$2n\pi=\frac{2\pi L}{h}\left(\sqrt{2m(E_0-mgL\sin\theta_n)}-\sqrt{2mE_0}\right)$, n = 0, 1, …

So the number of maximum reading is $N=\frac{L}{h}\left(\sqrt{2m(E_0-mgL)}-\sqrt{2mE_0}\right)$.

Hong Kong Physics Olympiad 2010
2010 年香港物理奧林匹克競賽

Written Examination
筆試

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
香港特區政府教育局
The Hong Kong Physical Society
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

May 23, 2010
2010 年 5 月 23 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for further announcement.

   選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID number in the field of "I.D. No., and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of "Course number & Section No."

   請依照選擇題答題紙的指示，用HB鉛筆在選擇題答題紙的I.D. No.欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格上填上你的英文姓名，最後於"Course & Section No. "欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.
   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用HB鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant ID number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.
   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.
   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitational constant, 6.67×10^{-11} (N m^2/kg^2)
e – charge of an electron, -1.6×10^{-19} (A s)
ε_0 – electrostatic constant, 8.85×10^{-12} (A s)/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance (= 1 Astronomical Unit (AU)) = 1.5×10^{11} m
Mass of Jupiter = 1.9×10^{27} kg
Mass of the sun = 1.99×10^{30} kg
Air Density = 1.2 kg/m^3
Water Density = 1.0×10^3 kg/m^3
Standard atmosphere pressure $p_0 = 1.0 \times 10^5$ N/m^2

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 萬有引力常數, 6.67×10^{-11} (N m^2/kg^2)
e – 電子電荷, -1.6×10^{-19} (A s)
ε_0 – 靜電常數, 8.85×10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 (= 1 天文單位) = 1.5×10^{11} m
木星質量 = 1.9×10^{27} kg
太陽質量 = 1.99×10^{30} kg
空氣密度 = 1.2 kg/m^3
水密度 = 1.0×10^3 kg/m^3
標準大氣壓 $p_0 = 1.0 \times 10^5$ N/m^2

The following conditions will be applied to all questions unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除非特別注明，否則下列條件將適用于本卷所有問題：

1) 所有物體都處于地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每道題 2 分，每道題選擇一個答案。）

The MC questions with the '*' sign may require information on page-3.

帶 * 的選擇題可能需要用到第三頁上的資料。

MC1*

An opened parachute of mass 1.0 kg is coming straight down from the sky. Attached to the parachute is the upper end of a light spring scale, while a block of mass 10 kg is attached to its lower end of the scale. The scale reading is 80 N. The air resistance at the moment is approximately ________.

一個張開的質量爲 1.0 kg 且豎直下降的降落傘下，掛有一個顯示讀數爲 80N 的無重彈簧秤，彈簧秤的低端懸挂著一個質量爲 10 kg 的重物。此時空氣阻力大約爲________。

(a) 55 N (b) 66 N (c) 77 N (d) 88 N (e) 99 N

MC2*

The wind speed is 100 Km/h. Estimate the force of the wind on an adult human being standing facing the wind. Choose the answer below that is closest to your result.

在風速爲 100 公里/小時的情况下，一個迎風正面站著的成年人所受的風力大約是多少？選擇以下最接近你的結果的答案。

(a) 0 N (b) 4630 N (c) 1.9 N (d) 39 N (e) 463 N

MC3*

The first extra-solar system discovered in 1995 consists of a solar type star and a planet with mass similar to Jupiter in the solar system. The period of the circular orbit of the planet is 4.2 days. Find the distance between the star and its planet **in terms of astronomical unit (A. U.).**

于 1995 年首次發現的太陽系以外的行星系統由一個類似太陽的恒星和一個質量與木星相若的行星組成。行星的圓形軌道的周期爲 4.2 天。求恒星-行星之間的距離（**以天文單位表達**）。

(a) 0.2 (b) 9.5 (c) 0.05 (d) 15 (e) 3.0

MC4*

Following MC3, find the orbital speed of the star around the center of mass of the star-planet system.

接上題，求該恒星繞恒星-行星系統的質心的運動速率。

(a) 2 m/s (b) 84 m/s (c) 62 m/s (d) 126 m/s (e) 312 m/s

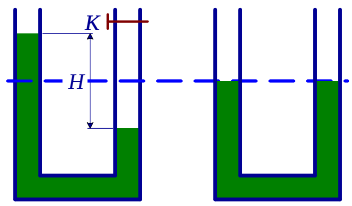
MC5*

A small wooden sphere of density 0.8 g/cm^3 is attached to the end of a string of length 4.0 m in water. The other end of the string is attached to the bottom, forming a reverse pendulum. Ignore water friction. Find the period of the pendulum.

密度爲 0.8 g/cm^{3}的小木球系于長度爲 4.0 m 的細繩的上端，細繩的下端系于水底，形成一個倒單擺。若不計阻力，求木球簡諧振動的周期。

(a) 4 s (b) 2 s (c) 5 s (d) 1 s (e) 8 s

MC6

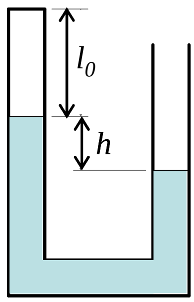


A U-tube of small and uniform cross section contains water of total length $4H$. The height difference between the water columns on the left and on the right is H when the valve K is closed. The valve is suddenly open, and water is flowing from left to right. Ignore friction. Find the speed of water when the heights of the left and the right water columns are the same.
橫截面均勻的 U 形管裝有總長爲 $4H$ 的水，開始時閥門 K 閉合，左右支管內水面高度差爲 H。管內部橫截面積很小，摩擦阻力忽略不計。試求：突然打開閥門 K 後左右水面高度相同時，水的流動速率。

(a) $\frac{1}{4}\sqrt{gH}$　　(b) $\sqrt{\frac{gH}{8}}$　　(c) $\frac{1}{2}\sqrt{gH}$　　(d) $\sqrt{\frac{gH}{2}}$　　(e) $\sqrt{gH}$

MC7

As shown, the left end of an upright U-shaped glass tube of uniform cross section is sealed and the right end is open. In the atmospheric pressure of $p_0 = 76$ cmHg and at $T_0 = 15^0$C, the length of the gas column sealed by the mercury is $l_0 = 8$ cm and the height difference between the left and the right mercury surfaces is $h = 4$ cm. When the temperature is raised to T_1, the length of the gas column becomes $l_1 = 9$ cm. Find T_1.



如圖所示，粗細均勻豎直放置的 U 形玻璃管左上端閉封，右上端開口，在標準大氣壓 $p_0 = 76$ cmHg 下，當溫度 $T_0 = 15^0$C 時，管內用水銀封閉著長 $l_0 = 8$ cm 的氣柱，這時兩管水銀面高度差 $h = 4$ cm。當溫度升高至 T_1 時被封閉的氣柱長爲 $l_1 = 9$ cm，則 T_1 =______ 。

(a) 40^0C　　(b) 60^0C　　(c) 70^0C　　(d) 50^0C　　(e) 80^0C

MC8

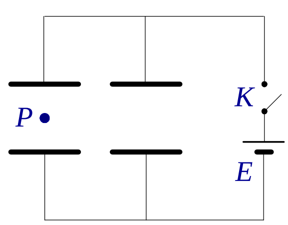
Following MC7, keep the temperature at T_1, fill in mercury in the amount of length x from the open end such that the gas column length returns to $l_0 = 8$ cm. Find x.
上題中若保持溫度 T_1 不變，再從開口端注入高度爲 x 的水銀使左上端氣柱長變回 $l_0 = 8$ cm，則 x =______ 。

(a) 11.25 cm　　(b) 13.25 cm　　(c) 14.35 cm　　(d) 8.25 cm　　(e) 7.25 cm

MC9

As shown, two parallel plate capacitors are placed horizontally and connected to a battery E. After charging the capacitors, the switch K is open. A charged particle is placed in the left capacitor and is at rest. If one reduces the distance between the plates of the right capacitor, the particle will then ________.

(a) move horizontally　　(b) move upwards　　(c) move downwards
(d) move in circle　　(e) remain at rest



兩個水平放置的平行板電容器連接如圖，電池 E 對它們充電，然後將電鍵 K 斷開。在左邊電容器的兩板間放一帶電微粒 P，它處於靜止平衡狀態。若將右邊電容器的兩極板間距離減小，微粒 P 將________ 。

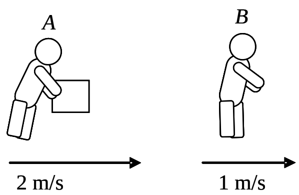
(a) 做水平直線運動　　(b) 做向上直綫運動
(c) 做向下直綫運動　　(d) 做圓周運動　　(e) 靜止不動

MC10

A 30 μF capacitor charged to 2.0 kV and a 60 μF capacitor charged to 1.0 kV are connected to each other, with the positive plate to the positive plate, and the negative plate to the negative plate. What is the final charge on the 30 μF capacitor?
一個 30 μF 的電容充電至 2.0 kV 的電壓，一個 60 μF 的電容充電至 1.0 kV 的電壓。現將兩電容的正極與正極相連，負極與負極相連，問在 30 μF 的電容上有多少電荷?

(a) 50 mC (b) 200 mC (c) 40 mC (d) 60 mC (e) 0 mC

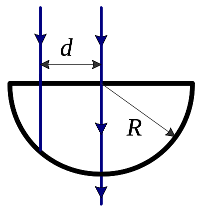
MC11



Two astronauts, *A* and *B*, both with mass of 60 kg, are moving along a straight line in the same direction in a “weightless” spaceship. Relative to the spaceship the speed of *A* is 2 m/s and that of *B* is 1 m/s. *A* is carrying a bag of mass 5 kg with him. To avoid collision with *B*, *A* throws the bag with a speed v relative to the spaceship towards *B* and *B* catches it. Find the minimum value of v.
兩個宇航員 A、B，每個的質量爲 60 kg ，在一‘無重’的飛船裹沿同一直綫向同一方向飄移。A 相對于飛船的速率爲 2 m/s，B 相對于飛船的速率爲 1 m/s。爲避免相撞，A 將質量爲 5 kg 的背包以相對于飛船 v 的速度扔向 B。B 接住背包。求 v 的最小値。

(a) 7.8 m/s (b) 26.0 m/s (c) 14.0 m/s (d) 9.2 m/s (e) 0

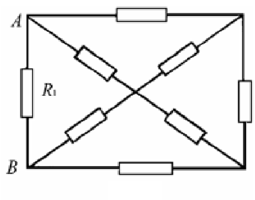
MC12



As shown, a narrow beam of light is incident onto a semi-circular glass cylinder of radius R. Light can exit the cylinder when the beam is at the center. When the beam is moved parallely to a distance d from the central line, no light can exit the cylinder from its lower surface. Find the refractive index of the glass.
如圖所示，一束細光線從中間射入半徑爲 R 的半圓柱形玻璃磚，可以觀察到有光綫從玻璃磚射出。現把入射光線平行地移動，當入射光距離中心綫爲 d 時，從玻璃磚下表面射出的光綫剛好消失。求玻璃的折射率。

(a) $\dfrac{R}{d}$ (b) $\dfrac{d}{R}$ (c) $\dfrac{R}{\sqrt{R^2-d^2}}$ (d) $\dfrac{\sqrt{R^2-d^2}}{R}$ (e) $\dfrac{R^2}{R^2-d^2}$

MC13



As shown, the circuit is made of 8 different resistors. It is found that when $R_1 = 4\ \Omega$, the resistance between A and B is 2 Ω. Now replace R_1 by a 6 Ω resistor, what is the resistance between A and B?
如圖所示電路由 8 個不同的電阻組成，已知 $R_1 = 4\ \Omega$ 時，測得 *A*、*B* 間的總電阻爲 2 Ω。現將 R_1 換成 6 Ω 的電阻，則 *A*、*B* 間的總電阻是多少Ω?

(a) 1 (b) 2 (c) 6 (d) 2.4 (e) 1.5

MC14*

To reach the fire at the height of 30 meters, what should be the initial speed of water jet on the ground?
爲將水射到 30 米高處的火場，在地面的水的初速度須爲多少?

(a) 30 m/s (b) 12 m/s (c) 300 m/s (d) 5 m/s (e) 24 m/s

MC15*

Following MC14, the area of the cross session of the water hose is 10 cm^2. What is the recoil force?
接上題，消防水槍口的截面積爲 10 cm^2，求水的反衝力。

(a) 588 N (b) 60 N (c) 705 N (d) 24 N (e) 5 N

MC16

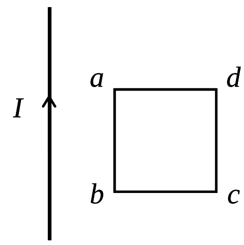
Three point charges, each with charge Q, are placed on the apexes of an equilateral triangle of length R. The potential energy of the system is then $\frac{K}{4\pi\varepsilon_0}\frac{Q^2}{R}$, where K is__________.

三個點電荷，每個帶電 Q,，分別放在一個邊長爲 R 的等邊三角形的三個頂點上。系統的勢能可表達爲 $\frac{K}{4\pi\varepsilon_0}\frac{Q^2}{R}$ 。求 K。

(a) 2 (b) 3 (c) 6 (d) 1 (e) 12

MC17

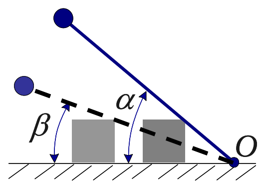
As shown, the electric current in a straight wire is increasing with time. The direction of the electric current in the square wire loop is _______.
豎直電綫裏的電流 I 隨時間增加。電綫邊上的四方綫圈裏的電流的方向是_______ 。



(a) *abcd* (b) *adcb* (c) *adbc* (d) *acbd* (e) *bcad*

MC18

A cubic block of mass m and side length b is placed on a smooth floor. A smooth and rigid rod of length L and with negligible mass is leaning against the block. A sphere of mass M is attached to the upper end of the rod. The lower end of the rod is fixed at point-O, but the rod can rotate freely around the point within the vertical plane. Initially the angle between the rod and the floor is α while the system is at rest. After releasing, find the speed of the block when the angle between the rod and the floor is β.



在光滑水平面上放置一個質量爲 m、邊長爲 b 的正方體滑塊。滑塊上擱有一長爲 L 的無重光滑直杆，杆的一端用光滑鉸鏈連結于地面上 O 點，使桿可繞 O 點在豎直平面內自由轉動；另一端鑲著一個質量爲 M 的金屬球。開始時杆和滑塊均靜止，杆和水平面夾角爲α。試求：放開杆後當杆轉動到杆和水平面夾角變爲 β 時滑塊的速度。

(a) $b\sqrt{\frac{MgL(\sin\alpha-\sin\beta)}{mb^2+ML^2\sin^4\beta}}$ (b) $b\sqrt{\frac{2MgL(\sin\alpha-\sin\beta)}{mb^2+ML^2\sin^4\beta}}$

(c) $b\sqrt{\frac{2MgL(\sin\alpha-\sin\beta)}{mb^2+ML^2\cos^4\beta}}$ (d) $b\sqrt{\frac{MgL(\sin\alpha-\sin\beta)}{mb^2+ML^2\cos^4\beta}}$ (e) 0

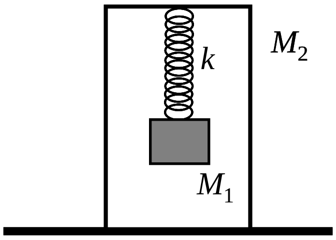
MC19

In a traffic accident, a taxi runs into a bus initially at rest. The two vehicles are locked together after impact and slide for a distance of 4 m. It is known that the frictional force between the locked vehicles and the ground is 25000 N. Find the speed of the taxi just before impact. Mass of the taxi = 1500 kg. Mass of the bus = 6000 kg.
在一起交通事故中，一輛質量爲 1500 kg 的的士撞上一輛停著的巴士。巴士的質量爲 6000 公斤。相撞後兩車貼在一起向前沖了 4 m。已知兩車與地面的磨擦力爲 25000N。求的士在撞車前的速度。

(a) 23.1 km/h (b) 46.2 km/h (c) 69.5 km/h (d) 93 km/h (e) 0

MC20

A block of mass M_1 is hanged by a light spring of force constant k to the top bar of a reverse U-frame of mass M_2 on the floor. The block is pooled down from its equilibrium position by a distance x and then released. Find the minimum value of x such that the reverse U-frame will leave the floor momentarily.



一質量爲 M_1 的物塊吊在一無重彈簧的下端。彈簧的彈力係數爲 k，上端系在質量爲 M_2 的倒 U-架的橫梁上。倒 U-架放在地面上。現將物塊從它的平衡位置向下拉一距離 x，然後釋放，求能使倒 U-架瞬時離開地面的 x 值。

(a) $x=(M_1+M_2)g/k$ (b) $x=(2M_1+M_2)g/k$

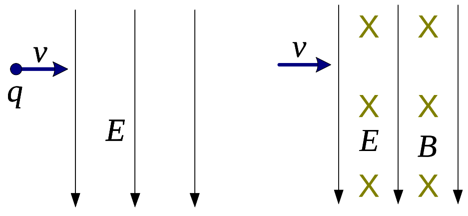
(c) $x=(M_1+2M_2)g/k$ (d) $x=M_1g/k$ (e) $x=M_2g/k$

《**End of MC's** 選擇題完》

Open Problems 開放題
Total 5 problems 共 5 題

The Open Problem(s) with the ‘*’ sign may require information on page-3.
帶 * 的開放題可能需要用到第三頁上的資料。

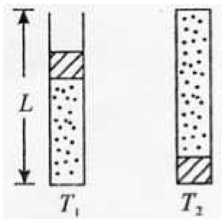
Q1 (5 points)
As shown in Fig. (a), a charged particle enters a region of uniform electric field E with initial horizontal velocity v. When it exits the region, its speed is v_a. In the second case, the same particle with the same initial velocity enters the region where a magnetic field B ($v < E/B$) is added perpendicularly to the original electric field E, as shown in Fig. (b). When it exits the region, its speed is v_b.Compare v_a with v_b, which one is larger? Give brief explanation in no more than 5 lines.



(a) (b)

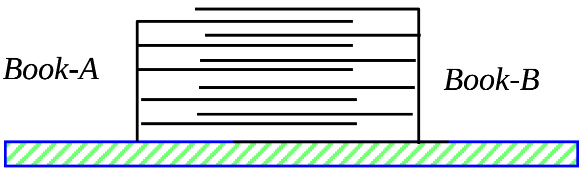
題 1（**5** 分）
如(*a*)圖所示，一個帶電粒子以水平速度 v 進入勻強電場區域。當粒子離開電場區時，粒子的速率爲 v_a。若加磁場 B 在同一區域，($v < E/B$)，電場和磁場正交重疊，如(*b*)圖所示，粒子仍以初速度 v 進入場區。當粒子離開場區時，粒子的速率爲 v_b。比較 v_a 和 v_b，哪個大？（需簡單給出理由。答案最好不要超過 5 行。）

Q2 (15 points)
Air is filled in a vertical cylinder of length L. The lower end of the cylinder is sealed and the upper end is open. The air pressure outside is p_0. A piston, which can move freely up and down the cylinder without gas leakage, is placed from the open end and sits down on the gas column. At equilibrium the length from the upper end of the cylinder to the outer surface of the piston is $L/4$. The temperature of the air inside is kept at T_1 in the process. Now, lower the temperature of the air inside to $T_2 = T_1/2$, and keep the temperature constant while turning the cylinder upside down. At equilibrium the outer surface of the piston is right at the open end edge of the cylinder. Find the thickness and the mass density of the piston.



題 2（**15** 分） 一個豎直放置的、長度爲 L 的圓筒下端封閉，上端與壓强爲 p_0 的大氣相通。初始時筒內氣體溫度爲 T_1。現將一個可沿筒壁自由滑動的、厚度爲 d 的活塞從上端放進圓筒，活塞下滑過程中氣體溫度保持不變且沒有氣體漏出，平衡後活塞外表面比圓筒上端低 $L/4$。將圓筒下部氣體溫度降至 $T_2 = T_1/2$，在保持溫度不變的條件下將筒倒置，平衡後活塞下端與圓筒下端剛好平齊。求活塞厚度 d 和活塞質量密度 ρ。

Q3 (15 points)
As shown in the figure, the pages of two identical books A and B are overlapping on one another. The mass of each book is 1000 g and the number of pages of each book is 200. The friction coefficient between the pages is $\mu = 0.3$. Book-A is fixed on the table. A horizontal force F is applied to book-B. Determine the minimum value of F to pull book-B out.

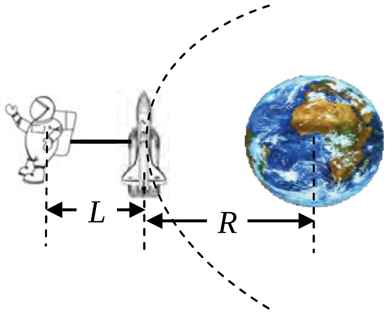
Book-A

Book-B

題 3（**15 分**）

如圖，兩本相同的書，每本 200 頁，重 1000 g，每頁互相交叉疊在一起，頁面間的摩擦係數爲 $\mu = 0.3$。書-A 固定在桌面上。求能將書-B 拉出的最小水平力 F。

Q4* (15 points)

As shown in the figure, an astronaut of total mass m = 110 kg was doing a spacewalk when his jetpack failed. His only connection to the spaceship was the communication cable wire of length L = 100 m. Assume the radius of the orbit R is the Earth's radius. Also assume that the astronaut and the spaceship underwent circular motion with the common angular velocity of $\boldsymbol{\omega}$ and remained on a straight line projecting from the Earth's center. The mass of the spaceship is much larger than that of the astronaut. Find the tention in the cable.



[Hint: For $R >> L$, $(R+L)^3 - R^3 \simeq 3R^2L$]

題 4*（**15 分**）

如圖，一質量爲 m = 110 kg 的宇航員在空間漫步時，推進系統突然失靈，只剩下一條長爲 L = 100 m 的通訊電纜與飛船連接。設飛船軌道半徑約等于地球的半徑，宇航員和飛船以同樣的角速度 $\boldsymbol{\omega}$ 作圓周運動，宇航員、飛船、地球中心始終保持在同一直綫上，飛船質量比宇航員大很多，求通訊電纜的張力。

[提示：當 $R >> L$ 時，$(R+L)^3 - R^3 \simeq 3R^2L$]

Q5 (10 points)

A photon of angular frequency ω carries energy $h\omega/2\pi$ and momentum $h\omega/2\pi c$, where h is the Planck Constant, and c is the speed of light in vacuum. When a photon meets an atom, it may be absorbed by the atom. The probability P for an atom to absorb a photon depends on the photon frequency ***in the rest frame*** of the atom ω_a, namely $P(\omega_a) = \dfrac{A}{(\omega_a - \omega_0)^2 + \gamma^2}$,

where A, γ, and ω_0 are constants. Shortly after absorbing the photon, the atom re-emits a photon of the same frequency in an arbitary direction. Within a very short time the atom absorbs many photons from a laser beam in one direction and re-emits photons in all directions. Therefore, on average over time, the atom experiences a force which can slow down its speed. This is the principle of cooling atoms with laser.

(a) An atom is moving in the positive x direction with speed v (<< c), and a photon with frequency ω is moving in the negative x direction. What is the frequency of the photon ω_a in the rest frame of the atom? (2 points)

(b) Suppose the atom is moving in the positive x direction with velocity v, and two identical laser beams shine along the positive x direction and the negative x direction, respectively. There are n photons from each beam colliding with the atom per unit time. Find the expression for the average force F acting on the atom. If the expression contains ω_a, you should replace it with the answer in (a). (6 points)

(c) For small v, we have $|\omega - \omega_a| << |\omega - \omega_0|$, the force in (b) can then be expressed as $F = -\beta v$. Find the expression for β. (2 points)

(Hint: For small $\delta\omega$, $\dfrac{A}{(\omega+\delta\omega)^2 + \gamma^2} \simeq \dfrac{A}{\omega^2+\gamma^2} - \dfrac{2\omega A \delta\omega}{\left(\omega^2+\gamma^2\right)^2}$.)

題 5 的中文版在下頁，敬請留意。

題 5（**10** 分）
一角頻率爲 ω 的光子具有能量 $h\omega/2\pi$ 、動量 $h\omega/2\pi c$ ，其中 h 爲普郎克常數， c 是真空中的光速。當光子遇上原子時，光子可能被原子吸收。光子被原子吸收的機率（或然率, Probability）與***在原子是靜止的參照系***裏的光子的角頻率 ω_a 有關，其表達式爲 $P(\omega_a)=\dfrac{A}{(\omega_a-\omega_0)^2+\gamma^2}$, 其中 A、 γ 、 ω_0 爲常數。原子在吸收了一個光子之後，很快就朝一任意方向發出一個頻率相同的光子。在一短時間內，該原子可以吸收很多從同一方向來的雷射光子，同時向各個方向不斷發射光子。因此，經時間平均，原子受到一個力，使它的運動減慢，這就是用雷射冷却原子的工作原理。

(a) 一原子以速度 v ($<< c$)沿正 x 方向運動，一頻率爲 ω 的光子沿反方向運動，求在原子是靜止的參照系裏光子的頻率 ω_a 。 (2 分)

(b) 設一原子仍以速度 v 沿正 x 方向運動，兩束相同的雷射分別沿正 x 方向和反 x 方向照射，每束雷射每單位時間有 n 個光子與原子相遇。求該原子受到的平均力 F。 若你的表達式裏含有 ω_a ，你必須用 (a) 的答案將它代替。(6 分)

(c) 若 v 很小，則 $|\omega-\omega_a| << |\omega-\omega_0|$ ，(b) 中的力可表達成 $F=-\beta v$ 。求 β。 (2 分)

(提示：當 $\delta\omega$ 很小時, $\dfrac{A}{(\omega+\delta\omega)^2+\gamma^2}\simeq\dfrac{A}{\omega^2+\gamma^2}-\dfrac{2\omega A\delta\omega}{\left(\omega^2+\gamma^2\right)^2}$ 。)

《**END** 完》

MCQ's
1(d), 2(e), 3(c), 4(d), 5(e), 6(b), 7(b), 8(a), 9(c), 10(c), 11(a), 12(a), 13(d), 14(e), 15(a), 16(b), 17(a), 18(b), 19(d), 20(a)

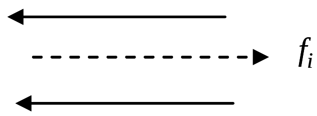
Open Q's

Q1 **(5 points)**
The work done by the electric field is proportional to the vertical displacement of the particle in the E-field. In case-B the magnetic field force has an upwards component so the vertical displacement of the particle is less than in case-A. Then, follow the energy conservation, we have $v_a > v_b$.

Q2. **(15 points)**
Solution:
$PV = NRT$, (2 points)
$NRT_1 = P_0 LS$ (2 points)
$(P_0 + d\rho)(\frac{3}{4}L - d)S = P_0 LS$ (3 points)
$(P_0 - d\rho)(L - d)S = P_0 LS / 2$ (3 points)
Solving the two equations, $\rho = 4\sqrt{3}P_0 / L$, and $d = \frac{1}{4}(2 - \sqrt{3})L$ (5 points)

Q3. **(15 points)**

Solution:
As shown in figure below, the ith page of book *B* subjects two frictional forces, $f_i^{'}$ (the page above ith page) and $f_i^{''}$ (the page below ith page)



The force required to pull out the ith page from book *A* is given by $f_i = f_i^{'} + f_i^{''}$ (3 points)
Since $f_i^{'} = 2(i-1)\mu mg$ and $f_i^{''} = (2i-1)\mu mg$, (3 points for each equation) thus we have
$f_i = (4i - 3)\mu mg$. .... (3 points)

From equation 1, when $i = 1, f_1 = 1\mu mg$; when $i = 200, f_{200} = 797\ \mu mg$

Summing up all the forces, $F = f_1 + f_2 + .. f_{200} = \frac{f_1 + f_{200}}{2} \times 200 = 79800\mu mg$
$\therefore F = 79800 \times 0.3 \times 0.005 \times 9.8 = 1173N$ (3 points)

Q4. **(15 points)**
Solution:
The spaceship moves under the influence of the Earth's gravity, given by
$F = G\frac{MM_E}{R_1^2}$ (2 points)

The net force acting on the spaceship is $G\frac{MM_E}{R_1^2} - T = MR_1\omega^2$ ....(1) (2 points)

where T is the tension of the communication cable. Similarly, for the astronaut,

$G\frac{mM_E}{R_2^2} + T = mR_2\omega^2$ ....(2) (2 points)

Equation ω from (1) and (2), we obtain

$\frac{1}{MR_1}(G\frac{MM_E}{R_1^2} - T) = \frac{1}{mR_2}(G\frac{mM_E}{R_2^2} + T)$ ...(3) (3 points)

From equation (3), we can easily find the tension T:

$T = G(\frac{mM}{MR_1 + mR_2})(\frac{R_2^3 - R_1^3}{R_1^2 R_2^2})M_E$ (2 points)

Using $R_1 \approx R_2 \approx R$; $R_2^3 - R_1^3 = (R_2 - R_1)(R_2^2 + R_1R_2 + R_1^2) \approx 3R^2L$

We can now rewrite T in the following form:

$$T = 3\left(\frac{mM}{m+M}\right)\cdot L \cdot \frac{GM_E}{R^3} = 3\left(\frac{L}{R}\right)\left(\frac{mM}{m+M}\right)g \ ; \text{ where } g = \frac{GM_E}{R^2}$$ (2 points)

Since $M >> m$, we can write an even simpler formula as an estimate:

$$T = 3\frac{mLg}{R} = 3\frac{(110)(9.8)(100)}{6400\times10^3} \approx 0.05N$$ (2 points)

5. **(10 points)**

Solution:

a) $\omega_\alpha = \frac{\sqrt{1-v^2/c^2}}{1+\frac{v\cos\theta}{c}}\omega \simeq \left(1 - \frac{v\cos\theta}{c}\right)\omega$

For $\theta = 0$, $\omega_\alpha = (1 - v/c)\omega$,

For $\theta = \pi$, $\omega_\alpha = (1 + v/c)\omega$. (2 points)

b) $F = \Delta Np/t$, (2 points)

$\frac{\Delta N}{t} = nA\left(\frac{1}{(\omega_\alpha^{(1)} - \omega_0)^2 + \gamma^2} - \frac{1}{(\omega_\alpha^{(2)} - \omega_0)^2 + \gamma^2}\right)$,(2 points)

$F = nA\left(\frac{1}{(\omega_\alpha^{(1)} - \omega_0)^2 + \gamma^2} - \frac{1}{(\omega_\alpha^{(2)} - \omega_0)^2 + \gamma^2}\right)\frac{h\omega}{2\pi c}$.(2 points)

c) $F = nA\frac{h\omega}{\pi c^2}\frac{2\omega(\omega - \omega_0)}{\left(\gamma^2 + (\omega - \omega_0)^2\right)^2}v$, hence, $\beta = nA\frac{h\omega}{\pi c^2}\frac{2\omega(\omega - \omega_0)}{\left(\gamma^2 + (\omega - \omega_0)^2\right)^2}$.(2 points)

Hong Kong Physics Olympiad 2011
2011 香港物理奧林匹克

(Junior Level 初級組)

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
香港特區政府教育局
The Physical Society of Hong Kong
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

March 19, 2011
2011 年 3 月 19 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID Number in the field of "I. D. No.", and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of "Course & Section No."

   請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 "I. D. No." 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格填上你的英文姓名，最後於 "Course & Section No. " 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant I. D. Number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 m/s^2
G – gravitational constant, 6.67×10^{-11} N m^2/kg^2
e – charge of an electron, -1.6×10^{-19} C
ε_0 – electrostatic constant, 8.85×10^{-12} C/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Radius of Earth = 6378 km
Sun-Earth distance (= 1 Astronomical Unit (AU)) = 1.5×10^{11} m
Mass of Jupiter = 1.9×10^{27} kg
Mass of the sun = 1.99×10^{30} kg
Air Density = 1.2 kg/m^3
Water Density = 1.0×10^3 kg/m^3
Standard atmosphere pressure $p_0 = 1.013 \times 10^5$ N/m^2

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 m/s^2
G – 萬有引力常數, 6.67×10^{-11} N m^2/kg^2
e – 電子電荷, -1.6×10^{-19} C
ε_0 – 靜電常數, 8.85×10^{-12} C/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
地球半徑 = 6378 km
太陽-地球距離 (= 1 天文單位(AU)) = 1.5×10^{11} m
木星質量 = 1.9×10^{27} kg
太陽質量 = 1.99×10^{30} kg
空氣密度 = 1.2 kg/m^3
水密度 = 1.0×10^3 kg/m^3
標準大氣壓 $p_0 = 1.013 \times 10^5$ N/m^2

The following conditions will be applied to all questions unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除特別註明外，下列條件將適用于本卷所有問題：

1) 所有物體都處于地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

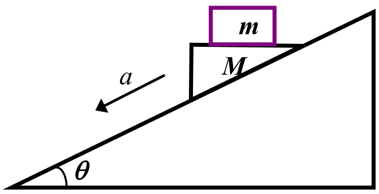
選擇題（每題 2 分，每題選擇一個答案。）

The MC questions with the „*" sign may require information on page-3.

帶 * 的選擇題可能需要用到第三頁上的資料。

MC1

As shown in the figure, a wedge of mass M is placed on a smooth inclined ramp that makes an angle θ to the horizontal. An object of mass m rests on top of the wedge. The system is sliding down the ramp at acceleration a. Determine the apparent weight of the object as it slides down. Note that there is friction between the object and the wedge so that the object remains relatively at rest on the wedge.



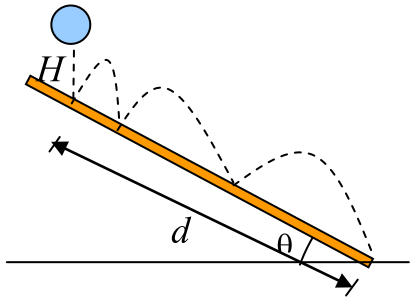
如圖所示，質量為 M 的楔子被置於平滑的斜坡上，斜坡相對於水平的傾角為θ。一件質量為 m 的物體置於楔子上。這系統以加速度 a 滑落斜坡。求該物體的表面重量。注意：物體和楔子之間的摩擦力令物體與楔子相對靜止。

(a) $mg\cos\theta$ (b) $mg\cos^2\theta$ (c) $mg\sin\theta\cos\theta$ (d) $mg\tan\theta$ (e) mg

MC2

A ball is released vertically from a height H above an inclined plane and makes several bounces. The angle of the inclined plane is θ. Assume the ball bounces elastically in each hit. Calculate the distance from the first hit to the fourth hit on the inclined plane.

一個球體從高於斜坡 H 的位置垂直掉落到斜坡上，並反彈數次。斜坡的傾角為θ。假設球體與斜坡的碰撞為彈性碰撞。求第一次碰撞到第四次碰撞的落點之間的距離。



(a) $3H\sin\theta$
(b) $24H\sin\theta$
(c) $30H\cos\theta$
(d) $36H\cos\theta$
(e) $48H\sin\theta$

MC3*

The first two extra-solar planets were discovered in 1992 to be revolving around a pulsar of 1.5 solar mass. The period of the circular orbit of one of the two planets is 98 days. Ignore the gravity interaction between the planets. Find the distance between the pulsar and the planet in terms of astronomical units (AU).

首兩個太陽系外行星於 1992 年被發現。其中一顆行星環繞 1.5 太陽質量的脈衝星作圓形軌跡運動，週期為 98 日。忽略行星之間的引力。求脈衝星與該行星的距離，以天文單位(AU)表達。

(a) 0.11 (b) 0.17 (c) 0.36 (d) 0.40 (e) 0.48

MC4

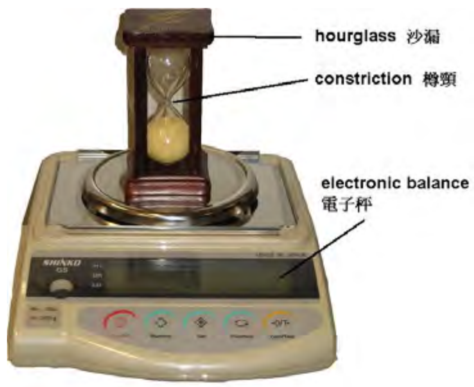
A ball hits a horizontal plane in a direction making an angle α with the horizontal, where $\sin\alpha = 3/5$. The coefficient of friction between the ball and the plane surface is 1/2. If 5/16 of the kinetic energy of the ball remains after the impact, then the ball bounces off the plane in a direction making at an angle θ with the horizontal. Find θ.

一球體與水平平面碰撞，方向與水平的夾角為α ， $\sin\alpha = 3/5$。球體與平面的摩擦系數為 1/2。如果碰撞過程後餘下的動能只有原來的 5/16，求球體反彈的方向與水平的夾角θ 。

(a) 23^{o} (b) 37^{o} (c) 53^{o} (d) 63^{o} (e) 83^{o}

MC5

As shown in the figure, an hourglass is put on an electronic balance. At time $t < 0$, a clot at the constriction prevents the sand from passing through. At time $t = 0$, the clot clears and the sand begins to drip down from the upper part to the lower part at a constant rate, until all sand is collected in the lower part after one hour. Assuming that the sand powder is extremely fine, how does the force acting on the electronic balance depend on the dripping process?

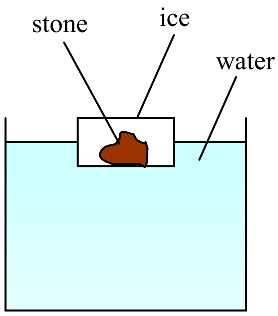


(a) The force increases to a higher value during the dripping process and then returns to the equilibrium value after the process.
(b) The force decreases to a lower value during the dripping process and then returns to the equilibrium value after the process.
(c) The force increases momentarily at the beginning and restores to the equilibrium value, and remains the same thereafter.
(d) The force increases momentarily at the beginning and restores to the equilibrium value, and decreases momentarily at the end of the dripping process, and remains at the equilibrium value thereafter.
(e) The force decreases momentarily at the beginning and restores to the equilibrium value, and increases momentarily at the end of the dripping process, and remains at the equilibrium value thereafter.

如圖所示，一沙漏放在電子秤上。在時間 $t < 0$ 時，沙漏的樽頸位置被堵塞，沙粉無從通過。在時間 $t = 0$ 時，堵塞被移走，沙粉開始已均勻速率從上方漏向下方，直至一小時後，所有沙粉都漏到下方。假設沙粉極度幼滑，問作用在電子秤上的作用力在漏沙過程中如何改變？
(a) 作用力在漏沙過程中增至一個高值，然後在過程後回復至平衡值。
(b) 作用力在漏沙過程中降至一個低值，然後在過程後回復至平衡值。
(c) 作用力在過程開始時瞬間增加，然後回復至平衡值，其後再沒有變動。
(d) 作用力在過程開始時瞬間增加，然後回復至平衡值，再在過程完結時瞬間降低，然後在過程後回復至平衡值。
(e) 作用力在過程開始時瞬間降低，然後回復至平衡值，再在過程完結時瞬間增加，然後在過程後回復至平衡值。

MC6

A piece of ice with an embedded stone floats on the surface of water in a glass. After the ice has melted, the stone sinks to the bottom of the glass. Compared with the initial water level, what is the change of the water level in the glass, first during the period the ice is melting, and second after the stone sinks to the bottom?



一片冰塊(ice)浮於杯中的水(water)面，冰塊內有一塊石子(stone)。當冰塊融解後，石子沉到杯底。與初始的水面高度比較，水面高度在冰塊融解時，和石子沉到杯底後有何改變？
(a) Remains the same then rises. 先不變，然後升高。
(b) Remains the same then falls. 先不變，然後降低。
(c) Remains the same all the way. 從始至終都不變。
(d) Rises then falls. 先上升，然後降低。
(e) Falls then rises. 先降低，然後上升。

MC7

A student observed the presence of strong winds generated by moving trains in the tunnels of subway. She suggested that wind turbines could be installed in the tunnels to generate electricity. Which of the following statement(s) is(are) correct?

I. The electricity generated by the turbines can be supplied to the air conditioners of the underground train stations, thereby saving energy.

II. The electricity generated by the turbines can be supplied to the electric motors of the underground trains, thereby saving energy.

III. The air resistance experienced by the trains will not be affected.

有學生觀察到地鐵隧道內，地鐵火車的運動產生很強的氣流。她提議在隧道內裝設渦輪機，利用氣流發電。下列哪些句子是正確的？

I. 渦輪機產生的電力可以供應給地鐵站的空調系統，以節省能源。

II. 渦輪機產生的電力可以供應給地鐵火車的電動機，以節省能源。

III. 地鐵火車遇到的空氣阻力不會受影響。

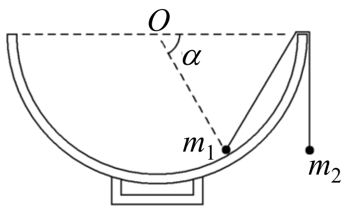
(a) All are not true 所有都不正確　　(b) Only I is true 只有 I 正確

(c) Only II is true 只有 II 正確　　(d) Only III is true 只有 III 正確

(e) All are true 所有都正確

MC8

As shown in the figure, a hemispherical bowl is placed horizontally on a table. Point-O is the center of the hemisphere. The edge and the surface of the bowl are smooth. A particle of mass m_1 is placed in a bowl and is tied to a string with negligible mass. The other end of the string is tied to another particle of mass m_2 hanging outside the bowl. When the system is in equilibrium, the line joining the particle m_1 and Point-O makes an angle $\alpha = 60^o$ with the horizontal. Find the ratio m_1/m_2.

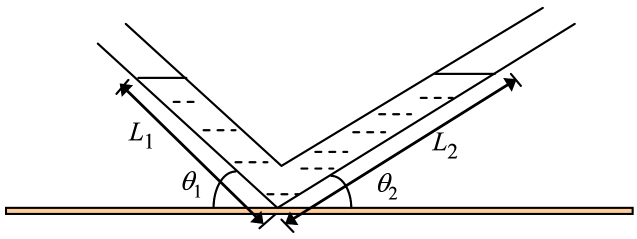


如圖所示，一個半球形的碗放在桌面上，碗口水平，O 點為其球心，碗的內表面及碗口是光滑的。一根細線跨在碗口上，線的兩端分別繫有質量為 m_1 和 m_2 的小球，當它們處於平衡狀態時，小球 m_1 與 O 點的連線與水平線的夾角為 $\alpha = 60^0$。求兩小球的質量比 m_1/m_2。

(a) 0.71　(b) 0.87　(c) 1.15　(d) 1.41　(e) 1.73

MC9

A non-viscous liquid of density ρ, as shown in figure, is filled in a „**V**‟ shape tube with A, L_1 and L_2 being the area of cross section and arm lengths respectively. If the liquid is slightly depressed in one of the arms, find the oscillation frequency of the liquid column.



如圖所示，„**V**‟形管內有非粘滯的液體，密度為 ρ。管的截面積和兩臂長度分別為 A, L_1 和 L_2。如令液體水平在其中一臂輕微降低，求液體柱產生振動的頻率。

(a) $\dfrac{1}{2\pi}\sqrt{\dfrac{g(\sin\theta_1+\sin\theta_2)}{L_1+L_2}}$　(b) $\dfrac{1}{2\pi}\sqrt{\dfrac{\rho g A(\sin\theta_1-\sin\theta_2)}{L_1+L_2}}$　(c) $\dfrac{1}{2\pi}\sqrt{\dfrac{L_1\sin\theta_1+L_2\sin\theta_2}{\rho g A}}$

(d) $\dfrac{1}{2\pi}\sqrt{(\dfrac{L_1\sin\theta_1}{L_2\sin\theta_2})\cdot\rho g A}$　(e) $\dfrac{1}{2\pi}\sqrt{\dfrac{g}{\dfrac{L_1}{\sin\theta_1}+\dfrac{L_2}{\sin\theta_2}}}$

MC10

The kinetic energy of a particle in a simple harmonic motion is $\frac{1}{2}av^2$, its potential energy is $\frac{1}{2}bx^2$, where x is the coordinate for the position of the particle and v is its speed. Find the frequency of the motion.

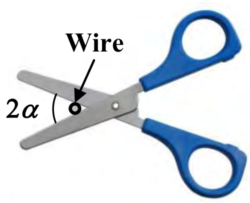
一質點在簡諧運動中的動能為$\frac{1}{2}av^2$，位能為$\frac{1}{2}bx^2$，其中 x 為質點的位置，v 為其速度。求運動的頻率。

(a) $\frac{1}{2\pi}\sqrt{\frac{a}{b}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{b}{a}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{a}{b}+\frac{b}{a}}$ (d) $\frac{1}{2\pi}\sqrt{ab}$ (e) $\frac{1}{2\pi}\sqrt{\frac{1}{ab}}$

MC11

Someone is using a scissors to cut a wire of circular cross section and negligible weight. The wire slides in the direction away from the hinge until the angle between the scissors blades becomes 2α . Find the coefficient of friction between the blades and the wire.

用一把剪刀，去剪一條圓截面的導綫。導綫先會向外滑動，直到剪刀之間的角度爲2α ，求剪刀和導綫之間的摩擦係數。

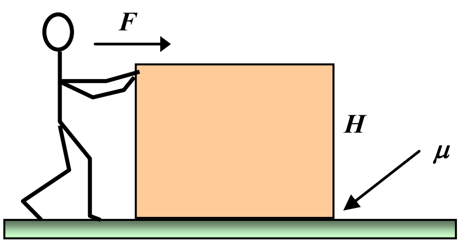


(a) $\sqrt{1-\tan\alpha}$ (b) $2\cos\alpha$ (c) $2\tan\alpha$ (d) $\tan\alpha$ (e) $\sqrt{2\cos^2\alpha-1}$

MC12

A person exerts a horizontal force F at the upper edge of a box to push the box of uniform mass density, length L, and height H across the floor. The friction coefficient between the box and the floor is μ. If $\mu > \mu_o$, the box will overturn before it slides. Determine the value of μ_o.

有人嘗試以作用力 F 沿水平方向作用於箱子的頂邊去推動箱子。箱子質量密度均勻，長度為 L，高度為 H。箱子與地面的摩擦系數為μ 。若$\mu > \mu_o$ ，箱子不會滑動，反而會翻倒。求μ_o的值。

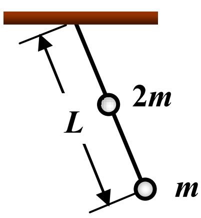


(a) $\frac{L}{2H}$ (b) $\frac{L}{H}$ (c) $\frac{H}{L\mu}$ (d) $\frac{\mu L}{H}$ (e) $\frac{2H}{\mu L}$

MC13

A compound pendulum is made of a light and rigid rod of length L with one end attached to a hinge on the ceiling. A small ball of mass m is attached to the other end of the rod, and another small ball of $2m$ is attached to the middle of the rod. Find the frequency of the simple harmonic oscillation of the pendulum.

有一複合擺錘，一端繫於天花板上，錘身長度為 L ，重量可忽略。錘另一端繫有質量為 m 的小球，錘身中央另繫有質量為 $2m$ 的小球。求擺錘簡諧振動的頻率。



(a) $\frac{1}{2\pi}\sqrt{\frac{g}{2L}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{4g}{3L}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{3g}{2L}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{9g}{4L}}$

(e) $\frac{1}{2\pi}\sqrt{\frac{9g}{2L}}$

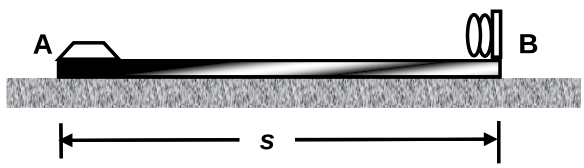
MC14

A small object is initially at the bottom of a plane inclined at an angle α with the horizontal. It is projected upward along the inclined plane with an initial velocity, and reaches the maximum height after time t_1. It then slides downward and returns to the initial position after time t_2. If the coefficient of sliding friction between the object and the surface is μ, find the ratio t_2/t_1.

一個傾角為 α 的斜面底部有一小物塊。現給物塊一個初速率，使它沿斜面向上滑動，經過時間 t_1 到達最高點，之後它又自動滑回到底部，所用時間為 t_2。若物體與斜面間的滑動摩擦係數為 μ，求比例 t_2/t_1。

(a) $\sqrt{\dfrac{\sin\alpha-\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}}$ (b) $\sqrt{\dfrac{\sin\alpha+\mu\cos\alpha}{\sin\alpha-\mu\cos\alpha}}$ (c) $\dfrac{\sin\alpha-\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}$ (d) $\dfrac{\sin\alpha+\mu\cos\alpha}{\sin\alpha-\mu\cos\alpha}$ (e) $\sqrt{\dfrac{\sin\alpha}{\mu\cos\alpha}}$

C15

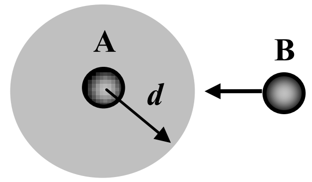


As shown in the figure, AB is a board of mass M = 4 kg and length s = 2 m, placed on a smooth horizontal surface. A bumper of negligible mass is fixed at end-B. A peg of mass m = 1 kg is placed at end-A. The coefficient of kinetic friction between the peg and the board is μ = 0.2. With both the board initially at rest, the peg is ejected with an initial velocity of v_0 = 10 m/s in contact with the board surface until it hits the bumper at end-B. After the collision, it just returns to end-A without falling off the board. Find the mechanical energy loss in the process.

如圖所示，AB 為位於光滑水平面上的長木板，質量為 M = 4 kg，長度為 s = 2 m。B 端有一固定擋板，A 端放有一小滑塊，其質量 m = 1 kg。小滑塊與木板間的動摩擦係數為 μ = 0.2。當木板處於靜止的初始狀態時，小滑塊以初速度 v_0 = 10 m/s 緊貼木板表面射出，直到和 B 端的擋板相撞。碰撞後，小滑塊恰好回到 A 端而不脫離木板。求此過程中損失的機械能。

(a) 20 J (b) 24 J (c) 28 J (d) 32 J (e) 40 J

MC16

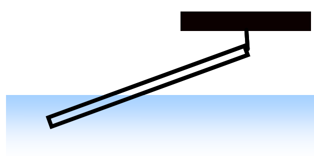


The elastic collision between two bodies, A and B, can be considered using the following model. A and B are free to move along a common line without friction. When their distance is greater than d = 1 m, the interacting force is zero; when their distance is less than d, a constant repulsive force F = 6 N is present. The mass of body A is m_A = 1 kg and it is initially at rest; the mass of body B is m_B = 3 kg and it is approaching body A head-on with a speed v_0 = 2 m/s. Find the minimum distance between A and B.

兩物體 A 和 B 的彈性碰撞過程可以如下的模型考慮。A 和 B 可沿同一直線自由運動，不用考慮摩擦力。當它們之間的距離大於 d = 1 m 時， 相互作用力為零; 當它們之間的距離小於 d 時， 存在大小恒為 F = 6N 的斥力。設 A 物體質量為 m_A = 1 kg， 開始時靜止在直線上某點; B 物體質量 m_B = 3 kg, 以速率 v_0 = 2 m/s 從遠處沿該直線向 A 運動。求 A、B 間的最小距離。

(a) 0.25 m (b) 0.50 m (c) 0.75 m (d) 1 m (e) 1.25 m

MC17



A uniform rod is hung at one end and is partially submerged in water. If the density of the rod is 5/9 that of water, find the fraction of the length of the rod above water.

一端懸掛的均勻竿，部分浸在水中。若竿密度是水密度的 5/9，求在平衡狀態時，竿在水面上部分的長度比例。

(a) 0.25 (b) 0.33 (c) 0.50 (d) 0.67 (e) 0.75

MC18

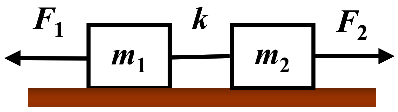
An observer stands next to the front end of the first carriage of a train. When the train starts to accelerate uniformly, it takes 5 seconds for the first carriage to pass the observer. Assuming that all carriages are of the same length, what is the time taken by the tenth carriage to pass the observer?

一位觀察者站在靜止的列車第一節車廂的前端。當列車以等加速度開動時，第一節車廂經過其旁需 5 s。假設所有車廂長度相同，求第十節車廂經過其旁的時間。

(a) 1.18 s (b) 1.07 s (c) 0.98 s (d) 0.91 s (e) 0.81 s

MC19

As shown in the figure, two blocks of masses m_1 and m_2 are connected by a light string, and are placed on a horizontal smooth surface. Forces of magnitude F_1 and F_2 act on them respectively, causing them to move linearly. The force constant of the light string is k, and $F_1 > F_2$. What is the extension x of the light string?

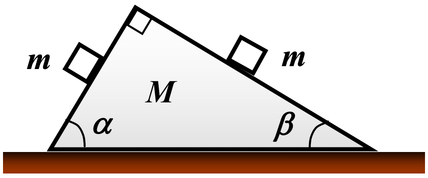


如圖所示，兩個用輕線相連的位於光滑水平面上的物塊，質量分別為 m_1 和 m_2，拉力 F_1 和 F_2 方向相反，令系統沿同一水平直線運動。輕線的剛度係數為 k，且 $F_1 > F_2$。求在兩個物塊運動過程中輕線伸長的長度 x。

(a) $\dfrac{F_1 m_1 + F_2 m_2}{k(m_1 + m_2)}$ (b) $\dfrac{F_1 m_2 + F_2 m_1}{k(m_1 + m_2)}$ (c) $\dfrac{|F_1 m_2 - F_2 m_1|}{k(m_1 + m_2)}$ (d) $\dfrac{|F_1 m_1 - F_2 m_2|}{k(m_1 + m_2)}$ (e) $\dfrac{F_1 m_1 + F_2 m_2}{k|m_1 - m_2|}$

MC20

As shown in the figure, a triangular wooden block of mass M is fixed on a horizontal table. Its top angle is 90°, and the base angles are α and β. Two small pieces of wood, each of mass m, are located on the inclined smooth surfaces. When the wood pieces slide down the inclined surfaces, what is the normal force acting on the table by the triangular block?



如圖所示，一質量為 M 的三角形木塊固定在水平桌面上，它的頂角爲 90°，兩底角為 α、β，兩個質量均為 m 的小木塊位于両側光滑的斜面上。當兩小木塊沿斜面下滑時，求三角形木塊對水平桌面的垂直作用力。

(a) Mg (b) $2mg$ (c) $Mg+mg$ (d) $Mg+2mg$ (e) $Mg+mg(\sin\alpha + \sin\beta)$

《END OF MC's 選擇題完》

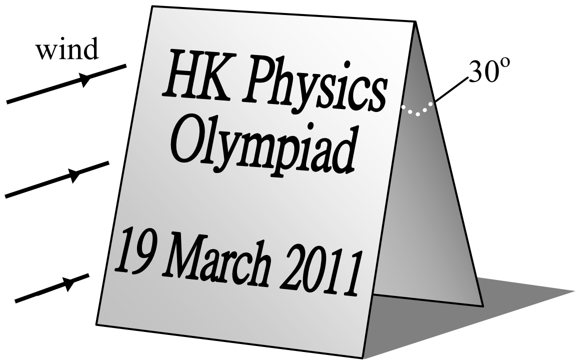
Open Problems 開放題

Total 5 problems 共 5 題

The Open Problem(s) with the „*" sign may require information on page-3.
帶 * 的開放題可能需要用到第三頁上的資料。

Q1 (10 points) **題 1 (10 分)**

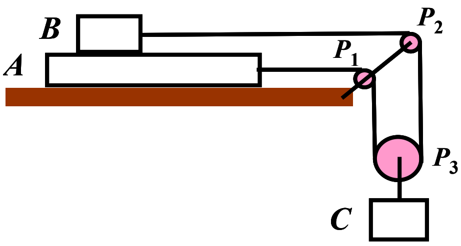
A "HK Physics Olympiad sandwich board" sign consists of two uniform pieces of metal后50.0 cm wide and 1.0 m high, each of mass M = 3.0 kg, joined at the top forming an angle $\theta = 30^o$. The sign looks like an inverse letter „V" when viewed sideways. Assume that the friction between the sign and the ground is sufficiently strong to prevent the sign from sliding on the ground, and that the board surface can stop any horizontal movement of air hitting it. The density of air is ρ = 1.2 kg/m^3. Find the minimum wind speed v_{min} above which the sign will turn over.



"香港物理奧林匹克"廣告牌由兩塊均勻金屬板組成，每塊金屬板的質量為 M = 3.0 kg，寬 50.0 cm，高 1.0 m，在頂端以$\theta = 30^o$的角度連接。 從側面看，廣告牌仿似倒置的„V"字。假設廣告牌與地面的摩擦足夠強，令廣告牌不致在地面滑行，而當空氣吹向它時，牌面可以終止空氣的任何水平運動。空氣的密度為ρ = 1.2 kg/m^3。計算可以把廣告牌吹翻的最低風速 v_{min}。

Q2 (10 points) **題 2 (10 分)**

Rectangular block A of mass 3m is placed on the rough surface of a table. Another block B of mass m with rough surfaces is placed on top of block A. A light inextensible string connects blocks A and B, and winds through the massless smooth pulleys P_1 and P_2 fixed at the edge of the table. A massless smooth pulley P_3 hangs from the string segment between pulleys P_1 and P_2, and block C of mass m hangs on pulley P_3. Let μ be the coefficient of friction between all contact surfaces.

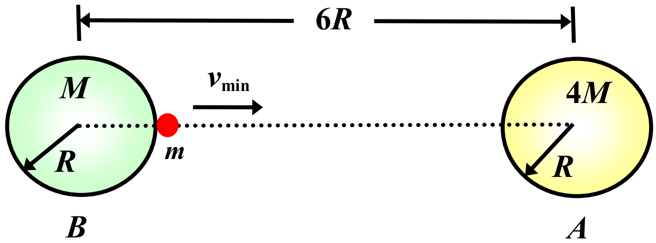


(a) Find the range of μ within which the system stays in equilibrium.
(b) Find the range of μ within which block C moves downwards, while block A remains stationary.

將一塊質量為 3m 的粗糙長方體物塊 A 置於粗糙水平桌上，並將一塊質量為 m 的粗糙物塊 B 置於 A 之上。一根不可伸長的輕繩連接 A 和 B。繩子繞過兩個固定於桌邊的無重光滑小滑輪 P_1 和 P_2，兩滑輪之間的繩子懸掛著一個光滑的滑輪 P_3。一質量為 m 的物塊 C 懸於 P_3之下。設μ為所有接觸面之間的摩擦係數。

(a) 若該系統保持平衡，求摩擦係數μ可取值的範圍。
(b) 若 A 不動而 C 向下運動，求摩擦係數μ可取值的範圍。

Q3 (15 points) **題3 (15分)**



As shown in the figure, spheres A and B have equal radii R and mass $4M$ and M respectively. They are separated center to center by a distance of $6R$. A projectile of mass m is ejected from the surface of sphere B in the direction towards the center of sphere A.

(a) Find the minimum speed v_{min} of the projectile so it can reach the surface of sphere A.

(b) If the projectile is ejected with the minimum speed v_{min}, calculate the speed of the projectile when it reaches the surface of sphere A.

(c) If the projectile can be launched from the surface of sphere B in any direction so as to escape to infinity, find the speed of the projectile when it leaves sphere B.

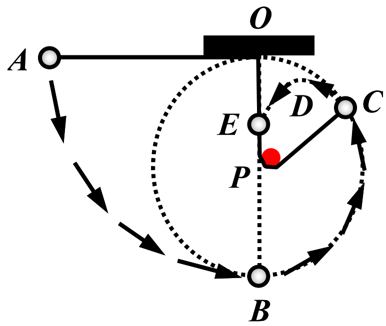
如圖所示，兩均勻球體 A 和 B 半徑均為 R，質量分別為 $4M$ 和 M，兩球中心距離為 $6R$。有拋射體質量為 m，在 B 球表面朝 A 球中心發射。

(a) 計算拋射體可以達到 A 球表面的最小速率 v_{min}。

(b) 若拋射體以最小速度 v_{min} 發射，計算拋射體到達 A 球表面時的速率。

(c) 若拋射體可以從 B 球朝任何方向發射以逃逸至無限遠，計算拋射體離開 B 球表面時的速率。

Q4 (15 points) **題 4 (15 分)**



As shown in the figure, a pendulum is made up of a bob A suspended from a fixed point O by a light inextensible string of length L. A nail P is located at a distance $L/2$ vertically below O. The pendulum is lifted with the string taut until line OA is horizontal and then released. When the pendulum swings to the vertical position at point B, only the portion below point P can swing further.

(a) When the interrupted pendulum swings further, the string becomes loose at point C. Find the angle between the line PC and the vertical direction.

(b) After the string becomes loose, the bob continues to move and reaches its maximum height at point D. Find the maximum height of the bob above point P.

(c) The bob then passes through point E which is right below O. Find the distance between E and O.

如圖所示的擺錘，擺線一端固定在點 O，另一端吊掛著擺球 A，擺線不能伸展，長度為 L，質量可忽略。在 O 點正下方距離 $L/2$ 處有一固定釘子 P。現將擺球拉到 OA 線成水平的位置，擺線保持拉直狀態，然後放手。當擺球運動到垂直位置 B 時，釘子擋住擺線，結果只有釘子以下的部分可以繼續運動。

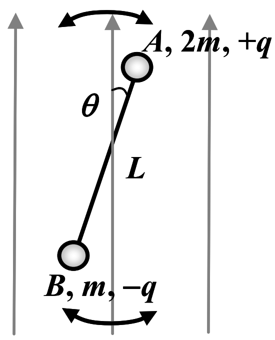
(a) 當受釘子阻擋的擺錘到達 C 點時，擺線開始鬆弛。求 PC 線與垂直方向的夾角。

(b) 擺線鬆弛後，擺球繼續運動，到達最高位置 D 點。求擺球相對於 P 點的最高高度。

(c) 擺球通過位於 O 點正下方的 E 點，求 O 點與 E 點之間的距離。

Q5 (10 points) **題 5 (10 分)**

As shown in the figure, a particle A of mass $2m$ and carrying charge q is connected by a light rigid rod of length L to another particle B of mass m and carrying charge $-q$. The system is placed in an electric field $\vec{E}$. The electric force on a charge q in an electric field $\vec{E}$ is $\vec{F} = q\vec{E}$. After the system settles into equilibrium, one particle is given a small push in the transverse direction so that the rod makes a small angle θ_0 with the electric field.



(a) Find the period of the angular oscillation.

(b) Find the maximum tension in the rod.

如圖所示，質點 A 的質量為 $2m$，帶電荷 q。另一質點 B 的質量為 m， 帶電荷 $-q$。兩質點由一輕而剛硬的棒連接，棒的長度為 L。現把系統放在電場 $\vec{E}$ 中，電場 $\vec{E}$ 作用於電荷 q 上的作用力為 $\vec{F} = q\vec{E}$。系統達到平衡後，把一質點向橫向輕輕推一下，使棒與電場形成夾角 θ_0。

(a) 求夾角的振動頻率。

(b) 求連接棒的最大張力。

《**END** 完》

Hong Kong Physics Olympiad 2011 (Junior Level)
2011 香港物理奧林匹克(初級組)
Suggested Solutions 答案及建議題解

Multiple Choice Questions

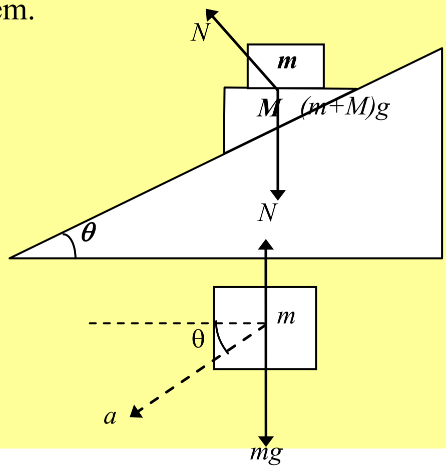
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	e	e	d	e	b	a	e	a	b	d	a	b	b	e	c	d	e	b	c

MC1

We may consider the wedge and the object as one system.
By using Newton's 2nd law of motion, we have

$(m + M)g\sin\theta = (m+M)a$

$\therefore\ a = g\sin\theta$ ....(1)



Consider the object:

$mg - N = ma\sin\theta$ .... (2)

sub (1) into (2), we have

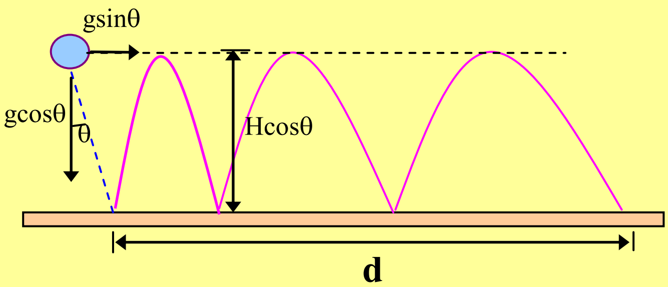
$mg - N = m(g\sin\theta)\sin\theta = mg\sin^2\theta$

$N = mg - mg\sin^2\theta = mg\cos^2\theta$

Answer: (b)

MC2

In the frame rotated by an angle θ, we can consider only the horizontal component of the gravitational force that accelerates the ball forward. In the vertical motion, the ball bounces elastically with an effective acceleration $g\cos\theta$. To find the distance between the first and the last bounce, say the fourth, we should also account for the velocity the ball acquires before the first hit:



Denote v_o as the horizontal component of the initial velocity of the ball before the first hit with respect to the inclined, t_{01} as the time between releasing the ball and the first hit and t_{14} as the time between the first to the fourth hit.

The horizontal component of the initial velocity of the ball before the first hit is given by $v_o = g\sin\theta \cdot t_{01}$ … (1)

The time t_{01}: $t_{01} = \sqrt{\dfrac{2H\cos\theta}{g\cos\theta}} = \sqrt{\dfrac{2H}{g}}$ …(2)

Since the ball makes 3 hits and each hit takes 2 times of t_{01}, therefore,

$$t_{14} = \left[2\sqrt{\frac{2H}{g}}\right]\cdot 3 = 6\sqrt{\frac{2H}{g}} \quad \ldots(3)$$

The total distance between the first and the last hit, d, is

$$d = v_0 t_{14} + \frac{(g\sin\theta)}{2}\cdot t_{14}^2 \quad \ldots.(4)$$

By substituting equation (1), (2) and (3) into (4), we have

$$\begin{aligned} d &= \left[g\sin\theta\cdot\sqrt{\frac{2H}{g}}\right]\left(6\sqrt{\frac{2H}{g}}\right) + \frac{(g\sin\theta)}{2}\cdot\left[6\sqrt{\frac{2H}{g}}\right]^2 \\ &= 12H\sin\theta + 36H\sin\theta \\ &= 48H\sin\theta \end{aligned}$$

Answer: (e)

<u>MC3*</u>

Using Newton's law of universal gravitation and Newton's second law for circular orbits, $M \propto R^3/T^2$. Comparing with Earth's orbit, where M = 1 solar mass, R = 1 AU and T = 1 year, $R = \sqrt[3]{1.5\left(\frac{98}{365}\right)^2} = 0.48$ AU . **Answer: (e)**

<u>MC4</u>

Let the x axis points in the direction along the plane, and the y axis is normal to the plane. Let m be the mass of the ball. Let u be the velocity of the ball immediately before it touches the surface. Then the impulse of the impact is given by

$$I_x = mv_x - mu\cos\alpha$$

$$I_y = mv_y + mu\sin\alpha$$

Due to the presence of friction, $I_x = -\mu I_y$

Hence $mv_x - mu\cos\alpha = -\mu(mv_y + mu\sin\alpha)$

$$v_x = u(\cos\alpha - \mu\sin\alpha) - \mu v_y \qquad (1)$$

Balance of energy:

$$\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 = \frac{f}{2}mu^2 \text{ where } f = \frac{5}{16}$$

$$v_x^2 + v_y^2 = fu^2$$

Substituting (1),

$$[u(\cos\alpha - \mu\sin\alpha) - \mu v_y]^2 + v_y^2 = fu^2$$

we arrive at a quadratic equation in v_y:

$$(1+\mu^2)v_y^2 - 2\mu(\cos\alpha - \mu\sin\alpha)uv_y + u^2(\cos\alpha - \mu\sin\alpha)^2 - fu^2 = 0$$

$$\frac{5}{4}\left(\frac{v_y}{u}\right)^2 - \frac{1}{2}\left(\frac{v_y}{u}\right) - \frac{1}{16} = 0$$

$v_y = \frac{u}{2}$ $v_x = \frac{u}{4}$

Angle with the x axis: $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1} 2 = 63^\circ$ **Answer: (d)**

MC5

Consider the motion of the center of mass of the sand. At $t = 0$, the center of mass starts to move downwards. Hence there is a downward acceleration. Using Newton's second law, the reaction force of the balance on the hourglass should decrease, so that it cannot completely cancel the weight of the sand, and the net force on the sand is downward. Throughout the dripping process, the center of mass of the sand is moving downwards at a constant velocity. There is no acceleration and hence the reaction force stays at the equilibrium value. At the end of the dripping process, the downward motion of the center of mass is stopped. There is a deceleration and the reaction force increases momentarily. **Answer: (e)**

MC6

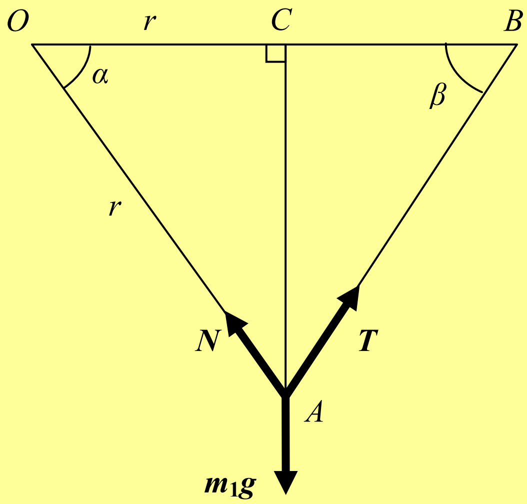
Using Archimedes' principle, volume of water displaced $V_{\text{dis}} = (m_{\text{stone}} + \rho_{\text{ice}} V_{\text{ice}}) / \rho_{\text{water}}$. Suppose a volume of ΔV_{ice} melts, then the volume of water displaced decreases by $\rho_{\text{ice}} \Delta V_{\text{ice}} / \rho_{\text{water}}$. At the same time, since mass is conserved, the volume of water increases by $\rho_{\text{ice}} \Delta V_{\text{ice}} / \rho_{\text{water}}$. Hence there is no change in the water level. However, when all ice melts and the stone sinks to the bottom, the volume of water displaced is $m_{\text{stone}} / \rho_{\text{stone}} < m_{\text{stone}} / \rho_{\text{water}}$. Hence the water level falls. **Answer: (b)**

MC7

The key is the conservation of energy. The electrical energy generated by the turbines comes from the kinetic energy of the wind, which in turn comes from the kinetic energy of the trains. Hence no energy is saved, and the air resistance experienced by the trains will increase. **Answer: (a)**

MC8

Sol. 考慮球 1 的平衡，作出受力圖如下。



$T = m_2 g$, $\beta = 90^0 - \frac{\alpha}{2}$.

$\Sigma F_y = 0, T\sin\beta + N\sin\alpha - m_1 g = 0$, $N\sin\alpha = m_1 g - m_2 g\sin\beta$ (i)

$\Sigma F_x = 0, T\cos\beta - N\cos\alpha = 0$, $N\cos\alpha = m_2 g\cos\beta$ (ii)

(i)÷(ii): $\dfrac{m_1}{m_2} = \dfrac{\tan\alpha}{\sqrt{2(1-\cos\alpha)}} = \sqrt{3}$. **Answer: (e)**

MC9

The force which is responsible for restoring the liquid levels in two arms of the tube is $F = -\Delta pA = -(h_1 + h_2)\rho gA$, where Δp is the pressure difference and h_1 & h_2 being the rise and fall of liquid levels in the two arms in vertical direction respectively.

Denote x as the change in length of the liquid along the tube. (note: the change in length of the liquid in both arms, either rise or fall, is the same)

Using this restoring force can enable us to have the following equation:

$F = -(x\sin\theta_1 + x\sin\theta_2)\rho gA = -\rho gA(\sin\theta_1 + \sin\theta_2)x = ma$

$$\Rightarrow a = -\frac{\rho gA(\sin\theta_1 + \sin\theta_2)}{m}\cdot x = -\omega^2\cdot x$$

By using S.H.M. equation, we have

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\rho gA(\sin\theta_1 + \sin\theta_2)}{\rho A(L_1 + L_2)}} = \frac{1}{2\pi}\sqrt{\frac{g(\sin\theta_1 + \sin\theta_2)}{L_1 + L_2}}$$ **Answer: (a)**

where mass of liquid in the tube $m = \rho V = \rho A(L_1 + L_2)$

MC10

For simple harmonic motion, $x = A\cos\omega t$ and $v = -\omega A\sin\omega t$. Substituting into the expression of the total energy,

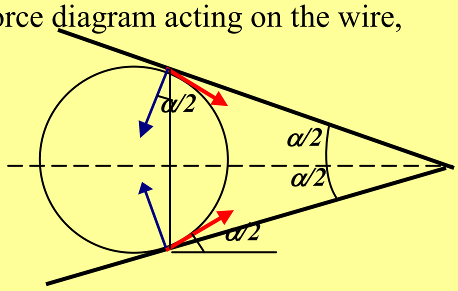
$$E = \frac{1}{2}av^2 + \frac{1}{2}bx^2 = \frac{1}{2}bA^2 + \frac{1}{2}(a\omega^2 - b)A\sin^2\omega t.$$

For energy to conserve, we have $\omega^2 = \dfrac{b}{a}$ and $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{b}{a}}$. **Answer: (b)**

MC11

Denote the friction force and normal force acting on each blade as F_f and N, respectively. At the critical angle the static friction force should reach its maximal value which is $F = \mu N$.

The condition for the wire not to move is when all components of the forces adding up equal to zero. As shown in the force diagram acting on the wire,



the vertical components cancel each other by symmetry, and the horizontal components cancel if $2F_f \cos\frac{\alpha}{2} = 2N\frac{\sin\alpha}{2}$.

Thus, from the condition of the critical angle we obtain: $\mu = \tan\frac{\alpha}{2}$. **Answer: (d)**

Note that the angle in the question is twice the angle here.

MC12

Let M be the mass of the box.
If the box is not sliding, we have $F < \mu Mg$ …(1)
If the box tips over, it will happen at the lower right hand corner, so it is better to measure torques about this point. For tipping, clockwise moment must exceed counterclockwise moment:

i.e. $FH > \frac{MgL}{2}$ …. (2)

Combining (1) and (2), we have (tipping before sliding)

$$\frac{MgL}{2H} < F < \mu Mg$$

$$\Rightarrow \mu > \frac{L}{2H}$$

Thus, the critical condition in this case ($\mu > \mu_o$) is $\mu_o = \frac{L}{2H}$ **Answer: (a)**

MC13

Let v be the velocity of the particle of mass m. Kinetic energy of the compound pendulum:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{3}{4}mv^2$$

Potential energy of the compound pendulum:

$$U = -mgL\cos\theta - 2mg\frac{L}{2}\cos\theta = -2mgL\cos\theta$$

For small oscillations, $U \approx -2mgL\left(1-\frac{\theta^2}{2}\right) = \text{cosntant} + mgL\theta^2$

For small oscillations, we can approximate the linear displacement by $x = L\theta$.

Hence $U = \frac{mg}{L}x^2$

Using the result of MC12, we can substitute $a = \frac{3}{2}m$, $b = \frac{2mg}{L}$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{2mg/L}{3m/2}} = \frac{1}{2\pi}\sqrt{\frac{4g}{3L}}$$. **Answer: (b)**

MC14

Using Newton's second law, we find that the acceleration of the upward displacement is $-g(\sin\alpha + \mu\cos\alpha)$. Hence $t_1 = \frac{v_1}{g(\sin\alpha + \mu\cos\alpha)}$ and $L = \frac{v_1^2}{2g(\sin\alpha + \mu\cos\alpha)}$.

The acceleration of the downward discplacement is $g(\sin\alpha - \mu\cos\alpha)$ (in the downward direction). Hence the time is given by $L = \frac{1}{2}g(\sin\alpha - \mu\cos\alpha)t_2^2$. Hence

$$t_2^2 = \frac{2L}{g(\sin\alpha - \mu\cos\alpha)} = \frac{v_1^2}{g^2(\sin\alpha - \mu\cos\alpha)(\sin\alpha + \mu\cos\alpha)} = t_1^2 \frac{\sin\alpha + \mu\cos\alpha}{\sin\alpha - \mu\cos\alpha}$$

$$\frac{t_2}{t_1} = \sqrt{\frac{\sin\alpha + \mu\cos\alpha}{\sin\alpha - \mu\cos\alpha}}$$ **Answer: (b)**

MC15

Sol. $mv_0 = (m+M)v$, $v = \frac{m}{m+M}v_0$=2m/s,

$$\Delta E_k = \frac{1}{2}mv_0{}^2 - \frac{1}{2}(m+M)v^2 = \frac{1}{2}\times 1\times 10^2 - \frac{1}{2}\times(1+4)\times 2^2 = 40 \text{ J}$$

Answer: (e)

MC16

Sol. 已知 m_1=1kg, m_2=3kg, F=6N, v_0 =2m/s, d =1m..

當兩物體速度相等時, 距離最小, 此時的速度為 v. 由動量守恆定律

$$m_2v_0 = (m_1+m_2)v,\ v = \frac{m_2}{m_1+m_2}v_0 = \frac{3}{1+3}\times 2 = 1.5 \text{ m/s}.$$

或 距離最小時所用時間為 T, $v_1 = a_1T = v_0 + a_2T$, $6T = 2 - 2T$, $T = 0.25\text{s}$, $v = 1.5\text{m/s}$.

$$\Delta E_k = \frac{1}{2}m_2v_0{}^2 - \frac{1}{2}(m_1+m_2)v^2 = \frac{1}{2}\times 3\times 2^2 - \frac{1}{2}(1+3)\times 1.5^2 = 1.5\text{J} = F\Delta s,\ \Delta s = 0.25\text{m}.$$

兩物體間的最小距離 = $d - \Delta s$ = 0.75 m. **Answer: (c)**

MC17

Let x be the fraction of the rod length submerged in water. Using Archimedes' principle, the buoyancy is given by $\rho_{\text{water}}AxLg$. Taking moments about the hanging end,

$$\rho_{\text{water}}AxLgL\left(1-\frac{x}{2}\right) = \rho_{\text{rod}}ALg\left(\frac{L}{2}\right)$$

$$x\left(1-\frac{x}{2}\right) = \frac{5}{9}\left(\frac{1}{2}\right) \quad\Rightarrow\quad x = \frac{1}{3}$$

The fraction of the rod length above the water is 2/3. **Answer: (d)**

MC18

Using $x = \frac{1}{2}at^2$, the time taken by the first carriage to pass the observer is the time difference between $x = L$ and $x = 0$, where L is the length of a carriage. Hence $L = \frac{1}{2}at_0^2$ where t_0 = 5 s. The time taken by the tenth carriage to pass the observer is the time difference between x = $10L$ and x = $9L$. Hence the time is $t_2 - t_1$, where $9L = \frac{1}{2}at_1^2$ and $10L = \frac{1}{2}at_2^2$ $\Rightarrow$ $t_2 - t_1 = (\sqrt{10}-3)t_0 = 0.81\text{ s}$. **Answer: (e)**

MC19

Let T be the tension in the light string, and a the acceleration. Using Newton's second law,

$F_1 - T = m_1 a$ (1)

$T - F_2 = m_2 a$ (2)

(1) + (2): $F_1 - F_2 = (m_1 + m_2)a \Rightarrow a = \dfrac{F_1 - F_2}{m_1 + m_2}$ and $T = \dfrac{m_2 F_1 + m_1 F_2}{m_1 + m_2}$

$x = \dfrac{T}{k} = \dfrac{m_2 F_1 + m_1 F_2}{k(m_1 + m_2)}$. **Answer: (b)**

MC20

Consider the forces acting on the two piece of wood. The normal reactions are $N_1 = mg\cos\alpha$ and $N_2 = mg\cos\beta$.

Consider the vertical component of the forces acting on the triangular block.

$F = Mg + N_1\cos\alpha + N_2\cos\beta = Mg + mg(\cos^2\alpha + \cos^2\beta)$

Since $\alpha + \beta = 90^\circ$, $F = Mg + mg(\cos^2\alpha + \sin^2\alpha) = Mg + mg$. **Answer: (c)**

Open Problems

Q1* (10 points)

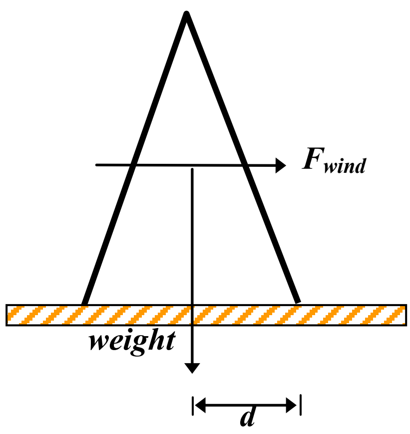
Solution:

Two forces act on the sign, namely the horizontal force, F_{wind}, from the wind and the force of gravity, *mg*, of the sign.

If the sign turns over, it will be pivoting at the lower right edge and there will be no longer be a force between the ground and the lower left edge. This means we need to balance the torques, about the lower right edge.

The force of gravity, acting from the center of mass, provides a stabilizing torque of

$\tau_{\text{gravity}} = F_{wind} \cdot d = (mg)(h)\tan(\theta/2) = (6\text{ kg})(9.8)(1.0)\tan 15^\circ = 15.8$ Nm



The force from the wind acts in the center of the left board, causing a tipping torque of

$\tau_{wind} = F_{wind}\left(\dfrac{h}{2}\right)$ ..(1)

By using Newton's 2nd law of motion, F_{wind} can be expressed as:

$F_{wind} = \dfrac{\Delta(mv)}{\Delta t} = \dfrac{\rho A(v\Delta t)}{\Delta t} \cdot v = \rho A v^2$ ….(2) , where A is the cross-sectional area of the board.

Combining equation (1) & (2), which yields $\tau_{gravity} = \tau_{wind} = (\rho A v^2)\left(\dfrac{h}{2}\right)$

This suggests the minimum speed of the wind v_{min} is

$$v_{\min} = \sqrt{\frac{2\tau_{gravity}}{\rho A h}} = \sqrt{\frac{2(15.8)}{(1.2)(0.5)(1.0)}} = 7.26\text{ m/s}$$

Q2 (10 points)

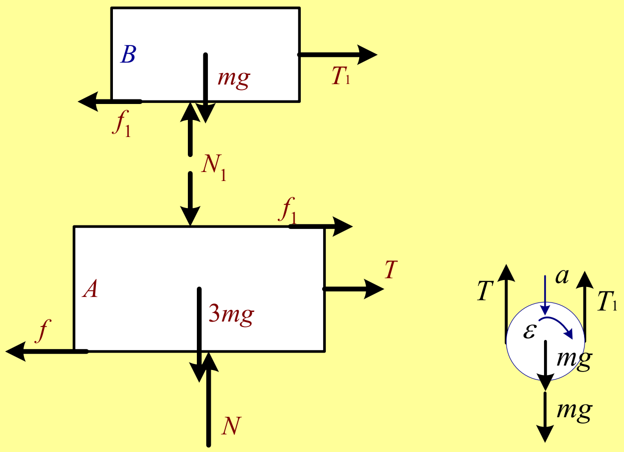
Sol.

(1) 考慮 C: $2T = mg, T = \frac{1}{2}mg$; 考慮 B: $N_1 = mg, f_1 = T = \frac{1}{2}mg$;

考慮 A: $N = N_1 + 3mg = 4mg, f = f_1 + T = \frac{1}{2}mg + \frac{1}{2}mg = mg$.

系統保持平衡，則滿足: $f_1 \le \mu N_1$ 和 $f \le \mu N$ ，即

$\frac{1}{2}mg \le \mu mg$ 和 $mg \le \mu(4mg)$ ，$\mu \ge \frac{1}{2}$ 和 $\mu \ge \frac{1}{4}$ ，$\therefore\ \underline{\mu \ge \frac{1}{2}}$.



設 B 加速度為 $2a$, 則 C 加速度為 a.

(2) 考慮 C: $mg - 2T = ma$; (i)

考慮 B: $N_1 = mg, T - f_1 = 2ma$, $f_1 = \mu N_1 = \mu mg$, (ii)

$\therefore \quad T - \mu mg = ma$ (iii)

由(i)和(iii), 得 $a = \frac{1-2\mu}{5}g$, $T = \frac{2+\mu}{5}mg$ (iv)

考慮 A: $N = 4mg, f = f_1 + T$ (v)

(ii)和(iv)代入(v), 得 $f = \mu mg + \frac{2+\mu}{5}mg = \frac{2+6\mu}{5}mg$.

C 向下做勻加速運動而 A 靜止，則滿足: $a > 0$ 和 $f \le \mu N$ ，即

$\frac{1-2\mu}{5}g > 0$ 和 $\frac{2+6\mu}{5}mg \le \mu(4mg)$ ，$\mu < \frac{1}{2}$ 和 $\mu \ge \frac{1}{7}$ ，$\therefore\ \underline{\frac{1}{2} > \mu \ge \frac{1}{7}}$

Q3 (15 points)

(a) First we calculate the position of the neutral point, where the attractive forces due to the planet and the satellite are equal and opposite. Let x be the distance of the neutral point from the center of sphere B. Then

$$\frac{GMm}{x^2} = \frac{G4Mm}{(6R-x)^2} \quad \Rightarrow \quad x = 2R$$

It is sufficient to send the projectile to the neutral point. Beyond the point, the projectile can fall to the surface of sphere A under free fall. Using the conservation of energy,

$$\frac{1}{2}mv_{\min}^2 - \frac{GMm}{R} - \frac{G4Mm}{5R} = -\frac{GMm}{2R} - \frac{G4Mm}{4R} \quad \Rightarrow \quad v_{\min} = \sqrt{\frac{3GM}{5R}}$$

(b) When the projectile reaches the surface of sphere A,

$$-\frac{GMm}{2R}-\frac{G4Mm}{4R}=\frac{1}{2}mv^2-\frac{GMm}{5R}-\frac{G4Mm}{R} \quad\Rightarrow\quad v=\sqrt{\frac{27GM}{5R}}$$

(c) Escape velocity:

$$\frac{1}{2}mv_{\text{esc}}^2-\frac{GMm}{R}-\frac{G4Mm}{5R}=0 \quad\Rightarrow\quad v_{\text{esc}}=\sqrt{\frac{18GM}{5R}}$$

Q4 (15 points)

(a) Let θ be the angle between line PC and the vertical. Using the conservation of energy, the velocity of the bob at point C:

$$0=-mg\frac{L}{2}(1-\cos\theta)+\frac{1}{2}mv_C^2 \quad\Rightarrow\quad v_C=\sqrt{gL(1-\cos\theta)}$$

Using Newton's second law, $T+mg\cos\theta=m\frac{v_C^2}{L/2}=2mg(1-\cos\theta)$

$T=0$ at point C. Therefore, $\cos\theta=\frac{2}{3} \quad\Rightarrow\quad \theta=48^\circ$

(b) At point C, the velocity is $v=\sqrt{gL(1-\cos\theta)}=\sqrt{\frac{gL}{3}}$ and makes an angle θ with the horizontal. Hence after the sting becomes loose, using point P as the origin, the coordinate of the bob is given by

$$x=\frac{L}{2}\sin\theta-v_C\cos\theta t$$

$$y=\frac{L}{2}\cos\theta+v_C\sin\theta t-\frac{1}{2}gt^2$$

Using $v_y=v_C\sin\theta-gt$, the maximum height is given by $v_y=0 \quad\Rightarrow\quad t=\frac{v_C\sin\theta}{g}$

$$y=\frac{L}{2}\cos\theta+\frac{v_C^2\sin^2\theta}{2g}=\frac{L}{3}+\frac{gL/3}{2g}\left(\frac{5}{9}\right)=\frac{23}{54}L$$

Hence the maximum height is $23L/54$ above point P.

(c) At point E, $x=0$. Hence

$$t=\frac{L\sin\theta}{2v_C\cos\theta}$$

$$y=\frac{L}{2}\cos\theta+\frac{\sin^2\theta}{2\cos\theta}L-\frac{3\sin^2\theta}{8\cos^2\theta}L=\frac{9}{32}L$$

Hence point E is at a distance $7L/16$ below point O.

Q5 (10 points)

(a) The system oscillates about the center of mass, which is at a distance of $L/3$ from particle A. Let a be the acceleration of A. Using Newton's second law,

$2ma=-qE\sin\theta$.

For small oscillations, $\sin\theta\approx\theta=\frac{3x}{L}$.

Hence $a=-\frac{3qE}{2mL}x$. This is the equation of a simple harmonic motion, with $\omega^2=\frac{3qE}{2mL}$.

Period of oscillations: $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2mL}{3qE}}$

(b) The amplitude of x: $x_0 = \frac{L\theta_0}{3}$

The amplitude of the velocity: $v_0 = \omega x_0 = \frac{\omega L\theta_0}{3}$

Using Newton's second law, $T - qE\cos\theta = 2m\frac{v^2}{L/3}$

$T = qE\cos\theta + \frac{6mv^2}{L}$. Both terms are largest when $\theta = 0$, where $v = v_0$. Hence the maximum tension is $T_{\max} = qE + \frac{6mv_0^2}{L} = qE + qE\theta_0^2$

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2011 香港物理奧林匹克

(Senior Level 高級組)

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共同舉辦

March 19, 2011
2011 年 3 月 19 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID Number in the field of "I. D. No.", and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of "Course & Section No."

   請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 "I. D. No." 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格填上你的英文姓名，最後於 "Course & Section No. " 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant I. D. Number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitational constant, 6.67×10^{-11} (N m^2/kg^2)
e – charge of an electron, -1.6×10^{-19} (A s)
ε_0 – electrostatic constant, 8.85×10^{-12} (A s)/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Sun-Earth distance (= 1 Astronomical Unit (AU)) = 1.5×10^{11} m
Mass of the sun = 1.99×10^{30} kg
Air Density at 20 °C and 1 atm = 1.2 kg/m^3
Water Density = 1.0×10^3 kg/m^3
Ice Density = 9.2×10^2 kg/m^3
Standard atmosphere pressure $p_0 = 1.0 \times 10^5$ N/m^2 = 760 mmHg

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 萬有引力常數, 6.67×10^{-11} (N m^2/kg^2)
e – 電子電荷, -1.6×10^{-19} (A s)
ε_0 – 靜電常數, 8.85×10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
太陽--地球距離 (= 1 天文單位) = 1.5×10^{11} m
太陽質量 = 1.99×10^{30} kg
20 °C、一個大氣壓的空氣密度 = 1.2 kg/m^3
水密度 = 1.0×10^3 kg/m^3
冰的密度 = 9.2×10^2 kg/m^3
標準大氣壓 $p_0 = 1.0 \times 10^5$ N/m^2 = 760 毫米汞柱 (mmHg)

The following conditions will be applied to all questions unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light in vacuum.

除非特別注明，否則下列條件將適用于本卷所有問題：

1) 所有物體都處于地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於真空中的光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每題 2 分，每題選擇一個答案。）

The MC questions with the „*" sign may require information on page-3.
帶 * 的選擇題可能需要用到第三頁上的資料。

MC1

A glass bottle falls off a table onto the floor and breaks into pieces. Which of the following statements regarding the process is **incorrect**?
一個玻璃瓶，從桌上掉到地上摔成碎片。以下關于這一過程的陳述哪一個是**錯**的？

(a) The potential energy of the bottle has changed.
瓶子的勢能改變了。

(b) Some of the potential energy of the bottle has eventually been converted into the energy to break the bottle.
瓶子的一部分勢能最終轉化成將瓶子打碎所需的能量。

(c) Some of the kinetic energy of the bottle has eventually been converted into the energy to break the bottle.
瓶子的一部分動能最終轉化成將瓶子打碎所需的能量。

(d) The impact from the floor to the bottle is larger than the impact from the bottle to the floor.
地面對瓶子的衝量比瓶子對地面的衝量大。

(e) The impact from the floor to the bottle breaks the bottle.
地面對瓶子的衝量打碎了瓶子。

MC2

A small weight of mass $2M$ is attached to the bottom of a hollow sphere of mass M and radius R. Half of the sphere is submerged when floating in water with mass density ρ. Find the frequency of the simple harmonic oscillation of the sphere in the vertical direction.
一個空心的球半徑爲 R、質量爲 M，下方繫了一個質量爲 $2M$ 的小重物，球浮在水面上, 有一半浸在水裏，水的質量密度爲 ρ 。求球上下浮動的簡諧振動頻率。

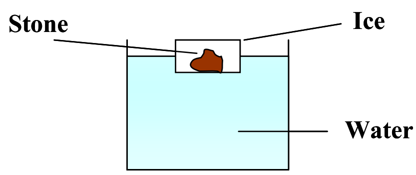
(a) $\omega = \sqrt{2\rho g\pi R^2 / M}$ (b) $\omega = \sqrt{\rho g\pi R^2 / (3M)}$ (c) $\omega = \sqrt{\rho g\pi R^2 / M}$

(d) $\omega = \sqrt{\rho g\pi R^2 / (2M)}$ (e) $\omega = \sqrt{3\rho g\pi R^2 / M}$

MC3

A piece of ice with an embedded stone floats on the surface of water in a glass. After the ice has melted, the stone sinks to the bottom of the glass. Compared with the initial water level, what is the change of the water level in the glass, first as the ice melts, and second after the stone sinks to the bottom?
一個裝水(water)的杯子裏有一塊冰(ice)浮在水面，冰塊裏有一塊石頭(stone)。冰融解後石頭沉到杯子底。與原來水面的高度比較，（1）當冰還在融化時，（2）當石塊沉到水底時，水面的高度應該是如何改變的？

(a) Remains the same and then rises. 不變，然後上升。
(b) Remains the same and then falls. 不變，然後下降。
(c) Remains the same all the way. 一直不變。
(d) Rises and then falls. 先升後降。
(e) Falls and then rises. 先降後升。



MC4*

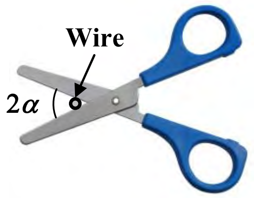
An extra-solar planet revolving around a pulsar of 1.5 solar mass was discovered in 1992. The period of the circular orbit of the planets is 98 days. Find the distance between the pulsar and the planet in terms of astronomical units (AU).

1992 年發現的太陽系以外的行星系統的中心爲一個質量爲太陽 1.5 倍的脉衝星。行星圓形軌道的周期爲 98 天。求脉衝星與行星之間的距離（**以天文單位表達**）。

(a) 0.11 (b) 0.17 (c) 0.36 (d) 0.40 (e) 0.48

MC5

Someone is using a scissors to cut a wire of circular cross section and negligible weight. The wire slides in the direction away from the hinge until the angle between the scissors blades becomes 2α. Find the friction coefficient between the blades and the wire.



用一把剪刀，去剪一條圓截面的導綫。導綫先會向外滑動，直到剪刀之間的角度爲 2α，求剪刀和導綫之間的摩擦係數。

(a) $\sqrt{1-\tan\alpha}$ (b) $2\cos\alpha$ (c) $\tan\alpha$ (d) $2\tan\alpha$ (e) $\sqrt{2\cos^2 2\alpha-1}$

MC6

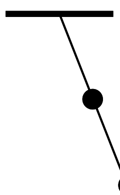
The kinetic energy of a particle in a simple harmonic motion is $0.5av^2$, its potential energy is $0.5bx^2$, where x is the coordinate for the position of the particle and v is its speed. The frequency of the motion is then__________.

一個質點做簡諧振動，它的動能爲 $0.5av^2$、勢能爲 $0.5bx^2$，期中 x 是粒子的位置坐標，v 是它的速度，則簡諧振動的頻率爲__________。

(a) $\frac{1}{2\pi}\sqrt{\frac{b}{a}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{a}{b}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{a}{b}+\frac{b}{a}}$ (d) $\frac{1}{2\pi}\sqrt{ab}$ (e) $\frac{1}{2\pi}\sqrt{\frac{1}{ab}}$

MC7

A compound pendulum is made of a light and rigid rod of length L with one end attached to a hinge on the ceiling. A small ball of mass m is attached to the other end of the rod, and another small ball of mass $2m$ is attached onto the middle of the rod. Find the frequency of the pendulum in small amplitude simple harmonic oscillation.



一個複合單擺的輕質量剛性杆，長度爲 L，一端繫在天花板的鉸鏈上，一質量爲 m 小球固定在杆的另一端點，另一個小球質量爲 $2m$ 固定在杆的中點。求擺的小幅簡諧振動頻率。

(a) $\frac{1}{2\pi}\sqrt{\frac{g}{2L}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{4g}{3L}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{3g}{2L}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{9g}{4L}}$ (e) $\frac{1}{2\pi}\sqrt{\frac{9g}{2L}}$

MC8

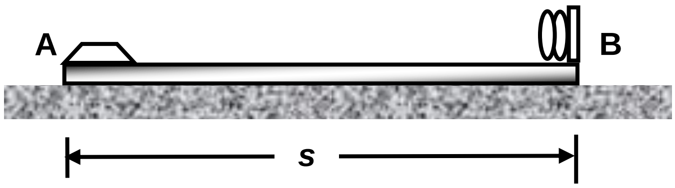
A small object is initially at the bottom of a slope inclined at an angle α with the horizontal. It is projected upward along the slope with an initial velocity, and reaches the maximum height after time t_1. It then slides downward and returns to the initial position after time t_2. If the coefficient of sliding friction between the object and the slope surface is μ, find the ratio t_2/t_1.

一個傾角為 α 的斜面底部有一小物塊。現給物塊一個初速率，使它沿斜面向上滑動，經過時間 t_1 後到達最高點，之後它又滑回到底部，所用時間爲 t_2。若物體與斜面間的滑動摩擦系數爲 μ，求比例 t_2/t_1。

(a) $\sqrt{\frac{\sin\alpha+\mu\cos\alpha}{\sin\alpha-\mu\cos\alpha}}$ (b) $\sqrt{\frac{\sin\alpha-\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}}$ (c) $\frac{\sin\alpha-\mu\cos\alpha}{\sin\alpha+\mu\cos\alpha}$ (d) $\frac{\sin\alpha+\mu\cos\alpha}{\sin\alpha-\mu\cos\alpha}$ (e) $\sqrt{\frac{\sin\alpha}{\mu\cos\alpha}}$

MC9

As shown in the figure, AB is a board of mass M = 4 kg and length s = 2 m, placed on a smooth horizontal surface. A bumper of negligible mass is fixed at end-B. A peg of mass m = 1 kg is placed at end-A. The coefficient of kinetic friction between the peg and the board is μ = 0.2. With both the board initially at rest, the peg is ejected with an initial velocity of v_0 = 10 m/s along the board until it hits the bumper at end-B. After the collision, it returns to end-A without falling of the board. Find the mechanical energy loss in the process.

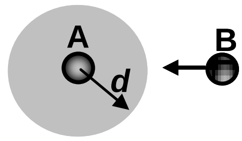


如圖所示，AB 爲位于光滑水平面上的長木板，質量爲 M = 4 kg，長度為 s = 2 m。B 端有一固定擋板。A 端放有一小滑塊，其質量 m = 1 kg。小滑塊與木板間的動摩擦系數爲 μ = 0.2。當木板處于靜止的初始狀態時，小滑塊以初速度 v_0 = 10 m/s 沿木板射出，直到和 B 端的擋板相撞。碰撞後，小滑塊恰好回到 A 端而不脫離木板。求過程中損失的機械能。

(a) 16 J (b) 24 J (c) 28 J (d) 32 J (e) 40 J

MC10

The elastic collision between two bodies, A and B, can be considered using the following model. A and B are free to move along a common line without friction. When their distance is greater than d = 1 m, the interacting force is zero; when their distance is less than d, a constant repulsive force F = 6 N is present. The mass of body A is m_A = 1 kg and it is initially at rest; the mass of body B is m_B = 3 kg and it is approaching body A head-on with a speed v_0 = 2 m/s. Find the minimum distance between A and B.

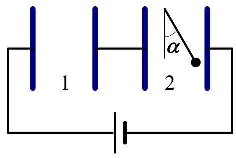


兩物體 A 和 B 的彈性碰撞過程可以如下的模型考慮。A 和 B 可沿同一直線自由運動，不用考慮摩擦力。當它們之間的距離大於 d = 1 m 時， 相互作用力為零; 當它們之間的距離小於 d 時， 存在大小恒為 F = 6N 的斥力。設 A 物體質量為 m_A = 1 kg， 開始時靜止在直線上某點; B 物體質量 m_B = 3 kg, 以速率 v_0 = 2 m/s 從遠處沿該直線向 A 運動。求 A、B 間的最小距離。

(a) 0.25 m (b) 0.50 m (c) 0.75 m (d) 1 m (e) 1.25 m

MC11

Two identical parallel plate capacitors 1 and 2 are placed vertically and connected in series to a battery. In capacitor-2 there is a charged small particle attached by a thin wire to a fixed point, as shown. Ignore the effect of the charge particle on the charge distribution on the capacitor plates. At equilibrium, the angle between the wire and the vertical direction is α . Now slowly pull a plate of capacitor-1 until the distance between its two plates is doubled. After equilibrium, find the angle between the wire and the vertical direction.



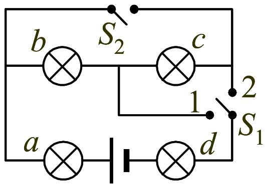
二個相同的豎直放置的平行板電容器 1 和 2 串聯後接上直流電源。電容器 2 內用細綫懸繫著一個帶電質點，平衡時懸綫與豎直方向所成的角爲α。不考慮帶電質點對電容器 2 內電荷分布的影響。現將電容器 1 的二板間距慢慢地改變爲初始時的 2 倍，電容器 2 內的帶電質點再次達到平衡。求懸綫與豎直方向所成的角。

(a) $\tan^{-1}(\frac{2}{3}\tan\alpha)$ (b) $\tan^{-1}(\frac{1}{3}\tan\alpha)$ (c) $\tan^{-1}(\frac{1}{2}\tan\alpha)$ (d) $\tan^{-1}(\frac{3}{2}\tan\alpha)$

(e) $\sin^{-1}(\frac{2}{3}\tan\alpha)$

MC12

Four lamps are connected in the way shown in the figure. When switch S_2 is open and switch S_1 is on position-2, Lamp-b is the brightest, and lamp-c and lamp-d are the dimmest and are of the same brightness. Now S_2 is closed and S_1 is on position-1, the sequence in brightness of the lamps is_______, with the first in the sequence being the brightest.



四盞燈按如圖所示的電路連接。當 S_2 斷開、S_1 接 2 時，燈 b 最亮，c 和 d 最暗且亮度相同。當 S_2 閉合、S_1 接 1 時，四盞燈按由明到暗次序排列，依次為_______ 。

(a) c, d, b, a (b) a, d, b, c (c) a, b, c, d (d) a, d, c, b (e) d, c, b, a

MC13

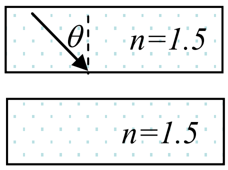
An open-end hot air balloon with fixed volume of 1000 m^3 and weighs 100 kg (including the payload but not the air inside the ballon) is floating in air. The air density outside is 1.2 kg/m^3, and the air temperature outside is 20°C. Find the temperature of the air inside the balloon.

一個開口熱氣球，體積爲 1000 m^3，重 100 kg（包括載重，但不包括氣球裏的空氣），漂浮在半空。氣球外邊空氣的密度爲 1.2 kg/m^3，氣球外邊空氣的溫度爲 20°C。求氣球裏空氣的溫度。

(a) 46.6 °C (b) 20 °C (c) 26.6 °C (d) 146.6 °C (e) 56.6 °C

MC14

A beam of light with wavelength 500 *nm* is incident from glass (refractive index = 1.5) to an air gap at an incidence angle θ = 75°. To get maximum reflection, the width of the air gap should be ________.



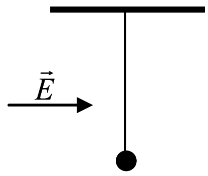
(a) 653 nm (b) 1306 nm (c) 327 nm (d) 980 nm (e) any of the listed values

一束波長爲 500 *nm* 的光，從玻璃（折射率等于 1.5）射到一個空氣間隙上，入射角爲 θ = 75°。爲達到最强反射，空氣間隙的厚度應該是________ 。

(a) 653 nm (b) 1306 nm (c) 327 nm (d) 980 nm (e) 任何所列答案都對

MC15

A simple pendulum is made of a small ball of mass m carrying charge q attached by a thin wire to the ceiling. Its original simple harmonic oscillation frequency is ω_0. After a uniform electric field E is applied horizontally to the pendulum, its frequence becomes $2\,\omega_0$. Find the strength of the electric field.

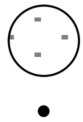


一個質量爲 m 的小球，由細繩繫在天花板上，成爲一個單擺。小球帶電 q，單擺的簡諧振動頻率爲 ω_0。現在水平方向加一電場 E，則單擺的頻率變爲 $2\omega_0$ ，求電場強度。

(a) $\sqrt{15}mg/q$ (b) $\sqrt{3}mg/q$ (c) $\sqrt{5}mg/q$ (d) mg/q (e) $2mg/q$

MC16

As shown, a uniform magnetic field B pointing out of the paper plane is confined in the shaded area of radius r. At a distance R from the center of the shaded area there is a point particle of mass m and carrying charge q. The magnetic field is then quickly changed to zero. Find the speed of the particle.

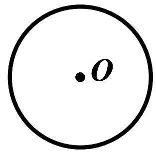


如圖，在一個半徑爲 r 的陰影區有一個指出紙面的均匀磁場 B，在離陰影區中心 R 的地方有一個質量爲 m、帶電爲 q 的質點。現將磁場很快下降到零，求質點的速度。

(a) $v=\dfrac{qBr^2}{2mR}$ (b) 0 (c) $v=\dfrac{qBr^2}{mR}$ (d) $v=\dfrac{2qBr^2}{mR}$ (e) $v=\dfrac{qBr^2}{4mR}$

MC17

Shown in the figure is the top view of a long and thin wall tube of radius R carrying uniform electric current I flowing out of the paper plane. Find the magnetic field at the center of the tube.

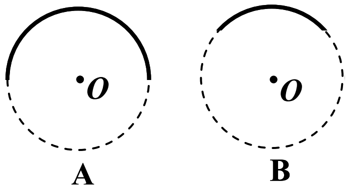


右圖所示的是一條半徑爲 R 的薄壁長管的俯視圖，管壁有均勻電流 I 向紙外流出，求管中心點處的磁場。

(a) 0　(b) $\frac{\mu_0 I}{2\pi R}$　(c) $\frac{\mu_0 I}{\pi R}$　(d) $\frac{2\mu_0 I}{\pi R}$　(e) $\frac{\mu_0 I}{4\pi R}$

MC18

Similar to MC17, structure-A is a half-circle sheet carrying uniform electric current I flowing out of the paper plane. Structure-B is a quarter-circle sheet carrying uniform electric current I flowing out of the paper plane. At the center of the circular arc point-O the total magnetic field is pointing to _____ in both cases and the field strength of structure-A is ______ than that of structure-B.



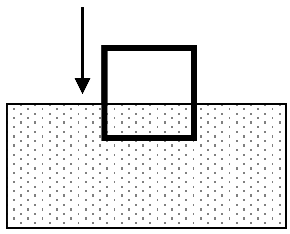
(a) up, larger　(b) left, larger　(c) left, smaller　(d) right, smaller　(e) right, larger

與 MC17 相似，結構-A 爲半條圓管，結構-B 爲 1/4 條圓管。它們的弧度半徑一樣，流過的均勻電流也一樣，幷流出紙面。則在圓弧中心 O 的磁場方向向_____，結構-A 的磁場比結構-B ____。

(a) 上, 大　(b) 左, 大　(c) 左, 小　(d) 右, 小　(e) 右, 大

MC19

A square coil of N turns with length L, mass m, and resistance R is falling through a region of uniform magnetic field pointing out of the paper plane. The coil quickly reaches the terminal speed v, which is ________.



一個有 N 匝的正方形綫圈，邊長爲 L，質量爲 m，電阻爲 R，下降到一個均勻磁場區域，磁場指出紙面。綫圈很快達到收尾（終止）速度 v。求速度 v。

(a) $v = mgR/\left(BL^2N\right)$　(b) $v = mgR/\left(B^2L\right)$　(c) $v = mgR/\left(B^2LN\right)$

(d) $v = mgR/\left(BLN\right)$　(e) $v = mgR/\left(B^2L^2N\right)$

MC20

Similar to MC19, two square coils are falling down at terminal speed through a magnetic field region. The materials of the wires of the coils are the same, and the cross section area of the wire in coil-A is half of that of coil-B. The number of turns of coil-A is twice of that of coil-B so the two coils are of the same mass m. After the two coils fall down by a distance H ($< L$), the energy dissipated by the electric heating of coil-A and coil-B are __________, respectively.

與 MC19 類似，兩個正方形綫圈，以收尾速度下降到一個均勻磁場區域，綫圈用的材料是一樣的，綫圈 A 導綫的截面積爲綫圈 B 的一半，綫圈 A 的匝數是綫圈 B 的兩倍，因此兩個綫圈具有同樣的質量 m。兩綫圈在磁場區域下降了高度 H ($< L$)，則綫圈 A 和 B 中電熱消耗的能量分別爲__________。

(a) 0, 0　(b) mgH, mgH　(c) $2mgH$, mgH　(d) mgH, $2mgH$

(e) $mgHR/\left(B^2L^2N\right)$, $2mgHR/\left(B^2L^2N\right)$

《End of MC's 選擇題完》

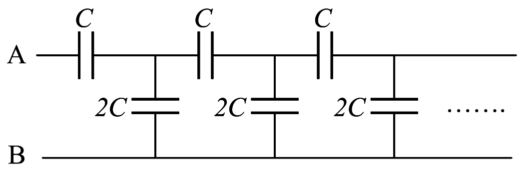
Open Problems 開放題
Total 5 problems 共 5 題

Q1 (10 points)　　　題 1（10 分）

(a) **Derive** the formulae for the total resistance of two resistors in parallel and serial connections.
推導兩個電阻串聯和并聯時總電阻的公式。

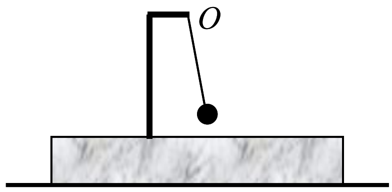
(b) **Derive** the formulae for the total capacitance of two capacitors in parallel and serial connections.
推導兩個電容串聯和并聯時總電容的公式。

(c) Find the capacitance between point-A and point-B of an infinitely long chain of capacitors as shown.
圖爲一個無限長電容鏈。求 A、B 點之間的電容。



Q2 (14 points)　　　題 2（14 分）

A small ball of mass M_1 is attached to one end of a light but rigid rod of length L. The other end of the rod is attached to a hinge O around which the rod can rotate freely in the vertical plane. The hinge is fixed on a rectangular block with mass M_2, which is on a smooth horizontal floor. An impulse is suddenly applied to the ball so that it acquires initial velocity v_0 to the right in the horizontal direction.



一個質量爲 M_1 的小球，繫在長度爲 L 的剛性細杆上，細杆的質量可以忽略，細杆的另一端繫在鉸鏈 O 點，細杆可繞 O 點在垂直平面內自由轉動，鉸鏈固定在一個質量爲 M_2 的長方形物塊上。物塊放在水平光滑地板上。現突然給小球一個衝量使它具有向右水平方向的初速度 v_0。

(a) Suppose the impulse is small enough that the ball remains near the bottom position. Find the position of the ball and the position of the block as a function of time. (7 points)
設衝量足够小，從而小球一直在底部附近運動，求小球和物塊之後任意時刻的位置。（7 分）

(b) Suppose the impulse is large enough that the ball can circle around the hinge. Find the force from the rod to the ball and the force from the block to the floor when the ball is (i) at the lowest position (ii) at the same height as the hinge; and (iii) at the highest position. (7 points)
設衝量足够大，使小球可以繞著鉸鏈做圓周運動，求當小球在下列位置時細杆對球的力和物塊對地板的力。(i) 在最低點。(ii) 在和鉸鏈同一高度的一點。(iii) 在最高點。（7 分）

Q3 (10 points)　　　題 3（10 分）

An electron has intrinsic angular momentum $\vec{S}$ called spin, and a permanent magnetic dipole moment $\vec{M} = -\frac{ge}{2m}\vec{S}$ associated with the spin, where e is the positive electron charge, m is the electron mass, and g is a number called g-factor. An electron with its spin aligned along its initial velocity in the x-direction enters a region of uniform magnetic field in the z-direction. Show that if g is exactly 2, then the spin is always in the same direction as the velocity of the electron. (Hints: (1) The relation between torque and angular momentum is the same as the relation between force and momentum. (2) Recall the way the acceleration of uniform circular motion is derived. (3) The torque to a dipole is $\vec{\tau} = \vec{M} \times \vec{B}$.)

電子具有固有的角動量 $\vec{S}$，稱爲自旋，和與自旋相關的磁偶極矩 $\vec{M} = -\frac{ge}{2m}\vec{S}$。其中 e 爲正電子電荷，m 爲電子質量，g 爲 g -因子。一電子初始速度和自旋都沿 X-方向，進入一均勻沿 Z-方向的磁場。證明若 g 剛好等于 2，則電子之後的速度和自旋始終在同一方向。(提示：(1)力矩與角動量的關係和力與動量的關係一樣。(2) 回顧一下勻速圓周運動的加速度的推導方法。(3) 磁偶極矩受到的力矩爲 $\vec{\tau} = \vec{M} \times \vec{B}$。)

Q4* (12 points) **題 4*（12 分）**

When the partial pressure of water vapor in air exceeds the saturated water vapor pressure (P_s) at a given temperature, the water vapor will condense into droplets which fall down as rain. P_s = 55.35 mmHg at 40°C, and P_s = 6.50 mmHg at 5 °C. The air + vapor mixture can be considered as ideal gas and the mass of a water molecule is approximately the same as an „air" molecule. In the humid air at sea level at 40°C the water vapor partial pressure is 90 % of P_s, and the pressure is P_0 = 1.0 atm.

(a) Find the air mass density ρ_0 at the sea level. (2 points)

The humid air then rises adiabatically to an altitude where the temperature is 5 °C. Ignore air pressure change due to the reduction of water vapor. For adiabatical processes of air, $PV^{7/5} = \text{Constant}$, where V is the volume, and the pressure at height H is $P = P_0(1-2\rho_0 gH/(7P_0))^{7/2}$, where g is the gravity acceleration near Earth surface.

(b) How much rain can one cubic meter of the humid air at sea level generate? (7 points)

(c) Find the altitude where the temperature is 5 °C. (3 point)

當空氣中水蒸汽的分壓強超過該溫度下的飽和水蒸汽壓(P_s)時，水蒸汽將凝聚成水滴導致下雨。已知 40°C 時 P_s = 55.35 mmHg，5 °C 時 P_s = 6.50 mmHg。空氣/水蒸汽的混合物可當作是理想氣體，水分子的質量近似等于‘空氣’分子的質量。海平面上 40°C 的潮濕空氣中，水蒸汽的分壓是 P_s的 90 %，氣壓 P_0 = 1 個大氣壓。

(a) 求海平面處空氣的質量密度 ρ_0。（2 分）

該潮濕空氣絕熱上升到某一高度，該處溫度爲 5 °C。忽略由于水蒸汽的減少導致的氣壓改變。空氣絕熱過程狀態方程爲 $PV^{7/5} =$ 常數，V 爲體積，在高度 H 的壓强爲 $P = P_0(1-2\rho_0 gH/(7P_0))^{7/2}$。

(b) 一立方米海平面上的潮濕空氣能够産生多少雨？(7 分)

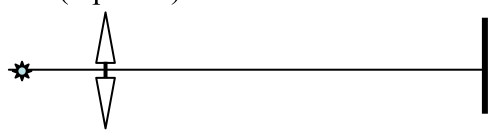
(c) 求溫度為 5 °C 處的高度。(3 分)

Q5 (14 points) **題 5（14 分）**

Refer to the figure which is not drawn to the correct scale. A positive lens of radius R = 1.0 cm and focal length f = 20 cm is cut in the middle. The upper part is lifted up by d = 1.0 mm and the lower part pulled down by the same distance. The gap in between is blocked by an opaque sheet. A point light source with wavelength $\lambda = 500$ nm is placed on the optical axis at 2f distance from the split lens. A large screen is placed at L = 1.0 m from the right focus point of the lens.

(a) Find the number of interference fringes on the screen. (7 points)

(b) The point source is pulled up by a small distance b from the optical axis. A second identical source is placed at a distance 2b below the first source. Find the minimum value of b to make all the fringes on the screen disappear. (7 points)



參考示意圖，一個正透鏡半徑爲 R = 1.0 cm，焦距爲 f = 20 cm，現將透鏡從中間割開，將上半部往上移 d = 1.0 mm，下半部往下移 d = 1.0 mm，兩個半透鏡之間的空隙由不透明物體封住。一個波長爲 $\lambda = 500$ nm 的點光源，放在光軸上離透鏡的距離 2f 處。一個大的屏幕放在離右焦點 L = 1.0 m 處。

(a) 求屏幕上干涉條紋的數目。(7 分)

(b) 將點光源向上移動一個小距離 b，另一個相同的光源放在第一個光源以下距離 2b 處，求能使屏幕上干涉條紋消失的 b 的最小值。(7 分)

《END 完》

Hong Kong Physics Olympiad 2011 (Senior Level)
2011 香港物理奧林匹克(高級組)
Suggested Solutions 答案及建議題解

Multiple Choice Questions

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
d	b	b	e	c	a	b	a	e	c	a	d	a	e	a	a	a	d	e	b

MC1: (d)

MC2: $F=-kx=(3m-\rho V)g$, $k=-\dfrac{dF}{dx}=\rho g\dfrac{dV}{dx}=\rho g\pi R^2$.

$\omega=\sqrt{k/M}=\sqrt{\rho g\pi R^2/(3m)}$.(b)

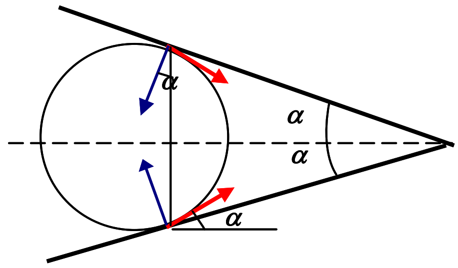
It can be further simplified to $\rho\dfrac{2}{3}\pi R^3=3m\Rightarrow\omega=\sqrt{\dfrac{3g}{2R}}$.

MC3: Using Archimedes' principle, volume of water displaced is $V_{\text{dis}}=(m_{\text{stone}}+\rho_{\text{ice}}V_{\text{ice}})/\rho_{\text{water}}$. Suppose a volume of ΔV_{ice} melts, then the volume of water displaced decreases by $\rho_{\text{ice}}\Delta V_{\text{ice}}/\rho_{\text{water}}$. At the same time, since mass is conserved, the volume of water increases by $\rho_{\text{ice}}\Delta V_{\text{ice}}/\rho_{\text{water}}$. Hence there is no change in the water level. However, when all ice melts and the stone sinks to the bottom, the volume of water displaced is $m_{\text{stone}}/\rho_{\text{stone}}<m_{\text{stone}}/\rho_{\text{water}}$. Hence the water level falls. **Answer: (b)**

MC4: Using Newton's law of universal gravitation and Newton's second law for circular orbits, $M\propto R^3/T^2$. Comparing with Earth's orbit, where M = 1 solar mass, R = 1 AU and T = 1 year,

$R=\sqrt[3]{1.5\left(\dfrac{98}{365}\right)^2}=0.48\ \text{AU}$. **Answer: (e)**

MC5: Denote the friction force and normal force acting on each blade as F_f and N, respectively. At the critical angle the static friction force should reach its maximal value which is $F=\mu N$.



The condition for the wire not to move is when all components of the forces adding up equal to zero. As shown in the force diagram acting on the wire, the vertical components cancel each other by symmetry, and the horizontal components cancel if

$2F_f\cos\alpha=2N\dfrac{\sin 2\alpha}{2}$.

Thus, from the condition of the critical angle we obtain:
$\mu=\tan\alpha$. **Answer: (c)**

MC6: (a)

MC7: Let v be the velocity of the particle of mass m. Kinetic energy of the compound pendulum:

$$K=\frac{1}{2}mv^2+\frac{1}{2}(2m)\left(\frac{v}{2}\right)^2=\frac{3}{4}mv^2$$

Potential energy of the compound pendulum:

$$U=-mgL\cos\theta-2mg\frac{L}{2}\cos\theta=-2mgL\cos\theta$$

For small oscillations, $U \approx -2mgL\left(1-\frac{\theta^2}{2}\right) = \text{cosntant} + mgL\theta^2$

For small oscillations, we can approximate the linear displacement by $x = L\theta$.

Hence $U = \frac{mg}{L}x^2$

Using the result of MC12, we can substitute $a = \frac{3}{2}m$, $b = \frac{2mg}{L}$ and $f = \frac{1}{2\pi}\sqrt{\frac{2mg/L}{3m/2}} = \frac{1}{2\pi}\sqrt{\frac{4g}{3L}}$.

Answer: (b)

MC8: Using Newton's second law, we find that the acceleration of the upward displacement is $-g(\sin\alpha + \mu\cos\alpha)$. Hence $t_1 = \frac{v_1}{g(\sin\alpha+\mu\cos\alpha)}$ and $L = \frac{v_1^2}{2g(\sin\alpha+\mu\cos\alpha)}$.

The acceleration of the downward displacement is $g(\sin\alpha - \mu\cos\alpha)$ (in the downward direction). Hence the time is given by $L = \frac{1}{2}g(\sin\alpha-\mu\cos\alpha)t_2^2$. Hence

$$t_2^2 = \frac{2L}{g(\sin\alpha-\mu\cos\alpha)} = \frac{v_1^2}{g^2(\sin\alpha-\mu\cos\alpha)(\sin\alpha+\mu\cos\alpha)} = t_1^2\frac{\sin\alpha+\mu\cos\alpha}{\sin\alpha-\mu\cos\alpha}$$

Answer: (a)

MC9: **Sol.** $mv_0 = (m+M)v$, $v = \frac{m}{m+M}v_0$=2m/s,

$$\Delta E_k = \frac{1}{2}mv_0^{\ 2} - \frac{1}{2}(m+M)v^2 = \frac{1}{2}\times1\times10^2 - \frac{1}{2}\times(1+4)\times2^2 = 40\text{ J}$$

Answer: (e)

MC10: **Sol.** 已知 m_1=1kg, m_2=3kg, F=6N, v_0 =2m/s, d =1m..

當兩物體速度相等時, 距離最小, 此時的速度為 v. 由動量守恆定律

$m_2v_0 = (m_1+m_2)v$, $v = \frac{m_2}{m_1+m_2}v_0 = \frac{3}{1+3}\times 2$ =1.5m/s.

或 距離最小時所用時間為 T, $v_1 = a_1T = v_0 + a_2T$, $6T = 2-2T$, $T = 0.25\text{s}$, $v = 1.5\text{m/s}$.

$\Delta E_k = \frac{1}{2}m_2v_0^{\ 2} - \frac{1}{2}(m_1+m_2)v^2 = \frac{1}{2}\times3\times2^2 - \frac{1}{2}(1+3)\times1.5^2$ =1.5J $= F\Delta s$, $\Delta s = 0.25\text{m}$.

兩物體間的最小距離= d–Δs= 0.75m. **Answer: (c)**

MC11: $\alpha = \tan^{-1}\left(\frac{2}{3}\tan\beta\right)$, (a)

MC12: $a > d > c > b$, (d)

MC13: $V(\rho_0 - \rho) = 100kg \Rightarrow \rho = 1.2 - 0.1 = 1.1\ kg/m^3$.

$T = T_0\rho_0/\rho = 293\cdot1.2/1.1 = 319.6K = 46.6\,°\text{C}$ (a)

MC14: Total reflection (e)

MC15: $E = \sqrt{15}mg/q$, (a)

MC16: $v = \frac{r^2Bq}{2mR}$, (a)

MC17: 0, (a)

MC18: (d)

MC19: $v = mgR/\left(B^2L^2N\right)$, (e)

MC20: By energy conservation, $Q_1 = Q_2 = mgH$. (b)

Open Questions

Q 1 (10 points):

(a) For parallel connection, the total current I is the sum of the currents I_1 and I_2 through the two resistors, and their voltage U is the same. So $R_p = \dfrac{U}{I_1 + I_2} = U / \left(\dfrac{U}{R_1} + \dfrac{U}{R_2} \right) = \dfrac{R_1 R_2}{R_1 + R_2}$, (1.5 points)

For serial connection, the current I through the resistors is the same, and the total voltage U across the two resistors is sum of the voltages across each resistor. So

$R_s = (U_1 + U_2)/I = R_1 + R_2$;(1.5 points)

(b) For parallel connection, the total charge Q is the sum of the charge Q_1 and Q_2 on the two capacitors, and the voltage U is the same for both capacitors. So $C_p = (Q_1 + Q_2)/U = C_1 + C_2$, (1.5 points)

For serial connection, the charge Q on each capacitor is the same, because in the section of circuit between the two capacitors the total charge should be zero. The total voltage U is the sum of voltage across each capacitor. So $C_s = \dfrac{Q}{U_1 + U_2} = Q / \left(\dfrac{Q}{C_1} + \dfrac{Q}{C_2} \right) = \dfrac{C_1 C_2}{C_1 + C_2}$.(1.5 points)

(c) As the chain is infinitely long, it does not matter if one section is taken away.

Therefore $\dfrac{1}{C^{\mathrm{T}}} = \dfrac{1}{2C + C^{\mathrm{T}}} + \dfrac{1}{C}$, (3 points)

Solving the equation one gets $C^{\mathrm{T}} = (\sqrt{3} - 1)C$.(1 point)

Q 2 (14 points):

(a)

Note that for the whole system the net force in the horizontal direction is zero. The impulse will give a total momentum to the system and its center of mass will move at constant speed $\dfrac{m_1 v_0}{m_1 + m_2}$, while the block and the ball will move back and forth relative to the center of mass.

Two ways to determine the vibration frequency are given below.

Method 1: (4 points)

Take the following coordinate system.

x_1 is the coordinate of the ball relative to the block. So $x_1 = L\theta$.

x_2 is the coordinate of the block relative to the floor.

$$\begin{cases} -m_1 g\theta = m_1(\ddot{x}_1 + \ddot{x}_2), \text{ (Newton's equation for the ball)} \\ m_1(\ddot{x}_1 + \ddot{x}_2) + m_2 \ddot{x}_2 = 0, \text{ (zero acceleration for center of mass.)} \end{cases},$$

Solving the two equations, $\ddot{x}_2 = -\dfrac{m_1}{m_1 + m_2}\ddot{x}_1 = -\dfrac{m_1}{m_1 + m_2} L\ddot{\theta}$

$$-g\theta = \left(1 - \frac{m_1}{m_1 + m_2}\right)\ddot{x}_1 = \frac{m_2}{m_1 + m_2} L\ddot{\theta} \Rightarrow \omega = \sqrt{\frac{g(m_1 + m_2)}{m_2 L}}.$$

Method 2: (4 points)

Both coordinates are relative to the floor.

$$\begin{cases} x_1 - x_2 = L\theta \\ m_1 x_1 + m_2 x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \dfrac{m_2 L\theta}{m_1 + m_2} \\ x_2 = -\dfrac{m_1 L\theta}{m_1 + m_2} \end{cases},$$

$$E_k = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 = \frac{m_1 m_2 L^2 \dot{\theta}^2}{2(m_1 + m_2)},$$

$U = \frac{1}{2} m_1 g L \theta^2$.

Thus $\omega = \sqrt{\frac{g\left(m_1 + m_2\right)}{m_2 l}}$.

We now proceed to get the equations of motion of the ball and the block.
The general solution is $\theta(t) = A\sin(\omega t) + B\cos(\omega t)$.
Consider the initial conditions $\theta(0) = 0, \quad \dot{\theta}(0) = v_0 / L$, thus

$\theta(t) = \frac{v_0}{\omega L}\sin(\omega t) = v_0\sqrt{\frac{m_2}{Lg\left(m_1 + m_2\right)}}\sin(\omega t)$, and

$x_1 = \frac{m_2 v_0}{m_1 + m_2}\sqrt{\frac{L m_2}{g\left(m_1 + m_2\right)}}\sin(\omega t)$,

or, in the reference system of the floor,

$$\begin{cases} x_1 = \frac{m_2 v_0}{m_1 + m_2}\sqrt{\frac{L m_2}{g\left(m_1 + m_2\right)}}\sin(\omega t) + \frac{m_1 v_0}{m_1 + m_2} t \\ x_2 = -\frac{m_1 v_0}{m_1 + m_2}\sqrt{\frac{L m_2}{g\left(m_1 + m_2\right)}}\sin(\omega t) + \frac{m_1 v_0}{m_1 + m_2} t \end{cases}$$. (3 points)

(b) Throughout the process, momentum and energy are always conserved. (7 points)
Take all coordinates to be relative to the floor. As the rod mass is ignored, the force of rod on the ball is always along the rod so it does not produce a torque to the rod. One can therefore replace the rod by a massless thread, or assuming the ball is moving on a smooth circular rail centered at the hinge.

$$\begin{cases} m_1 v_1 + m_2 v_2 = m_1 v_0 \\ \frac{1}{2} m_1 \left(v_1^2 + v_y^2\right) + \frac{1}{2} m_2 v_2^2 + mgh = \frac{1}{2} m_1 v_0^2 \end{cases}$$

(i) $v_y = 0, \quad h = 0 \Rightarrow \begin{cases} v_1 = v_0 \\ v_2 = 0 \end{cases}$, and $F = m_1 v_0^2 / L$, $T = F + m_1 g = m_1\left(\frac{v_0^2}{L} + g\right)$, and The force of the floor on the block is $N = m_2 g + T = \left(m_1 + m_2\right)g + m_1 v_0^2 / L$.(2 points)

(ii) $h = L, \quad v_1 = v_2 = v_x \Rightarrow \begin{cases} v_x = \frac{m_1 v_0}{m_1 + m_2} \\ v_y = \sqrt{\frac{m_2 v_0^2}{m_1 + m_2} - 2gL} \end{cases}$,

$T = F = \frac{m_1 v_y^2}{L} = \frac{m_1 m_2 v_0^2}{(m_1 + m_2)L} - 2m_1 g$

$N = (m_1 + m_2)g$ (2 points)

(iii) $v_y = 0, \quad h = 2L \Rightarrow \begin{cases} v_1 = \frac{m_1^2 v_0 - \sqrt{m_1^2 m_2\left(m_2 v_0^2 - 4\left(m_1 + m_2\right)gL\right)}}{m_1\left(m_1 + m_2\right)} \\ v_2 = \frac{m_1 m_2 v_0 - \sqrt{m_1^2 m_2\left(m_2 v_0^2 - 4\left(m_1 + m_2\right)gL\right)}}{m_2\left(m_1 + m_2\right)} \end{cases}$,

$$T = F - m_1 g = m_1\left(\frac{(v_1 - v_2)^2}{L} - g\right) = m_1\left(\frac{v_0^2}{L} - \left(\frac{4m_1}{m_2} + 5\right)g\right)$$

$N = m_2 g - T = (m_1 + m_2)(4m_1 + m_2)g / m_2 - m_1 v_0^2 / L$.(2 points)

Q 3: (10 points)

Method-1

$\vec{\tau} = \vec{M} \times \vec{B}$, and $\vec{F} = e\vec{v} \times \vec{B}$.

Maintaining the relative direction between spin and velocity requires simply that

$\tau / I = F / P$

$\frac{geB}{2m} = \frac{evB}{mv} = \frac{eB}{m}$, so $g = 2$.

Method-2

The electron will move in a circle in the magnetic field. We need to find the period of such circular motion first. Let the radius of the circle be *R*. Then $evB = mv^2 / R \Rightarrow v = eBR / m$. The period is $T_e = 2\pi R / v = \frac{2\pi mR}{eBR} = \frac{2\pi m}{eB}$.

For the electron spin $\vec{S}$, the torque is always perpendicular to the spin, so the spin will rotate. Suppose the spin rotate by a small angle $\Delta\theta$ over a short time period Δt, the change of angular momentum is $\Delta S = S\Delta\theta$, similar to circular motion where the change of velocity is $\Delta v = v\Delta\theta$. The angular speed of the spin rotation is then $\omega = \Delta\theta / \Delta t$.

Using the equation for torque and angular momentum, $\frac{geSB}{2m} = \tau = \frac{\Delta S}{\Delta t} = \frac{S\Delta\theta}{\Delta t} = \omega S$.

The rotation period of the spin is $T_s = \frac{2\pi}{\omega} = \frac{4\pi m}{geB}$.

Finally, $T_s = T_e \Rightarrow g = 2$.

Q4 (12 points)

(a) At the same pressure, $T\rho = \text{Constant}$, (1 point)

The density at 40°C is $\rho_0 = 1.2 \times \frac{293}{313} = 1.12 \text{ kg/m}^3$ (1 point)

(b) The fraction of water vapor at 40°C at sea level $\eta_1 = \frac{55.35}{760} \times 90\%$ (2 points)

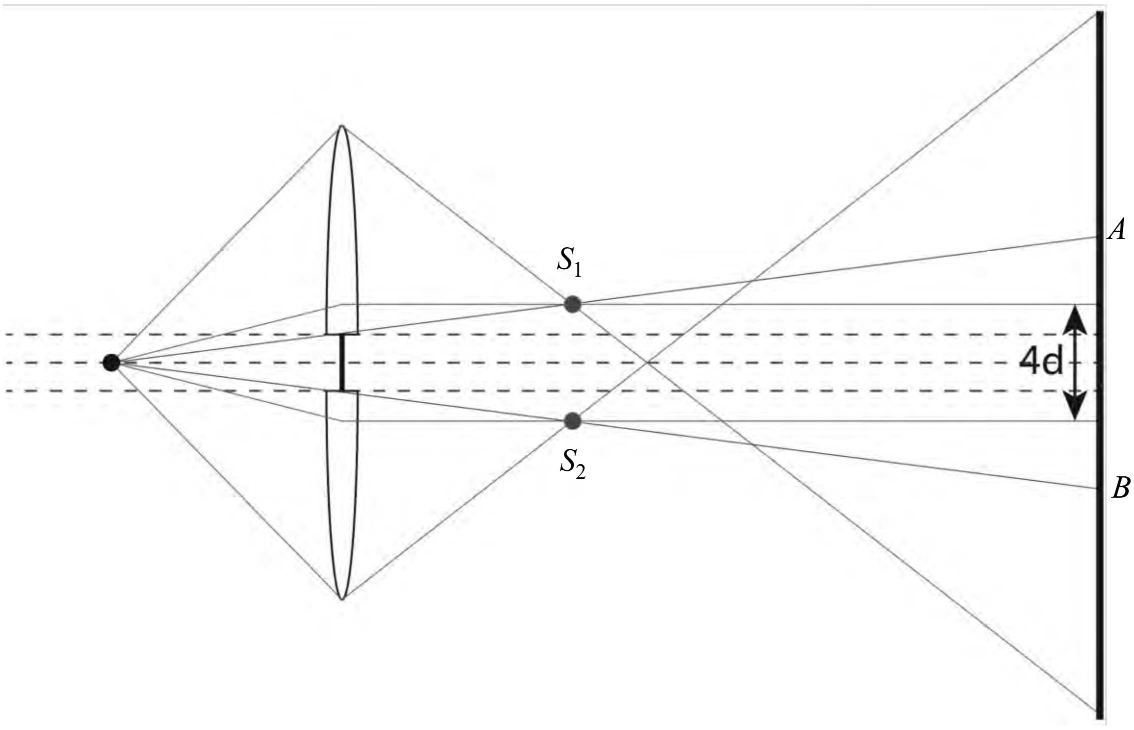
For adiabatic process, $PV^{7/5} =$ constant, and for ideal gas *PV* = *nkT*, so $P = CT^{7/2}$

The fraction of water at 5°C at high altitude $\eta_2 = \frac{6.5}{760(\frac{278}{313})^{\frac{7}{2}}}$ (2 points)

Rain = $(\eta_1 - \eta_2) \times \rho \times V = (\frac{55.35}{760} \times 90\% - \frac{6.5}{760(\frac{278}{313})^{\frac{7}{2}}}) \times 1.12 = (0.0665 - 0.013) \times 1.12 = 60g$ (3 points)

(c) Height： $278 = 313(1 - \frac{2 \times 1.12 \times 9.8 \times h}{7 \times 1.03 \times 10^5}) \Rightarrow h = 3673m$ (3 points)

Q 5: (14 points)



(a) (7 points)

Using the lens formula, we know that there are two images S_1 and S_2 at distance 2f from the split lens and their distance is $4d$. (1 point) The two images are equivalent to the two small holes illuminated by a point source in a typical Young's interference experiment. Their interference effect produces the fringes on the screen.

$\tilde{L} = L - f$. Let ξ be the distance of the observation position on the screen to the optical axis.

$$\delta_1 = \frac{1}{\lambda}\sqrt{(\xi - 2d)^2 + \tilde{L}^2} = \frac{\tilde{L}}{\lambda}\sqrt{\left(\frac{\xi - 2d}{\tilde{L}}\right)^2 + 1} \approx \frac{\tilde{L}}{\lambda}\left(1 + \left(\frac{\xi - 2d}{\tilde{L}}\right)^2\right),$$

$\delta_2 = \frac{\tilde{L}}{\lambda}\left(1 + \left(\frac{\xi + 2d}{\tilde{L}}\right)^2\right)$. And then optical path difference is $\Delta = |\delta_1 - \delta_2| = \frac{4d}{\tilde{L}}\xi$. (2 points)

The width of the fringe $\Delta\xi$ is then

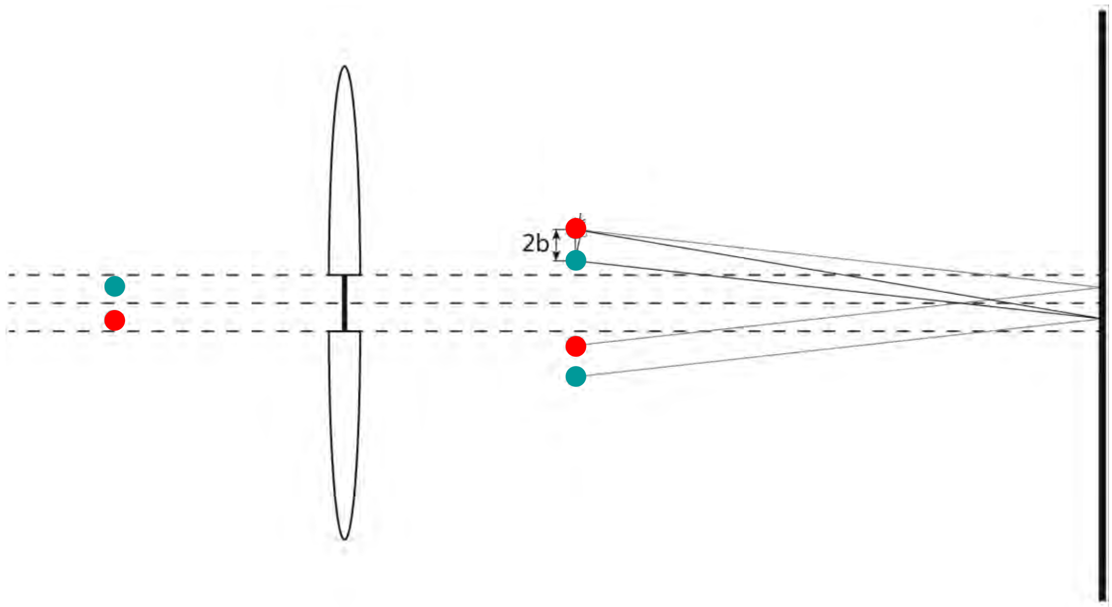
$$\frac{4d}{\tilde{L}}\Delta\xi = \lambda \Rightarrow \Delta\xi = \frac{\lambda\tilde{L}}{4d} = \frac{500\times 10^{9}\times(1-0.2)}{4\times 10^{-3}} = 1.0\times 10^{-4}\ \text{m} = 0.1\ \text{mm}.$$

Now we need to know the width of the area where interference occurs, or where the light from S_1 overlap with the light from S_2. From the figure we see that the area is between point-A and point-B. By simple geometry, we get the distance between point-A and point-B to be

$D = (3f + L)\frac{d}{f} = 1600\times 1/200 = 8\,\text{mm}$ (1 point)

So $\text{Number} = \frac{D}{\Delta\xi} = \frac{8}{0.1} = 80$. (2 points)

(b)



Each source (red or green) will form a pair of images, just like in (a). Each pair forms its own set of interference fringes on the screen. When the red set bright fringes overlap with the dark fringes of the green pair, then the light intensity on the screen is uniform and no fringes can be observed. This means that at any point on the screen, the optical path difference from the red pair should differ from that from the green pair by at least half the wavelength.

The distance between each pair of interfering images is still $4d$. The center position of the red pair from the optical axis is $((2d+b)-(2d-b))/2 = b$. At a position above the optical axis by ξ, the optical path difference is $\Delta_R = \frac{4d}{\tilde{L}}(\xi - b)$.

Likewise, the center position of the green pair from the optical axis is $-b$. At the same position above the optical axis by ξ the optical path difference is $\Delta_G = \frac{4d}{\tilde{L}}(\xi + b)$ (2 points)

For fringes to disappear, we must have $\frac{8db}{\tilde{L}} = \Delta_G - \Delta_R = \frac{\lambda}{2}$.

Finally, $b = \frac{\lambda\tilde{L}}{16d} = \frac{5\times10^{-4}\times800}{16} = 0.025mm$. (3 points)

Hong Kong Physics Olympiad 2012
2012 年香港物理奧林匹克競賽

(Junior Level 初級組)

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
香港特區政府教育局
The Physical Society of Hong Kong
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

March 17, 2012
2012 年 3 月 17 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID Number in the field of “I. D. No.”, and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of “Course & Section No.”

   請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 “I. D. No.” 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格填上你的英文姓名，最後於 “Course & Section No. ” 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of “Course Title”, and write your English name in the field of “Student Name” and your 8-dight Participant I. D. Number in the field of “Student Number”. You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別注明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m/s^2
Gravitational constant 萬有引力常數	G	6.67×10^{-11} N m^2/kg^2
Radius of Earth 地球半徑	R_E	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	r_E	1.5×10^{11} m
Mass of Sun 太陽質量	M_{Sun}	1.99×10^{30} kg
Mass of Earth 地球質量	M_E	5.98×10^{24} kg
Air Density 空氣密度	ρ_0	1.2 kg/m^3
Water Density 水密度	ρ_w	1.0×10^3 kg/m^3
Standard atmosphere pressure 標準大氣壓	p_0	1.013×10^5 N/m^2
Charge of an electron 電子電荷	e	1.6×10^{-19} C
Permittivity constant 真空電容率	ε_0	8.85×10^{-12} C/(V m)
Electron mass 電子質量	m_e	9.11×10^{-31} kg
Speed of light in vacuum 真空光速	c	3.0×10^8 m/s

Trigonometric identities:
三角學恆等式:

$$\sin(x+y) = \sin x\cos y + \cos x\sin y \qquad \cos(x+y) = \cos x\cos y - \sin x\sin y$$

$$\sin 2x = 2\sin x\cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$\sin x\cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right] \qquad \cos x\cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

$$\sin x\sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

The following conditions will be applied to all questions unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除特別註明外，下列條件將適用于本卷所有問題：

1) 所有物體都處于地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每題 2 分，每題選擇一個答案。）

The MC questions with the '*' sign may require information on page-3.
帶 * 的選擇題可能需要用到第三頁上的資料。

1. As shown in Fig. 1, a boy is riding on a bus. The bus moves with a uniform speed of 20 km/h in a horizontal circle, and the traffic light is located at the center of the circle. What is the velocity of the traffic light relative to the boy?
A) 20 km/h in the forward direction of the bus
B) 20 km/h in the backward direction of the bus
C) 20 km/h perpendicular to the forward direction of the bus and directed away from the bus
D) 20 km/h perpendicular to the forward direction of the bus and directed towards the bus
E) 0 km/h

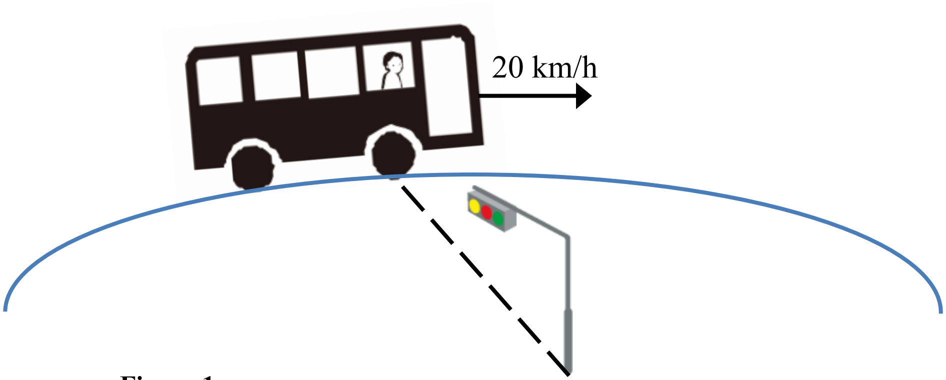


Figure 1

如圖 1 所示，一名男孩正在乘坐巴士，巴士以均速 20 km/h 繞著交通燈作水平圓周運動，交通燈位於圓圈的中心。交通燈相對於男孩的速度是多少？
A) 20 km/h 與巴士前進順向
B) 20 km/h 與巴士前進逆向
C) 20 km/h 與巴士前進方向垂直並遠離巴士
D) 20 km/h 與巴士前進方向垂直並趨近巴士
E) 0 km/h

2. A tennis ball machine is installed on the bus in Fig. 1. It projects tennis balls at a speed of 100 km/h in the direction perpendicular to the forward direction of the bus, and on the outward side of the circular path. The trajectories of the tennis balls observed by the boy are
A) straight lines perpendicular to the forward direction of the bus
B) straight lines slightly inclined towards the forward direction of the bus
C) straight lines slightly inclined towards the backward direction of the bus
D) curves slightly inclined towards the forward direction of the bus
E) curves slightly inclined towards the backward direction of the bus
圖 1 的巴士裝有一台網球發射器。發射器以時速 100 km/h 沿著與巴士垂直的方向，向圓形路徑外邊發射網球。男孩觀察到的網球軌跡是
A) 與巴士前進方向垂直的直線
B) 比巴士前進方向略爲前傾的直線
C) 比巴士前進方向略爲後傾的直線
D) 比巴士前進方向略爲前傾的曲線
E) 比巴士前進方向略爲後傾的曲線

3. A man sits in the back of a canoe in still water. He then moves to the front of the canoe and sits there. Neglecting the damping of water, the final position and the motion of the canoe is:
A) forward of its original position and moving forward
B) forward of its original position and moving backward
C) rearward of its original position and moving forward
D) rearward of its original position and moving backward
E) rearward of its original position and not moving
在靜止的水中，船夫坐在獨木舟的後方。他隨後向前移至獨木舟的前方然後坐下。若不計水的阻尼，獨木舟最終的位置和運動狀態是：
A) 比原來位置前移並向前動 B) 比原來位置前移並向後動
C) 比原來位置後移並向前動 D) 比原來位置後移並向後動
E) 比原來位置後移並不動

4. A boat is about to cross a river to the opposite bank. The river water flows at a speed of 4 km/h. If the speed of the boat is 3 km/h, what should be the angle between the boat velocity and the upstream direction of the river, so that the downstream displacement is minimum when the boat reaches the opposite bank?
一艘船正要橫渡一道河往對岸。河水流速爲 4 km/h。若船速爲 3 km/h，那麼船速應與河的上游方向成什麼方角度，才可以在抵達對岸時，令其向下游的位移爲最小？
A) 0^o B) 37^o C) 41^o D) 53^o E) 90^o

5. *In a talent show, a juggler juggles 4 balls simultaneously, as shown in Fig. 2. A spectator uses his high speed video tape and determines that it takes the juggler 0.9 s to cycle each ball through his hands (including catching, transferring and throwing) and to be ready to catch the next ball. It is noted that at most one ball must be in a hand of the juggler in each cycle of juggling. What is the minimum vertical speed the juggler must throw up each ball?
在天才表演中，一藝人同時要弄 4 個球，如圖 2 所示。有觀眾以高速攝錄機拍下過程，發現藝人需用 0.9 s 經手每個球(包括接球，傳球和拋球)，然後從新準備接下一球。他還留意到在要球的每一循環中，藝人的每隻手最多只有一個球。問藝人拋球的最低垂直速率是多少？


Figure 2

A) 9.3 m/s B) 11.4 m/s C) 12.8 m/s D) 13.2 m/s E) 17.6 m/s

6. As shown in Fig. 3, the block-spring system is in equilibrium provided that the left spring is stretched by x_1. The whole system rests on a smooth supporting surface. The coefficient of static friction between the blocks is μ_s, and the blocks have equal mass m. What is the maximum amplitude of the oscillations of the system such that the top block does not slide on the bottom one?

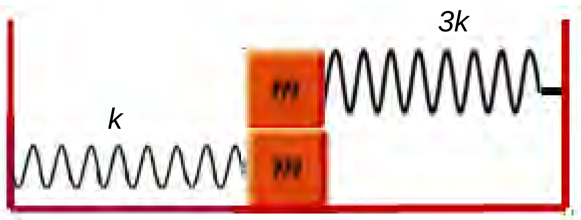


Figure 3

如圖 2 所示，一個方塊-彈簧系統處在平衡態中，其中左彈簧伸展長度爲 x_1。全系統安放在光滑面上。方塊之間的靜摩擦係數爲μ_s，方塊的質量均爲 m。問系統振動可達到的最大幅度，而上下方塊之間仍能保持不滑動。

A) $\frac{\mu_s mg}{2k}+x_1$ B) $4kx_1-\mu_s mg$ C) $\frac{\mu_s mg}{k}-x_1$ D) $2(2kx_1+\mu_s mg)$ E) $2(2kx_1-\mu_s mg)$

7. *An object is projected up an inclined plane with an initial speed of v_0 = 10 m/s, as shown in Fig. 4. The angle of the incline is θ = 30^o above the horizontal direction and the coefficient of the sliding friction μ_k = 0.1, determine the total time for the object to return to the point of projection.
一物體沿斜面以初速 v_0 = 10 m/s 射出，如圖 4 所示。斜面的水平傾角爲 θ = 30^o，動摩擦係數爲 μ_k = 0.1，求物體回到原點的總時間。
A) 3.81 s B) 4.26 s C) 4.54 s D) 4.94 s E) 5.32 s

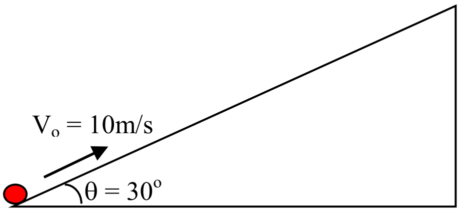


Figure 4

8. *As shown Fig. 5, a train with a length of L = 500 m moves by its inertia through the horizontal section of a railroad. However, the train encounters a small hill that slopes gently. With what minimum speed v can the train cross the hill? The base of hill has a length ℓ = 100 m, the lengths of the slopes are ℓ_1 = 80 m and ℓ_2 = 60 m. The slopes of hill can be considered as straight lines and the small section of rounding at the top of the hill can be ignored. Neglect any friction.
如圖 5 所示，一列長度爲 L = 500 m 的火車，以慣性在水平軌道上滑行，然後遇到一個小山丘。問火車需以什麼最低速度 v 越過山丘？山丘底長 l = 100 m，兩邊斜坡長 l_1 = 80 m 和 l_2 = 60 m。可以考慮斜坡爲直線，坡頂彎曲的一小段可以忽略。可以忽略摩擦力。
A) 9.6 m/s B) 11.5 m/s C) 13.2 m/s D) 15.0 m/s E) 16.2 m/s

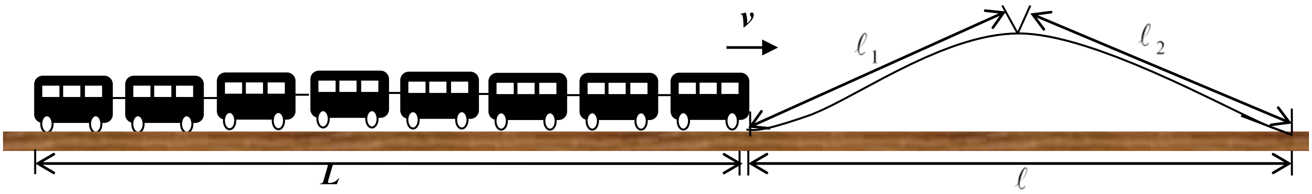


Figure 5

9. It is given the mass of Earth is a times that of Moon, and the radius of Earth is b times that of Moon. The period of a simple pendulum is T. When it is carried to Moon, the period of the simple pendulum becomes
已知地球的質量是月球的 a 倍，地球半徑是月球的 b 倍。現有一個單擺，在地球上的週期爲 T，若將它移到月球上，則單擺的週期等於
A) $\frac{a}{\sqrt{b}}T$ B) $\frac{\sqrt{b}}{a}T$ C) $\frac{\sqrt{a}}{b}T$ D) $\frac{b}{\sqrt{a}}T$ E) $\sqrt{\frac{b}{a}}T$

10. There exist some triple star systems in the universe. They are more distant from other stars, and are composed of three stars of equal mass M. The gravitational forces due to other stars can be neglected. A basic stable structure of triple star systems consists of three collinear stars, with two companion stars moving around a central star on a circular orbit with radius R. The linear velocity of the companion stars is $v_1 = k_1\sqrt{GM/R}$, where k_1 =
宇宙中存在一些離其他恒星較遠的、由質量 M 相等的三顆星組成的三星系統，通常可忽略其他星體對它們的引力作用。已觀測到穩定的三星系統存在的一種基本構成形式，是三顆星位於同一直線上，兩顆伴星圍繞中央星在半徑爲 R 的圓軌道上運行。伴星運動的線速度 $v_1 = k_1\sqrt{GM/R}$ ，式中 k_1 =
A) $\sqrt{10}$ B) $\sqrt{5}$ C) $\sqrt{5/2}$ D) $\sqrt{5}/2$ E) $\sqrt[3]{5/2}$

《END OF MC's 選擇題完》

Open Problems 開放題

Total 6 problems 共 6 題

The Open Problem(s) with the '*' sign may require information on page 3.
帶 * 的開放題可能需要用到第三頁上的資料。

Q1* (10 points) 題1*（10分）

The helicopter has a mass m and maintains its height by imparting a downward momentum to a column of air defined by the slipstream boundary as shown in Fig. 6. The propeller blades can project a downward air speed v, where the pressure in the stream below the blades is atmospheric and the radius of the circular cross-section of the slipstream is r. Neglect any rotational energy of the air, the temperature rise due to air friction and any change in air density ρ.

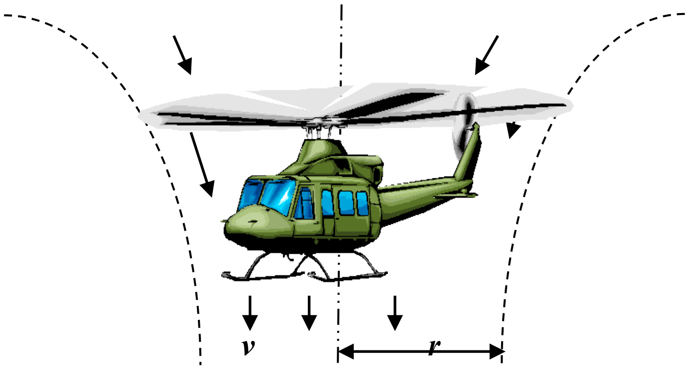


Figure 6

(a) Determine the power P required of the engine.
(b) If the power is doubled, calculate the acceleration of the helicopter.
直升機的質量爲 m，它傳輸下向的動量予空氣柱，用以維持其高度。氣流的邊界如圖 6 所示。它的引擎業片能鼓動空氣向下流，速度達到 v。氣流中的壓強爲大氣壓強，氣流的切面爲圓形，半徑爲 r。可以忽略空氣的轉動能、空氣摩擦引起的升溫、和空氣密度的改變。
(a) 試推導引擎所需的功率 P。
(b) 如果把功率增至兩倍，試計算直升機的加速度。

Q2* (10 points) 題 2*（10 分）

As shown in Fig. 7, in a ski jumping competition on a slope with inclination angle $\theta = 60^{o}$, an althlete jumps at point O with initial speed $v_0 = 25$ m/sand lands at point-A.
(a) Find the optimum jumping angle α so that the distance OA is maximum. (You may need to use trigonometric identities listed in page 3.)
(b) Find the maximum distance OA.
如圖 7 所示，在傾角$\theta = 60^0$的雪坡上舉行跳台滑雪比賽。運動員在起跳點 O 以速率 $v_0 = 25$ m/s、仰角α起跳，最後落在斜坡上 A 點。取 O、A 兩點的距離 L 爲比賽成绩。
(a)求運動員可以跳得最遠距離 OA 的起跳角α。(你或許會用到第 3 頁的三角學恆等式。)
(b) 求最遠距離 OA。

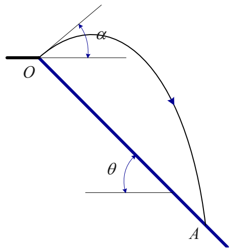


Figure 7

Q3* (15 points) 題 3*（15 分）

On a smooth horizontal surface shown in Fig. 8, there is a rectangular board AB with mass $2m$ and an arc-shaped structure BC with mass $3m$. The coefficient of kinetic friction of the upper surface of the rectangular board is $\mu = 0.35$, and the surface of the arc-shaped structure BC is smooth. The arc BC subtends a right angle with radius R. The two objects touch at point B.
(a) A small object of mass m moves from A to the right with an initial velocity $v_0 = 10$ m/s on the upper surface of the rectangular block. When it reaches point B, the velocity is $v = 5$ m/s. Calculate the velocity v_L of the rectangular board AB at this instant.
(b) The small object continues to move onto the arc-shaped surface BC. The rectangular board AB loses contact with the arc-shaped structure BC, and the small object continues to to move along the arc

surface, and finally just reaches the highest point C of the arc BC. Calculate the velocity of the arc-shaped structure v_R.
(c) Calculate the length L and the radius R.
質量爲 $2m$ 的長方體 AB 和質量爲 $3m$ 的圓弧體 BC 靜止在圖 8 中的光滑水平面上，長方體上表面的動摩擦系數爲$\mu = 0.35$，圓弧體 BC 表面則爲光滑圓弧。圓弧 BC 的夾角爲直角，半徑爲 R。兩物體於 B 點接觸。
(a) 質量爲 m 的小滑塊在 A 點以初速度 $v_0 = 10$ m/s 沿長方體方板上表面向右滑動，滑過 B 點的速率爲 $v = 5$m/s。試求此刻長方體 AB 的速率 v_L。
(b) 滑塊然後滑到圓弧體表面 BC 上，長方體 AB 和圓弧體 BC 失去接觸，滑塊則繼續在圓弧表面上滑動，最終恰好能滑到圓弧 BC 上的最高點 C。試求此刻圓弧體 BC 的速率 v_R。
(c) 試求長度 L 和圓弧半徑 R。

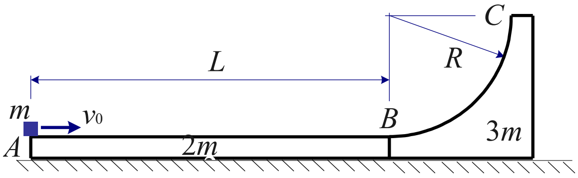


Figure 8

Q4* (10 points) 題 4*（10 分）

Geosynchronous satellites have the same period T as the Earth's rotation. They are at such a height above the earth's surface h that they remain always above the same spot. Suppose a geosynchronous solar satellite, as shown in Fig. 9a, sends radio signal directly to receivers on earth.

Figure 9a

When the satellite moves to the rear part of the earth (Fig. 9b), the light source is completely blocked by Earth in the shadow region (commonly known as an umbra). The length of the umbra is usually n times ($n \approx 200$) of Earth's radius R_E.
(a) Determine the height h. Express your answer in units of R_E.
(b) Determine the duration in each day that the satellite cannot receive sunlight. Express your answer in minutes.

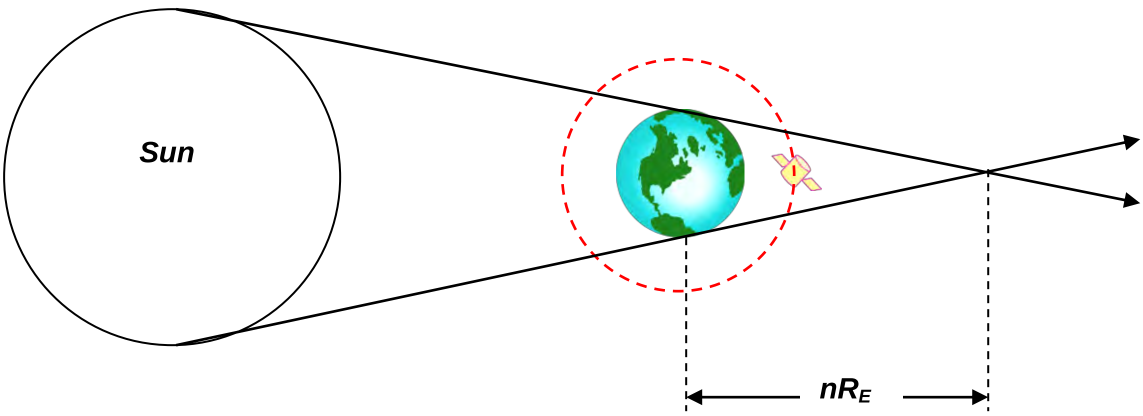


Figure 9b

地球同步衛星的軌道周期 T，與地球自轉週期相同。它們處於離地面高度 h 的位置，使它們經常位於同一地點上空。如圖 9a 所示，設有一地球同步的太陽能衛星，直接傳送無線電信號給地球上的接收站。
當衛星航行到地球背面(圖 9b)，光源被地球完全遮擋形成全影區。全影區的長度通常爲地球半徑 R_E的 n 倍($n \approx 200$)。
(a) 試計算高度 h，以 R_E 爲單位表達結果。
(b) 試計算衛星每日不能接收陽光的時間，以分鐘爲單位表達結果。

Q5* (20 points)　　題 5*（20 分）

Let r_E = 1.00 AU be the circular orbtal radius of Earth around Sun, and r_V = 0.72 AU be the circular orbital radius of Venus around Sun. To launch a space probe from Earth to Venus, the space probe first enters Earth's orbit and moves to a position sufficiently remote from Earth, so that the gravitational attraction of Earth is negligible when compared with the gravitational attraction of Sun. Then, the following manouvers as shown in Fig. 10 are performed:
(1) The kinetic energy of the space probe is reduced by $\Delta K = K(r_E - r_V)/(r_E + r_V)$, where K is the kinetic energy of the space probe at that instant. This is done by switching on the engine for a short time and then switching it off. The space probe then enters a transfer orbit around the Sun. The transfer orbit is tangential to Earth's orbit at its near end and to Venus's orbit at its far end.
(2) When the space probe touches Venus's orbit, the space probe is traveling too fast for it to land on Venus. For the second time, the kinetic energy is reduced by switching on the engine for a short time and then switching it off. The space probe then travels at the orbital velocity of Venus.

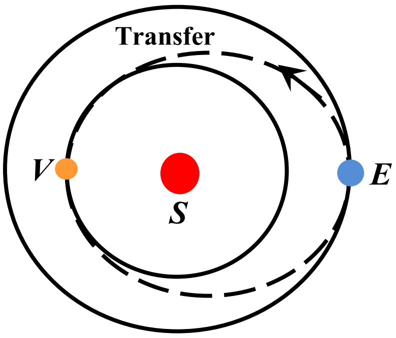


Figure 10

(a) Calculate the orbital speed of Earth and Venus. Express your answer in km/s.
(b) Calculate the speed (in km/s) of the space probe in the transfer orbit at point E.
(c) Calculate the speed (in km/s) of the space probe in the transfer orbit at point V.
(d) Calculate the fractional reduction of the kinetic energy of the space probe at point V.
設 r_E = 1.00 AU 爲地球環繞太陽圓形軌道的半徑，r_V = 0.72 AU 爲金星環繞太陽圓形軌道的半徑。發射一個太空探測器從地球到金星，可以先將探測器放進地球軌跡，令它處於離地球較遠的位置，以致地球的引力相對於太陽引力，可以忽略不計。然後對探測器進行如圖 11 的操作：
(1) 把探測器的動能降低 $\Delta K = K(r_E - r_V)/(r_E + r_V)$，其中 K 爲探測器在該時刻的動能，方法是開動引擎一段短時間，然後關掉引擎。探測器因而進入環繞太陽的轉移軌道。這轉移軌道在近端與地球軌道相切，在遠端則與金星軌道相切。
(2) 當探測器切入金星軌道，它的高速令它難以降落金星表面。它需要第二次開動引擎一段短時間，然後關掉引擎。探測器因而降低動能，以金星軌道速率航行。
(a) 試計算地球和金星在軌道上的速率，以 km/s 爲單位表達結果。
(b) 試計算探測器在轉移軌道 E 點的速率，以 km/s 爲單位表達結果。
(c) 試計算探測器在轉移軌道 V 點的速率，以 km/s 爲單位表達結果。
(d) 探測器在轉移軌道 V 點時，需降低的動能爲該時刻動能的什麼比例?

Q6 (15 points) **題 6（15 分）**

As shown in Fig. 11, a pendulum consists of a massive cubic block with side length b and density ρ_b hung by a light rigid rod with length $L >> b$. The hinge connecting the rod and the block maintains the block at the same orientation when it swings. The block is partially immersed in a liquid with density ρ_l, with ρ_l greater than ρ_b. The immersion depth of the block in the liquid is c.

(a) Calculate the period of the pendulum for small oscillation angles ϕ when c is sufficiently small. You may assume that the damping of the liquid during oscillations is negligible, and the change of the immersion depth during oscillations is negligible.

(b) When liquid is added, the liquid level rises, and the value of c reaches a threshold value c^*, above which the pendulum rod is no longer vertical. What is the espression of c^*?

(c) When c is above c^*, the pendulum oscillates about a non-zero angle ϕ. Calculate the period of the pendulum for small oscillations when the liquid level has risen to one that corresponds to $\phi = 30^o$.

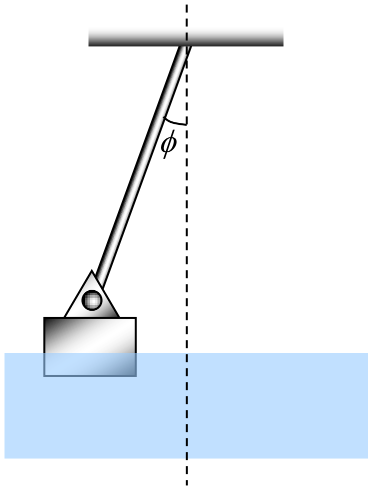


Figure 11

如圖 11 所示，在一個鐘擺中，一根無質量的剛體長桿懸掛著一個有質量的立方體。立方體邊長爲 b，密度爲 ρ_b 。長桿長度爲 $L >> b$。連接長桿和立方體的樞紐，在鐘擺運動時，把立方體維持在同一方向。立方體部分浸在液體中，液體密度爲 ρ_l ，數值比 ρ_b 大。立方體浸在液體中的深度爲 c。

(a) 試計算當 c 充分小的時候，鐘擺作小角度振動的周期。可以假設在振動時，液體的阻尼可以忽略，也可以假設在振動時，立方體浸在液體中的深度改變可以忽略。

(b) 當液體增加時，液體表面上升，c 的數值達到閾值 c^*。在閾值以上，鐘擺的長桿不再垂直。c^*的表達式爲何？

(c) 當 c 在 c^*以上時，鐘擺環繞非零值的ϕ振動。試計算當液面升至$\phi = 30^o$的位置時，鐘擺作小角度振動的周期。

《END 完》

Hong Kong Physics Olympiad 2012
2012 年香港物理奧林匹克競賽

(Junior Level 初級組)

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
香港特區政府教育局
The Physical Society of Hong Kong
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

March 17, 2012
2012 年 3 月 17 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將于比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID Number in the field of "I. D. No.", and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of "Course & Section No."

   請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 "I. D. No." 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格填上你的英文姓名，最後於 "Course & Section No. " 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant I. D. Number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別注明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m/s^2
Gravitational constant 萬有引力常數	G	6.67×10^{-11} N m^2/kg^2
Radius of Earth 地球半徑	R_E	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	r_E	1.5×10^{11} m
Mass of Sun 太陽質量	M_{Sun}	1.99×10^{30} kg
Mass of Earth 地球質量	M_E	5.98×10^{24} kg
Air Density 空氣密度	ρ_0	1.2 kg/m^3
Water Density 水密度	ρ_w	1.0×10^3 kg/m^3
Standard atmosphere pressure 標準大氣壓	p_0	1.013×10^5 N/m^2
Charge of an electron 電子電荷	e	1.6×10^{-19} C
Permittivity constant 真空電容率	ε_0	8.85×10^{-12} C/(V m)
Electron mass 電子質量	m_e	9.11×10^{-31} kg
Speed of light in vacuum 真空光速	c	3.0×10^8 m/s

Trigonometric identities:
三角學恆等式:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\sin 2x = 2\sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$
$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$
$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

The following conditions will be applied to all questions unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light.

除特別註明外，下列條件將適用于本卷所有問題：

1) 所有物體都處于地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於光速。

Multiple Choice Questions

(2 points each. Select one answer in each question.)

選擇題（每題 2 分，每題選擇一個答案。）

The MC questions with the '*' sign may require information on page-3.
帶 * 的選擇題可能需要用到第三頁上的資料。

1. As shown in Fig. 1, a boy is riding on a bus. The bus moves with a uniform speed of 20 km/h in a horizontal circle, and the traffic light is located at the center of the circle. What is the velocity of the traffic light relative to the boy?
A) 20 km/h in the forward direction of the bus
B) 20 km/h in the backward direction of the bus
C) 20 km/h perpendicular to the forward direction of the bus and directed away from the bus
D) 20 km/h perpendicular to the forward direction of the bus and directed towards the bus
E) 0 km/h

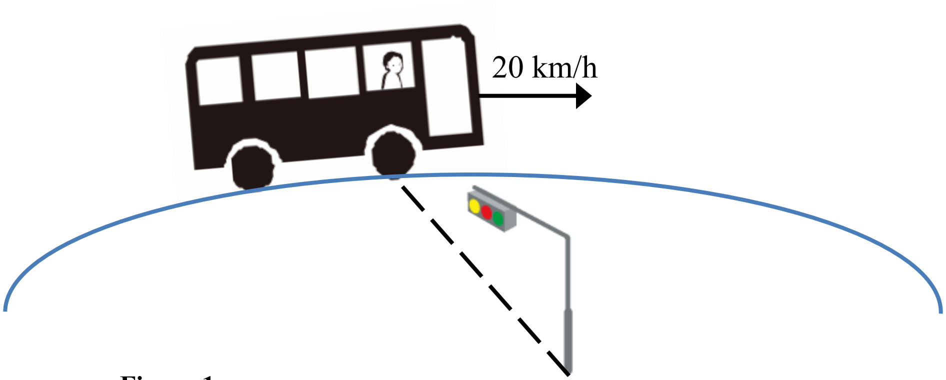


Figure 1

如圖 1 所示，一名男孩正在乘坐巴士，巴士以均速 20 km/h 繞著交通燈作水平圓周運動，交通燈位於圓圈的中心。交通燈相對於男孩的速度是多少？
A) 20 km/h 與巴士前進順向
B) 20 km/h 與巴士前進逆向
C) 20 km/h 與巴士前進方向垂直並遠離巴士
D) 20 km/h 與巴士前進方向垂直並趨近巴士
E) 0 km/h

Solution:
The circular motion only affects the relative acceleration. It does not affect the relative velocity.
(answer: B)

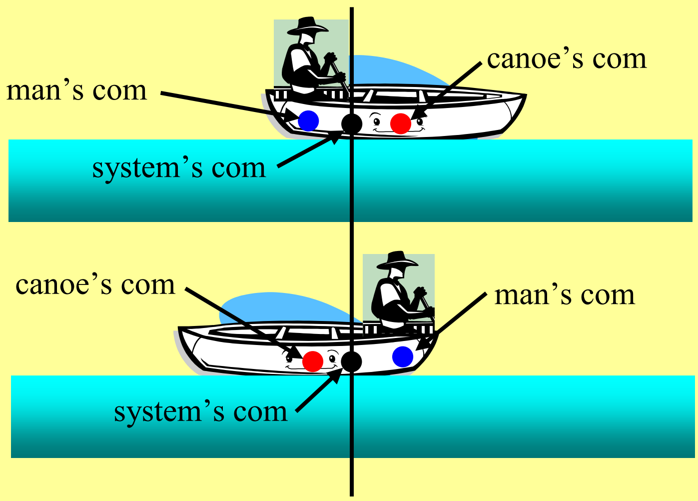
2. A tennis ball machine is installed on the bus in Fig. 1. It projects tennis balls at a speed of 100 km/h in the direction perpendicular to the forward direction of the bus, and on the outward side of the circular path. The trajectories of the tennis balls observed by the boy are
A) straight lines perpendicular to the forward direction of the bus
B) straight lines slightly inclined towards the forward direction of the bus
C) straight lines slightly inclined towards the backward direction of the bus
D) curves slightly inclined towards the forward direction of the bus
E) curves slightly inclined towards the backward direction of the bus
圖 1 的巴士裝有一台網球發射器。發射器以時速 100 km/h 沿著與巴士垂直的方向，向圓形路徑外邊發射網球。男孩觀察到的網球軌跡是
A) 與巴士前進方向垂直的直線
B) 比巴士前進方向略爲前傾的直線
C) 比巴士前進方向略爲後傾的直線

D) 比巴士前進方向略爲前傾的曲線
E) 比巴士前進方向略爲後傾的曲線

Solution:
Because of the curved motion path of the bus, the trajectory should be a curve. (answer: E)

3. A man sits in the back of a canoe in still water. He then moves to the front of the canoe and sits there. Neglecting the damping of water, the final position and the motion of the canoe is:
A) forward of its original position and moving forward
B) forward of its original position and moving backward
C) rearward of its original position and moving forward
D) rearward of its original position and moving backward
E) rearward of its original position and not moving
在靜止的水中，船夫坐在獨木舟的後方。他隨後向前移至獨木舟的前方然後坐下。若不計水的阻尼，獨木舟最終的位置和運動狀態是：
A) 比原來位置前移並向前動　B) 比原來位置前移並向後動
C) 比原來位置後移並向前動　D) 比原來位置後移並向後動
E) 比原來位置後移並不動

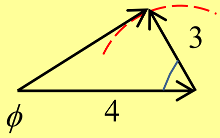
Solution:



Since there is no external force, the center of mass should stay at the same position. Hence the canoe is rearward of its original position and not moving. (answer: E).

4. A boat is about to cross a river to the opposite bank. The river water flows at a speed of 4 km/h. If the speed of the boat is 3 km/h, what should be the angle between the boat velocity and the upstream direction of the river, so that the downstream displacement is minimum when the boat reaches the opposite bank?
一艘船正要橫渡一道河往對岸。河水流速爲 4 km/h。若船速爲 3 km/h，那麼船速應與河的上游方向成什麼方角度，才可以在抵達對岸時，令其向下游的位移爲最小？
A) 0^o B) 37^o C) 41^o D) 53^o E) 90^o

Solution:
To minimize the downstream displacement, the resultant vector of the water and boat velocity should make an angle with the downstream direction as large as possible. Considering variable directions of the boat velocity, the tip of the boat velocity generates a circle as shown in the figure. The largest angle of the resultant velocity is given by the tangent to the circle. Hence $\phi = \cos^{-1}(3/4) = 41^o$. (answer: C)



5. *In a talent show, a juggler juggles 4 balls simultaneously, as shown in Fig. 2. A spectator uses his high speed video tape and determines that it takes the juggler 0.9 s to cycle each ball through his hands (including catching, transferring and throwing) and to be ready to catch the next ball. It is noted that at most one ball must be in a hand of the juggler in each cycle of juggling. What is the minimum vertical speed the juggler must throw up each ball?
在天才表演中，一藝人同時耍弄 4 個球，如圖 2 所示。有觀眾以高速攝錄機拍下過程，發現藝人需用 0.9 s 經手每個球(包括接球，傳球和抛球)，然後從新準備接下一球。他還留意到在耍球的每一循環中，藝人的每隻手最多只有一個球。問藝人抛球的最低垂直速率是多少？
A) 9.3 m/s B) 11.4 m/s C) 12.8 m/s D) 13.2 m/s E) 17.6 m/s

Figure 2

Solution:

One of the balls' height can be described by $y = v_o t - \frac{1}{2}gt^2$. The amount of time it takes to rise and fall to its initial height is therefore given by $\frac{2v_o}{g}$. If the time it takes to cycle the ball through the juggler's hands is τ = 0.9 s, then there must be 3 balls in the air during that time τ. A single ball must stay in the air for at least 3τ so the condition is $\frac{2v_o}{g} \geq 3\tau$, or $v_o \geq 13.2$ m/s. (answer: D)

6. As shown in Fig. 3, the block-spring system is in equilibrium provided that the left spring is stretched by x_1. The whole system rests on a smooth supporting surface. The coefficient of static friction between the blocks is μ_s, and the blocks have equal mass m. What is the maximum amplitude of the oscillations of the system such that the top block does not slide on the bottom one?
如圖 2 所示，一個方塊-彈簧系統處在平衡態中，其中左彈簧伸展長度爲 x_1。全系統安放在光滑面上。方塊之間的靜摩擦係數爲 μ_s，方塊的質量均爲 m。問系統振動可達到的最大幅度，而上下方塊之間仍能保持不滑動。

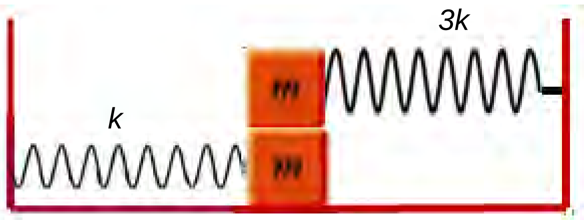


Figure 3

A) $\frac{\mu_s mg}{2k} + x_1$ B) $4kx_1 - \mu_s mg$ C) $\frac{\mu_s mg}{k} - x_1$ D) $2(2kx_1 + \mu_s mg)$ E) $2(2kx_1 - \mu_s mg)$

Solution:
The equivalent force constant of the springs in the system is $(3k+k) = 4k$
Applying Newton's 2[nd] law to the two-block system gives: $-4kx = 2ma$ (1)
Applying Newton's 2[nd] law to the lower block gives: $k(x_1 - x) - f = ma$, (2)
where f is the magnitude of the frictional force.

Solving the Eq. (1) for ma and substituting the result into Eq. (2) gives
$k(x_1 - x) - f = -2kx$.

Solving for f: $f = k(x_1 + x)$.

The maximum value for x is the amplitude A, and the maximum value for f is $\mu_s N = (\mu_s)mg$. Thus, $(\mu_s)mg = k(x_1 + A_{max})$. Solving for A_{max} gives

$$A_{max} = \frac{\mu_s mg}{k} - x_1 \quad \text{(answer: C)}$$

7. *An object is projected up an inclined plane with an initial speed of v_0 = 10 m/s, as shown in Fig. 4. The angle of the incline is θ = 30° above the horizontal direction and the coefficient of the sliding friction μ_k = 0.1, determine the total time for the object to return to the point of projection.
一物體沿斜面以初速 v_0 = 10 m/s 射出，如圖 4 所示。斜面的水平傾角為 θ = 30°，動摩擦係數為 μ_k = 0.1，求物體回到原點的總時間。
A) 3.81 s B) 4.26 s C) 4.54 s D) 4.94 s E) 5.32 s

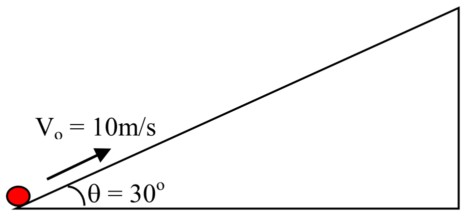


Figure 4

Solution:

Going up: $F_x = -mg\sin 30^\circ - \mu mg\cos 30^\circ = ma$

$$\therefore a = -g[\sin 30^o + (0.1)\cos 30^o] = -5.75\,\mathrm{m/s^2}$$

At the highest point $v = 0$ so $v = v_\circ + at \;\Rightarrow\; t_{up} = -\frac{v_o}{a} = -\frac{v_o}{-(5.75)} = 0.174 v_o$ s

The length of the inclined plane is given by

$$S_{up} = v_o t_{iu} + \frac{1}{2}at_{up}^2 = 0.174v_o^2 - \frac{1}{2}(5.75)(0.174v_o)^2 = 0.087v_o^2$$

Going down: $v_\circ' = 0$, $a = -g[\sin 30^o - (0.1)\cos 30^o] = -9.8(0.5 - 0.0866) = 4.05\ \mathrm{m/s^2}$

$$S_{down} = v_o' t_{down} + \frac{1}{2}at_{up}^2 = 0 - \frac{1}{2}(4.05)(t_{down}^2)$$

$$0.087v_o^2 = \frac{1}{2}(4.05)(t_{down}^2)$$

$$t_{down} = 0.207 v_\circ\ s$$

$$\therefore t_{total} = t_{up} + t_{down} = 0.381 v_o = 0.381(10) = 3.81\,\mathrm{s} \quad \text{(answer: A)}$$

8. *As shown Fig. 5, a train with a length of L = 500 m moves by its inertia through the horizontal section of a railroad. However, the train encounters a small hill that slopes gently. With what minimum speed v can the train cross the hill? The base of hill has a length ℓ = 100 m, the lengths of the slopes are ℓ_1 = 80 m and ℓ_2 = 60 m. The slopes of hill can be considered as straight lines and the small section of rounding at the top of the hill can be ignored. Neglect any friction.
如圖 5 所示，一列長度為 L = 500 m 的火車，以慣性在水平軌道上滑行，然後遇到一個小山丘。問火車需以什麼最低速度 v 越過山丘？山丘底長 l = 100 m，兩邊斜坡長 l_1 = 80 m 和 l_2 = 60 m。可以考慮斜坡為直線，坡頂彎曲的一小段可以忽略。可以忽略摩擦力。
A) 9.6 m/s B) 11.5 m/s C) 13.2 m/s D) 15.0 m/s E) 16.2 m/s

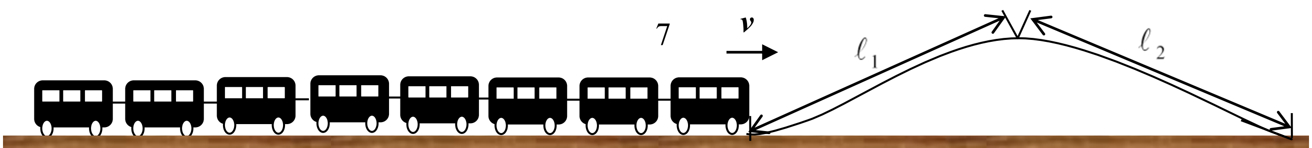


Figure 5

Solution:

The potential energy of the train is maximum when parts of the train completely occupy both slopes of hill.

Using cosine rule, the inclination of the left slope is given by

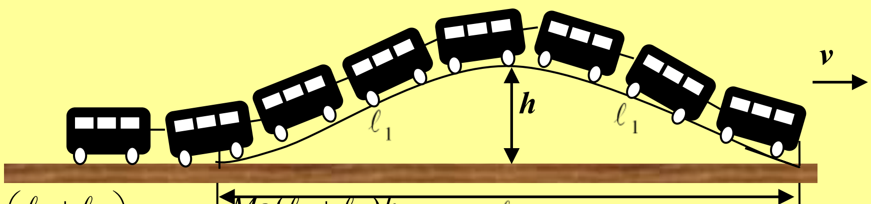
$$\cos\theta = \frac{l^2 + l_1^2 - l_2^2}{2ll_1} = 0.8$$

$$\sin\theta = \sqrt{1-\cos^2\theta} = 0.6$$

Height of the hill: $h = l_1 \sin\theta = 48\,\text{m}$

Alternatively, as can be seen from the given conditions, the lengths of slopes and base of hill satisfy the Pythagorean theorem: $\sqrt{\ell_1^2 + \ell_2^2} = \ell$. Hence it is a right-angled triangle. By equating the area of the triangle, the height of the hill $h = \ell_1 \ell_2 / \ell = 48$ m.

The center of mass of the lifted part of the train, which is located on the hill, gives $h_{c.g.} = h/2$. Since the mass of the parts of the train indicated is proportional to its length, it is possible to find potential energy of train relative to the foot of the hill:



$$P.E. = M\left(\frac{\ell_1 + \ell_2}{L}\right) g h_{c.g.} = \frac{Mg(\ell_1 + \ell_2)h}{2L}$$; where M is the mass of the entire train.

If the train can cross the hill, its initial kinetic energy $K.E. = Mv^2/2$ must be larger than the potential energy U. Hence the desired minimum speed of the train is

$$v_{\min} = \sqrt{gh\left(\frac{\ell_1 + \ell_2}{L}\right)} \approx 11.5\,\text{m/s}$$ (answer: B)

9. It is given the mass of Earth is a times that of Moon, and the radius of Earth is b times that of Moon. The period of a simple pendulum is T. When it is carried to Moon, the period of the simple pendulum becomes
已知地球的質量是月球的 a 倍，地球半徑是月球的 b 倍。現有一個單擺，在地球上的週期爲 T，若將它移到月球上，則單擺的週期等於

A) $\frac{a}{\sqrt{b}}T$ B) $\frac{\sqrt{b}}{a}T$ C) $\frac{\sqrt{a}}{b}T$ D) $\frac{b}{\sqrt{a}}T$ E) $\sqrt{\frac{b}{a}}T$

Solution:

$$mg = G\frac{mM}{R^2},\ g = \frac{GM}{R^2},\ T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{G}}\frac{R}{\sqrt{M}},\ \frac{T_2}{T_1} = \frac{R_2}{R_1}\sqrt{\frac{M_1}{M_2}} = \frac{\sqrt{a}}{b}.$$ (answer: C)

10. There exist some triple star systems in the universe. They are more distant from other stars, and are composed of three stars of equal mass M. The gravitational forces due to other stars can be

neglected. A basic stable structure of triple star systems consists of three collinear stars, with two companion stars moving around a central star on a circular orbit with radius R. The linear velocity of the companion stars is $v_1 = k_1\sqrt{GM/R}$, where k_1 =

宇宙中存在一些離其他恒星較遠的、由質量 M 相等的三顆星組成的三星系統，通常可忽略其他星體對它們的引力作用。已觀測到穩定的三星系統存在的一種基本構成形式，是三顆星位於同一直線上，兩顆伴星圍繞中央星在半徑爲 R 的圓軌道上運行。伴星運動的線速度 $v_1 = k_1\sqrt{GM/R}$ ，式中 k_1 =

A) $\sqrt{10}$ B) $\sqrt{5}$ C) $\sqrt{5/2}$ D) $\sqrt{5}/2$ E) $\sqrt[3]{5/2}$

Solution

$\Sigma F = F_1 + F_2 = \frac{Gm^2}{R^2} + \frac{Gm^2}{(2R)^2} = \frac{5Gm^2}{4R^2} = mR\omega^2$, $\omega = \sqrt{\frac{5Gm}{4R^3}}$, $v_1 = R\omega = \underline{\frac{\sqrt{5}}{2}\sqrt{\frac{Gm}{R}}}$. (answer: D)

《END OF MC's 選擇題完》

Open Problems 開放題

Total 6 problems 共 6 題

The Open Problem(s) with the '*' sign may require information on page 3.
帶 * 的開放題可能需要用到第三頁上的資料。

Q1* (10 points) **題1*（10分）**

The helicopter has a mass m and maintains its height by imparting a downward momentum to a column of air defined by the slipstream boundary as shown in Fig. 6. The propeller blades can project a downward air speed v, where the pressure in the stream below the blades is atmospheric and the radius of the circular cross-section of the slipstream is r. Neglect any rotational energy of the air, the temperature rise due to air friction and any change in air density ρ.

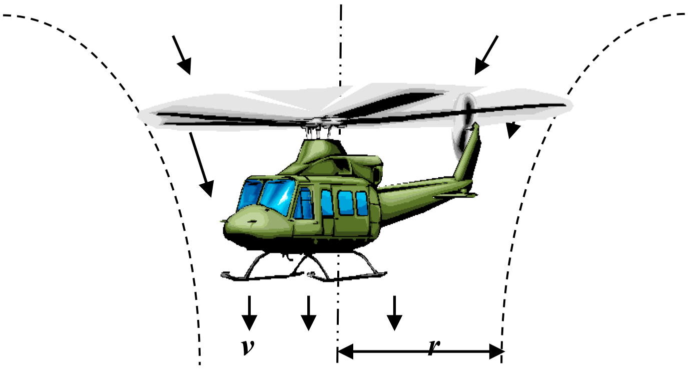


Figure 6

(a) Determine the power P required of the engine.
(b) If the power is doubled, calculate the acceleration of the helicopter.

直升機的質量爲 m，它傳輸下向的動量予空氣柱，用以維持其高度。氣流的邊界如圖 6 所示。它的引擎業片能鼓動空氣向下流，速度達到 v。氣流中的壓強爲大氣壓強，氣流的切面爲圓形，半徑爲 r。可以忽略空氣的轉動能、空氣摩擦引起的升溫、和空氣密度的改變。
(a) 試推導引擎所需的功率 P。
(b) 如果把功率增至兩倍，試計算直升機的加速度。

Solution:

(a) Using Newton's second law, the force exerted by the engine is equal to the rate of change of momentum of the air.

Mass of air propelled by the enginer in time $t = \rho vAt$

Increase in momentum of the air in time $t = (\rho vAt)v = \rho v^2 At$.

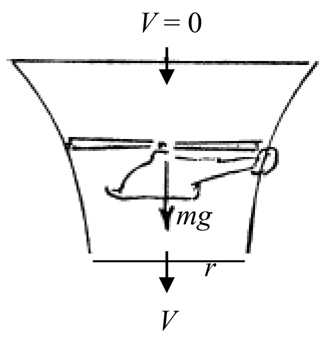
Hence the force is $F = \dfrac{\rho v^2 At}{t} = \rho v^2 A = \rho\pi r^2 v^2$.

For the helicopter to maintain its height, $F = mg$.

Hence $mg = \rho\pi r^2 v^2 \quad \Rightarrow \quad v = \dfrac{1}{r}\sqrt{\dfrac{mg}{\pi\rho}}$.

The increase of kinetic energy of the air in time t $K = \dfrac{1}{2}(\rho vAt)v^2$

Hence the power is $P = \dfrac{K}{t} = \dfrac{1}{2}\rho\pi r^2 v^3 = \dfrac{mg}{2r}\sqrt{\dfrac{mg}{\pi\rho}}$. (answer)



(b) Since P is proportional to v^3, the air velocity becomes $\sqrt[3]{2}v$ when the power is doubled.

Since F is proportional to v^2, the force becomes $\sqrt[3]{4}F = \sqrt[3]{4}mg$ when the power is doubled.

Using Newton's second law, the acceleration of the helicopter is given by

$\sqrt[3]{4}mg - mg = ma \quad \Rightarrow \quad a = \left(\sqrt[3]{4} - 1\right)g = 5.76\,\text{m/s}^2$ (answer)

Q2* (10 points) **題 2*（10 分）**

As shown in Fig. 7, in a ski jumping competition on a slope with inclination angle θ = 60°, an althlete jumps at point O with initial speed v_0 = 25 m/sand lands at point-A.
(a) Find the optimum jumping angle α so that the distance OA is maximum. (You may need to use trigonometric identities listed in page 3.)
(b) Find the maximum distance OA.
如圖 7 所示，在傾角$\theta = 60^0$的雪坡上舉行跳台滑雪比賽。運動員在起跳點 O 以速率 $v_0 = 25$ m/s、仰角α起跳，最後落在斜坡上 A 點。取 O、A 兩點的距離 L 爲比賽成绩。
(a)求運動員可以跳得最遠距離 OA 的起跳角α。(你或許會用到第 3 頁的三角學恆等式。)
(b) 求最遠距離 OA。

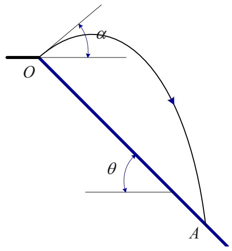
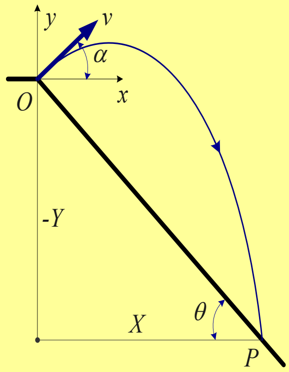


Figure 7

Solution:



Set the coordinate axes as shown in the figure. At time T,

$$X = vT\cos\alpha\,, \qquad (1)$$

$$Y = vT\sin\alpha - \frac{1}{2}gT^2\,. \qquad (2)$$

They satisfy the equation $$Y = -X\tan\theta\,. \qquad (3)$$

From Eqs. (1) to (3), $gT^2 - 2vT(\sin\alpha + \cos\alpha\tan\theta) = 0$.

We ontain $T = \frac{2v}{g}(\sin\alpha + \cos\alpha\tan\theta) = \frac{2v}{g}\frac{\sin(\alpha+\theta)}{\cos\theta}$ (4)

Substituting Eq. (4) into Eq. (1), $X = \frac{2v^2}{g}\frac{\sin(\theta+\alpha)\cos\alpha}{\cos\theta}$ (5)

The distance OP along the slope is given by $L(\alpha) = \frac{X}{\cos\theta} = \frac{2v^2}{g}\frac{\sin(\theta+\alpha)\cos\alpha}{\cos^2\theta}$. (6)

To calculate the best inclination and the maximum distance, we need to maximize L with respect to α.

From the trigonometric identity: $\sin x\cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$, we find

$$\sin(\theta+\alpha)\cos\alpha = \frac{\sin(\theta+2\alpha)+\sin\theta}{2}.$$

Hence $L(\alpha) = \frac{v^2}{g}\frac{\sin(\theta+2\alpha)+\sin\theta}{\cos^2\theta}$. When $\sin(\theta+2\alpha)$=1, that is, $\theta+2\alpha=90^0$, $\alpha_0 = \frac{90^0-\theta}{2}$=15°,

and the maximum distance is $L_{\max} = L(\alpha_0) = \frac{v^2}{g}\frac{1+\sin\theta}{\cos^2\theta} = \frac{25^2}{9.8}\frac{1+\sin 60^0}{\cos^2 60^0} = 255.1\left(1+\frac{\sqrt{3}}{2}\right)$.

The optimal jumping angle α is 15^0, and the maximum distance is 476 m. (answer)

An alternative solution can be obtained by setting the slope direction as the X axis, and the perpendicular direction as the Y axis. Then

$$x = v\cos(\theta+\alpha)t + \frac{1}{2}(g\sin\theta)t^2 \qquad \text{(a)}$$

$$y = v\sin(\theta+\alpha)t - \frac{1}{2}(g\cos\theta)t^2 \qquad \text{(b)}$$

When $y = 0$, $t = \dfrac{2v\sin(\theta+\alpha)}{g\cos\theta}$. Substituting into (a),

$$x = \frac{2v^2\sin(\theta+\alpha)}{g\cos^2\theta}\left[\cos(\theta+\alpha)\cos\theta + \sin\theta\sin(\theta+\alpha)\right].$$

Using the trigonometric identity $\cos(x-y) = \cos x\cos y + \sin x \sin y$, we have

$$x = \frac{2v^2\sin(\theta+\alpha)\cos\alpha}{g\cos^2\theta}.$$

Using the trigonometric identity $\sin x\cos y = \frac{1}{2}\left[\sin(x+y)+\sin(x-y)\right]$,

$x = \dfrac{v^2}{g\cos^2\theta}\left[\sin(\theta+2\alpha)+\sin\theta\right]$. When $\sin(\theta+2\alpha)=1$, that is, $\theta+2\alpha=90^0$, $\alpha_0 = \dfrac{90^0-\theta}{2}=15^{\circ}$

and the maximum distance is $L_{\max} = L(\alpha_0) = \dfrac{v^2}{g}\dfrac{1+\sin\theta}{\cos^2\theta} = \dfrac{25^2}{9.8}\dfrac{1+\sin 60^0}{\cos^2 60^0} = 255.1\left(1+\dfrac{\sqrt{3}}{2}\right)$.

The optimal jumping angle α is 15^0, and the maximum distance is 476 m. (answer)

Q3* (15 points) 題 3*（15 分）

On a smooth horizontal surface shown in Fig. 8, there is a rectangular board AB with mass $2m$ and an arc-shaped structure BC with mass $3m$. The coefficient of kinetic friction of the upper surface of the rectangular board is $\mu = 0.35$, and the surface of the arc-shaped structure BC is smooth. The arc BC subtends a right angle with radius R. The two objects touch at point B.

(a) A small object of mass m moves from A to the right with an initial velocity v_0 = 10 m/s on the upper surface of the rectangular block. When it reaches point B, the velocity is v = 5 m/s. Calculate the velocity v_L of the rectangular board AB at this instant.

(b) The small object continues to move onto the arc-shaped surface BC. The rectangular board AB loses contact with the arc-shaped structure BC, and the small object continues to to move along the arc surface, and finally just reaches the highest point C of the arc BC. Calculate the velocity of the arc-shaped structure v_R.

(c) Calculate the length L and the radius R.

質量爲 $2m$ 的長方體 AB 和質量爲 $3m$ 的圓弧體 BC 靜止在圖 8 中的光滑水平面上，長方體上表面的動摩擦系數爲μ = 0.35，圓弧體 BC 表面則爲光滑圓弧。圓弧 BC 的夾角爲直角，半徑爲 R。兩物體於 B 點接觸。

(a) 質量爲 m 的小滑塊在 A 點以初速度 v_0 = 10 m/s 沿長方體方板上表面向右滑動，滑過 B 點的速率爲 v = 5m/s。試求此刻長方體 AB 的速率 v_L。

(b) 滑塊然後滑到圓弧體表面 BC 上，長方體 AB 和圓弧體 BC 失去接觸，滑塊則繼續在圓弧表面上滑動，最終恰好能滑到圓弧 BC 上的最高點 C。試求此刻圓弧體 BC 的速率 v_R。

(c) 試求長度 L 和圓弧半徑 R。

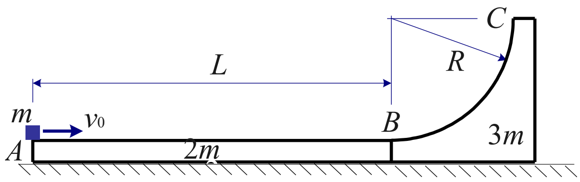


Figure 8

Solution:

v_0=10m/s, v=5m/s, μ=0.35

(a) Momentum conservation from A to B, $mv_0 = mv + (m_1 + m_2)v_L \Rightarrow v_L = \dfrac{m(v_0 - v)}{m_1 + m_2} = \dfrac{v_0}{10}$ =1 m/s (answer)

(b) Momentum conservation from B to C,

$$mv + m_2 v_L = (m + m_2)v_R,\ m(\frac{v_0}{2}) + 3m(\frac{v_0}{10}) = (m + 3m)v_R \Rightarrow$$

$$v_R = \frac{mv + m_2 v_L}{m + m_2} = \frac{1}{m + m_2}\left(mv + m_2\frac{m(v_0 - v)}{m_1 + m_2}\right) = \frac{m(m_1 v + m_2 v_0)}{(m + m_2)(m_1 + m_2)} = \frac{v_0}{5} = 2 \text{ m/s} \quad \text{(answer)}$$

(c) Energy conservation from A to B, $\dfrac{1}{2}mv_0^2 = \dfrac{1}{2}mv^2 + \dfrac{1}{2}(m_1 + m_2)v_L^2 + \mu mgL \Rightarrow$

$$L = \frac{m(v_0^2 - v^2) - (m_1 + m_2)v_L^2}{2\mu mg} = \frac{(m_1 + m_2)(v_0^2 - v^2) - m(v_0 - v)^2}{2\mu g(m_1 + m_2)} = \frac{7}{20}\frac{v_0^2}{\mu g} = 10.2 \text{ m} \quad \text{(answer)}$$

Energy conservation from B to C, $\dfrac{1}{2}mv^2 + \dfrac{1}{2}m_2 v_L^2 = \dfrac{1}{2}(m + m_2)v_R^2 + mgR$,

$$m(\frac{v_0}{2})^2 + 3m(\frac{v_0}{10})^2 = (m + 3m)(\frac{v_0}{5})^2 + 2mgR \Rightarrow$$

$$R = \frac{mv^2 + m_2 v_L^2 - (m + m_2)v_R^2}{2mg} = \frac{3}{50}\frac{v_0^2}{g} = 0.61 \text{ m} \quad \text{(answer)}$$

Q4* (10 points)　　題 4*（10 分）

Geosynchronous satellites have the same period T as the Earth's rotation. They are at such a height above the earth's surface h that they remain always above the same spot. Suppose a geosynchronous solar satellite, as shown in Fig. 9a, sends radio signal directly to receivers on earth.

Figure 9a

When the satellite moves to the rear part of the earth (Fig. 9b), the light source is completely blocked by Earth in the shadow region (commonly known as an umbra). The length of the umbra is usually n times ($n \approx 200$) of Earth's radius R_E.

(a) Determine the height h. Express your answer in units of R_E.

(b) Determine the duration in each day that the satellite cannot receive sunlight. Express your answer in minutes.

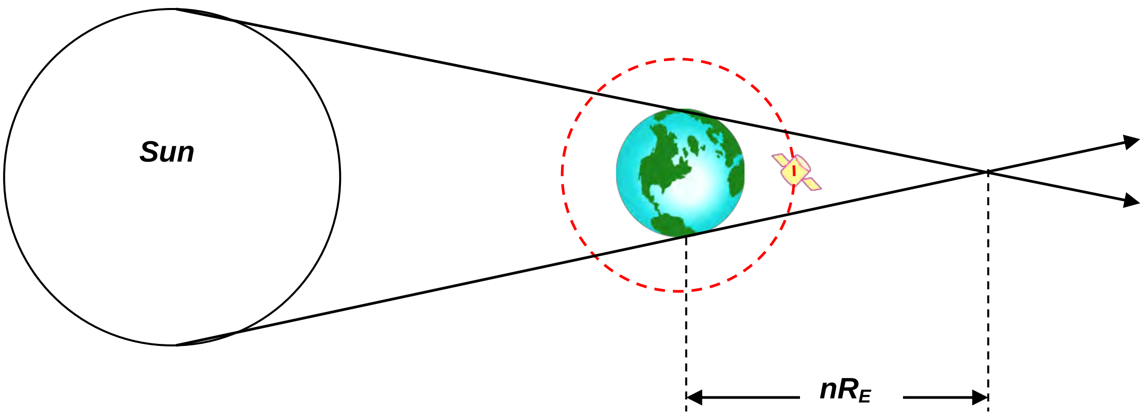


Figure 9b

地球同步衛星的軌道周期 T，與地球自轉週期相同。它們處於離地面高度 h 的位置，使它們經常位於同一地點上空。如圖 9a 所示，設有一地球同步的太陽能衛星，直接傳送無線電信號給地球上的接收站。

當衛星航行到地球背面(圖 9b)，光源被地球完全遮擋形成全影區。全影區的長度通常為地球半徑 R_E的 n 倍($n \approx 200$)。

(a) 試計算高度 h，以 R_E為單位表達結果。

(b) 試計算衛星每日不能接收陽光的時間，以分鐘為單位表達結果。

Solution:

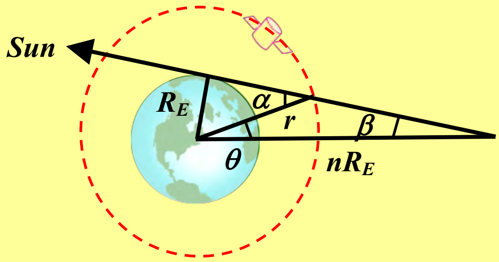
(a) Let r be the distance of the satellite measured from the centre of the earth. When the satellite moves in the circular orbit, its centripetal force is

$$G\frac{mM_e}{r^2} = ma = m(\frac{4\pi^2}{T^2})r; \quad \Rightarrow r = \sqrt[3]{\frac{GM_eT^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times10^{-11})(5.98\times10^{24})(86400)^2}{4\pi^2}} = 4.225\times10^7 \text{ m}$$

$$r = \frac{4.225\times10^7}{6.378\times10^6} = 6.62\,R_E$$

Hence $h = 5.62\ R_E$.   (answer)

(b)



As shown in the figure,

$$\sin\alpha = \frac{R_E}{r} = \frac{1}{6.6244} \quad \Rightarrow \quad \alpha = 0.1515\text{rad}$$

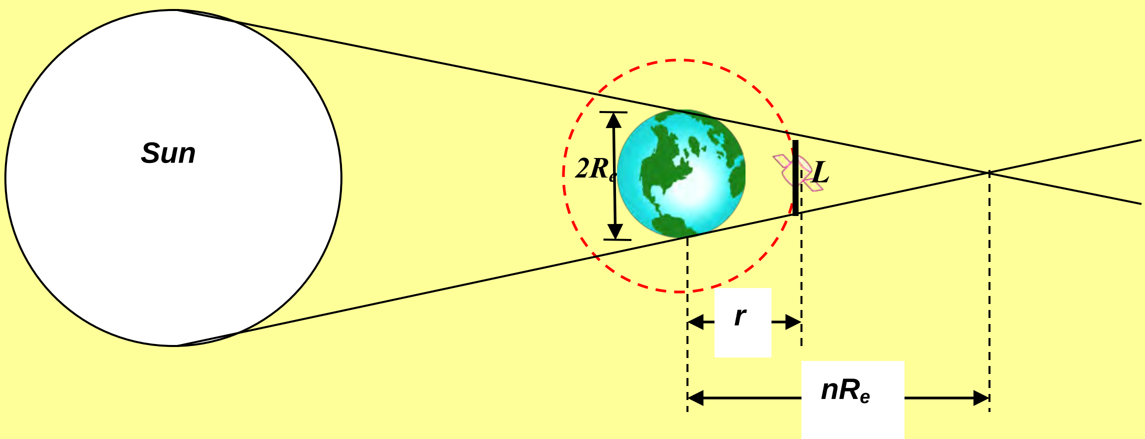
$$\sin\beta = \frac{R_E}{nR_E} = \frac{1}{200} \quad \Rightarrow \quad \beta = 0.005\text{rad}$$

$$\theta = \alpha - \beta = 0.1515 - 0.005 = 0.1465\text{rad}$$

Hence the length of the black-out period is

$$t = (24)(60)\frac{(2)(0.1465)}{2\pi} = 67.2 \text{ min} \quad \text{(answer)}$$

An alternative, but approximate, solution can be obtained by assuming nR_e is very large compared with L, so that the orbit in the region of umbra can be treated as a straight line L.



By using simple trigonometry, we have the following equations:

$$\frac{L}{2R_e} = \frac{nR_e - r}{nR_e}, \Rightarrow L = 2R_e - \frac{2r}{n}$$

The time the satellite moves in the region of umbra is

$$t = (\frac{L}{2\pi r})T = \left(\frac{R_e}{\pi r} - \frac{1}{n\pi}\right) \cdot T \quad \ldots(2)$$

Substitute equation (1) into (2) which yields

$$t = \left(\sqrt[3]{\frac{4R_e}{\pi g T^2}} - \frac{1}{n\pi}\right) \cdot T$$, where $T = 24 \times 3600 = 86400$ s, $R_e = 6{,}378{,}000$ m

$$= \left(\sqrt[3]{\frac{4(6378000)}{(3.14)(9.8)(86400)^2}} - \frac{1}{(200)(3.14)}\right) \cdot (86400)$$

$$= (0.048 - 1.59 \times 10^{-3}) \cdot (86400) = 4010s = 66.8\,\text{min}$$

Q5* (20 points)　　題 5*（20 分）

Let r_E = 1.00 AU be the circular orbtal radius of Earth around Sun, and r_V = 0.72 AU be the circular orbital radius of Venus around Sun. To launch a space probe from Earth to Venus, the space probe first enters Earth's orbit and moves to a position sufficiently remote from Earth, so that the gravitational attraction of Earth is negligible when compared with the gravitational attraction of Sun. Then, the following manouvers as shown in Fig. 10 are performed:

(1) The kinetic energy of the space probe is reduced by $\Delta K = K(r_E - r_V)/(r_E + r_V)$, where K is the kinetic energy of the space probe at that instant. This is done by switching on the engine for a short time and then switching it off. The space probe then enters a transfer orbit around the Sun. The transfer orbit is tangential to Earth's orbit at its near end and to Venus's orbit at its far end.

(2) When the space probe touches Venus's orbit, the space probe is traveling too fast for it to land on Venus. For the second time, the kinetic energy is reduced by switching on the engine for a short time and then switching it off. The space probe then travels at the orbital velocity of Venus.

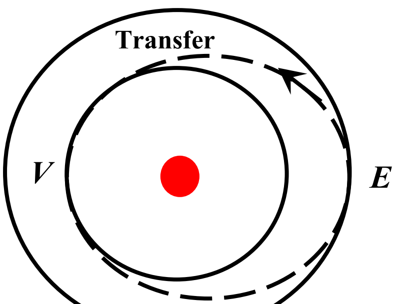


Figure 10

(a) Calculate the orbital speed of Earth and Venus. Express your answer in km/s.
(b) Calculate the speed (in km/s) of the space probe in the transfer orbit at point E.
(c) Calculate the speed (in km/s) of the space probe in the transfer orbit at point V.
(d) Calculate the fractional reduction of the kinetic energy of the space probe at point V.

設 r_E = 1.00 AU 爲地球環繞太陽圓形軌道的半徑，r_V = 0.72 AU 爲金星環繞太陽圓形軌道的半徑。發射一個太空探測器從地球到金星，可以先將探測器放進地球軌跡，令它處於離地球較遠的位置，以致地球的引力相對於太陽引力，可以忽略不計。然後對探測器進行如圖 10 的操作：

(1) 把探測器的動能降低 $\Delta K = K(r_E - r_V)/(r_E + r_V)$，其中 K 爲探測器在該時刻的動能，方法是開動引擎一段短時間，然後關掉引擎。探測器因而進入環繞太陽的轉移軌道。這轉移軌道在近端與地球軌道相切，在遠端則與金星軌道相切。

(2) 當探測器切入金星軌道，它的高速令它難以降落金星表面。它需要第二次開動引擎一段短時間，然後關掉引擎。探測器因而降低動能，以金星軌道速率航行。

(a) 試計算地球和金星在軌道上的速率，以 km/s 爲單位表達結果。
(b) 試計算探測器在轉移軌道 E 點的速率，以 km/s 爲單位表達結果。
(c) 試計算探測器在轉移軌道 V 點的速率，以 km/s 爲單位表達結果。
(d) 探測器在轉移軌道 V 點時，需降低的動能爲該時刻動能的什麼比例?

Solution:

(a) Using Newton's second law,

$$M_E \frac{v_E^2}{r_E} = \frac{GM_{Sun}M_E}{r_E^2} \quad \Rightarrow \quad v_E = \sqrt{\frac{GM_{Sun}}{r_E}} = \sqrt{\frac{(6.67\times10^{-11})(1.99\times10^{30})}{1.5\times10^{11}}} = 29747\,\text{m/s} \approx 29.75\,\text{km/s}$$

(answer)

Similarly, $v_V = \sqrt{\dfrac{GM_{Sun}}{r_V}} = \sqrt{\dfrac{(6.67\times10^{-11})(1.99\times10^{30})}{(0.72)(1.5\times10^{11})}} = 35057\,\text{m/s} \approx 35.06\,\text{km/s}$ (answer)

(b) At point E of Earth's orbit, kinetic energy: $K = \dfrac{1}{2}mv_E^2 = \dfrac{GM_{Sun}m}{2r_E}$.

At point E of the transfer orbit, kinetic energy:

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_E^2\left(1 - \frac{r_E - r_V}{r_E + r_V}\right) \quad \Rightarrow \quad v_1 = v_E\sqrt{\frac{2r_V}{r_E + r_V}} = 27218\,\text{m/s} \approx 27.22\,\text{km/s} \quad \text{(answer)}$$

(c) Using conservation of energy from point E to point V,

$$\frac{1}{2}mv_1^2 - \frac{GM_{Sun}m}{r_E} = \frac{1}{2}mv_2^2 - \frac{GM_{Sun}m}{r_V}$$

$$v_2 = \sqrt{v_1^2 - \frac{2GM_{Sun}}{r_E} + \frac{2GM_{Sun}}{r_V}} = \sqrt{\frac{2GM_{Sun}r_E}{r_V(r_E + r_V)}} = v_V\sqrt{\frac{2r_E}{r_E + r_V}} = 37803\,\text{m/s} \approx 37.80\,\text{km/s} \quad \text{(answer)}$$

(d) Fractional rduction of kinetic energy:

$$\frac{\Delta K_2}{K_2} = \frac{\frac{1}{2}mv_2^2 - \frac{1}{2}mv_V^2}{\frac{1}{2}mv_2^2} = \frac{v_2^2 - v_V^2}{v_2^2} = \frac{\frac{2r_E}{r_E + r_V} - 1}{\frac{2r_E}{r_E + r_V}} = \frac{r_E - r_V}{2r_E} = \frac{1 - 0.72}{2} = 0.14 \quad \text{(answer)}$$

Q6 (15 points) **題 6（15 分）**

As shown in Fig. 11, a pendulum consists of a massive cubic block with side length b and density ρ_b hung by a light rigid rod with length $L >> b$. The hinge connecting the rod and the block maintains the block at the same orientation when it swings. The block is partially immersed in a liquid with density ρ_l, with ρ_l greater than ρ_b. The immersion depth of the block in the liquid is c.

(a) Calculate the period of the pendulum for small oscillation angles ϕ when c is sufficiently small. You may assume that the damping of the liquid during oscillations is negligible, and the change of the immersion depth during oscillations is negligible.

(b) When liquid is added, the liquid level rises, and the value of c reaches a threshold value c^*, above which the pendulum rod is no longer vertical. What is the espression of c^*?

(c) When c is above c^*, the pendulum oscillates about a non-zero angle ϕ. Calculate the period of the pendulum for small oscillations when the liquid level has risen to one that corresponds to $\phi = 30^\circ$.

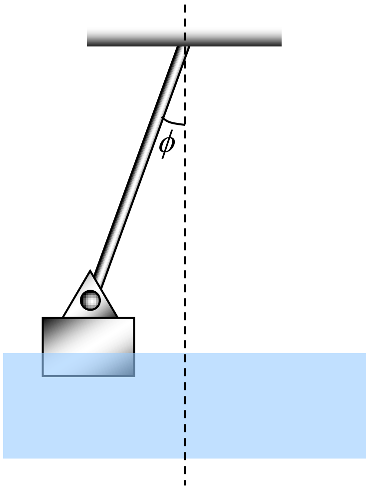


Figure 11

如圖 11 所示，在一個鐘擺中，一根無質量的剛體長桿懸掛著一個有質量的立方體。立方體邊長爲 b，密度爲 ρ_b。長桿長度爲 $L >> b$。連接長桿和立方體的樞紐，在鐘擺運動時，把立方體維持在同一方向。立方體部分浸在液體中，液體密度爲 ρ_l，數值比 ρ_b 大。立方體浸在液體中的深度爲 c。

(a) 試計算當 c 充分小的時候，鐘擺作小角度振動的周期。可以假設在振動時，液體的阻尼可以忽略，也可以假設在振動時，立方體浸在液體中的深度改變可以忽略。

(b) 當液體增加時，液體表面上升，c 的數值達到閾值 c^*。在閾值以上，鐘擺的長桿不再垂直。c^*的表達式爲何？

(c) 當 c 在 c^*以上時，鐘擺環繞非零值的ϕ振動。試計算當液面升至$\phi = 30^\circ$的位置時，鐘擺作小角度振動的周期。

Solution:

(a) Weight of the block: $W = \rho_b b^3 g$

Buoyancy of the block: $B = \rho_l b^2 cg$

Hence $W - B = \rho_b b^3 g - \rho_l b^2 cg = \rho_b b^3 g\left(1 - \dfrac{\rho_l c}{\rho_b b}\right)$.

This implies that the pendulum is equivalent to one that is placed in a gravitational field with gravitational acceleration $g\left(1 - \dfrac{\rho_l c}{\rho_b b}\right)$. Hence the period of the pendulum is $T = 2\pi\sqrt{\dfrac{L}{g\left(1 - \dfrac{\rho_l c}{\rho_b b}\right)}}$.

(answer)

(b) When c approaches $\dfrac{\rho_b b}{\rho_l}$, then we have B approaches W. Above this value, the block will float, and the pendulum is no longer vertical. Hence $c^* = \dfrac{\rho_b b}{\rho_l}$. (answer)

(c) Let x be the displacement of the block along the tangential direction of the rigid rod swing.
x is positive in the direction of increasing ϕ.
Then the vertical displacement of the block is $x\sin\phi$.
Change in buoyancy: $\Delta B = \rho_l b^2 x \sin\phi g$

Using Newton's second law,
$ma = -\Delta B \sin\phi = -\left(\rho_l b^2 g \sin^2\phi\right)x$
Hence the period of oscillations is $ma = -\Delta B \sin\phi = -\left(\rho_l b^2 g \sin^2\phi\right)x$.
Angular frequency: $\omega^2 = \dfrac{\rho_l b^2 g \sin^2\phi}{m} = \dfrac{g\sin^2\phi}{b}$.
Period: $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{b}{g\sin^2\phi}} = 2\pi\sqrt{\dfrac{4b}{g}}$. (answer)

《END 完》

Hong Kong Physics Olympiad 2012
2012 年香港物理奧林匹克競賽

(Senior Level 高級組)

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
香港特區政府教育局
The Hong Kong Physical Society
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

March 17, 2012
2012 年 3 月 17 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID Number in the field of “I. D. No.”, and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of “Course & Section No.”

   請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 “I. D. No.” 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格填上你的英文姓名，最後於 “Course & Section No. ” 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of “Course Title”, and write your English name in the field of “Student Name” and your 8-dight Participant I. D. Number in the field of “Student Number”. You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其它部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:

g – gravitational acceleration on Earth surface, 9.8 (m/s^2)
G – gravitational constant, 6.67×10^{-11} (N m^2/kg^2)
e – charge of an electron, -1.6×10^{-19} (A s)
ε_0 – electrostatic constant, 8.85×10^{-12} (A s)/(V m)
m_e – electron mass = 9.11×10^{-31} kg
c – speed of light in vacuum, 3.0×10^8 m/s
Mass of Earth = 5.97×10^{24} kg
Radius of Earth = 6,371 km
Mass of Moon = 7.4×10^{22} kg
Radius of Moon = 1,737 km
Density of water = 1.0×10^3 kg/m^3
Air Density at 20 °C and 1 atm = 1.2 kg/m^3

除非特別注明，否則本卷將使用下列符號和常數：

g – 地球表面重力加速度, 9.8 (m/s^2)
G – 萬有引力常數, 6.67×10^{-11} (N m^2/kg^2)
e – 電子電荷, -1.6×10^{-19} (A s)
ε_0 – 靜電常數, 8.85×10^{-12} (A s)/(V m)
m_e – 電子質量, 9.11×10^{-31} kg
c – 真空光速, 3.0×10^8 m/s
地球質量 = 5.97×10^{24} kg
地球半徑 = 6,371 km
月球質量 = 7.4×10^{22} kg
月球半徑 = 1,737 km
水的密度 = 1.0×10^3 kg/m^3
20 °C、一個大氣壓的空氣密度 = 1.2 kg/m^3

The following conditions will be applied to all questions unless otherwise specified:

1) All objects are near Earth surface and the gravity is pointing downwards.
2) Neglect air resistance.
3) All speeds are much smaller than the speed of light in vacuum.

除非特別注明，否則下列條件將適用於本卷所有問題：

1) 所有物體都處於地球表面，重力向下；
2) 忽略空氣阻力；
3) 所有速度均遠小於真空中的光速。

Multiple Choice Questions

(2 points each. Total 10 MC's. Select one answer in each question.)

選擇題（每題 2 分，共 10 題，每題選擇一個答案。）

The questions with the '*' sign may require information on page-3.
帶 * 的題可能需要用到第三頁上的資料。

MC1 選擇題 1

Two small blocks A and B with mass ratio $M_A/M_B = 3$ are connected by a light spring and on a smooth horizontal floor. The blocks are pressed towards each other a little and then released. The ratio of the maximum displacements from their initial positions of the two blocks A_A/A_B is ______.

二個物塊 A、B，其質量比爲 $M_A/M_B = 3$，以一輕彈簧相連，並放在水平的光滑平面上。現將兩物塊壓縮一點後放開，則物塊最大位移之比 A_A/A_B 爲______。

(a) 1/9 (b) 1/3 (c) 1 (d) 3 (e) 9

MC2 選擇題 2

Following MC1, the ratio of maximum momenta of the two blocks P_A/P_B is _______.

接上題。物塊最大動量之比 P_A/P_B 爲______。

(a) 9 (b) 3 (c) 1 (d) 1/3 (e) 1/9

MC3 選擇題 3

Following MC1, the ratio of maximum kinetic energy E_A/E_B is _______.

接上題。物塊最大動能之比 E_A/E_B 爲______ 。

(a) 9 (b) 3 (c) 1 (d) 1/3 (e) 1/9

MC4* 選擇題 4

The period of a simple pendulum on the ground is 1 s. If it is placed on the surface of the moon, the period of the pendulum will approximately be _____.

一個單擺，在地面的週期爲 1 秒。若將它放在月球表面，則它的週期約爲_____。

(a) 11.2 s (b) 1 s (c) 2.45 s (d) 0 (e) ∞

MC5 選擇題 5

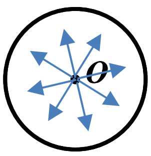
If the pendulum in MC4 is placed on a satellite on a circular orbit around Earth at 100 km above the ground, its period will approximately be _____.

若將 MC4 裡的單擺放在離地面 100 公里在圓形軌道上運行的衛星上，則它的週期約爲_____ 。

(a) 11.2 s (b) 1 s (c) 2.45 s (d) 0 (e) ∞

MC6 選擇題 6

Electric current I is flowing out from point-O and spread out radially on a conducting plane. Find the magnetic field amplitude on the axis through point-O and perpendicular to the plane, at distance H from the plane.



電流 I 從 O-點均勻地在一導電面上沿徑向向外流出。中心軸經過 O-點與平面垂直。求在軸上距離平面 H 處的磁場強度。

(a) 0 (b) $\frac{\mu_0 I}{2\pi H}$ (c) $\frac{\mu_0 I}{\pi H}$ (d) $\frac{2\mu_0 I}{\pi H}$ (e) $\frac{\mu_0 I}{4\pi H}$

MC7 選擇題 7

Two chambers of the same volume are filled with air and connected by a pathway. The temperatures in the two chambers are maintained at different values. Which chamber contains more air?

(a) the chamber with lower temperature
(b) the chamber with higher temperature
(c) the chamber with higher pressure
(d) neither, because both have the same pressure
(e) neither, because both have the same volume

二個相同體積的腔室充滿空氣，並由通道相連，如果兩個腔室的溫度不同，那麽哪個腔室含有較多的空氣？

(a) 溫度較低的腔室　(b) 溫度較高的腔室　(c) 氣壓較高的腔室
(d) 都不是，因爲它們具有相同的氣壓　(e)都不是，因爲它們具有相同的體積。

MC8 選擇題 8

Which of the following correctly describes the difference between microwaves and radio waves?

(a) Microwaves are used for communication but not radio waves.
(b) Microwaves are electromagnetic waves, while radio waves are sound waves.
(c) Microwaves travel faster in vacuum than radio waves.
(d) Microwaves carry energy but radio waves do not.
(e) Radio waves can go around a building but microwaves cannot.

以下哪一項正確地形容微波及無線電波的分別？

(a) 微波可用來通訊但無線電波不能。
(b) 微波是電磁波，無線電波是聲波。
(c) 在真空中，微波的傳播速度比無線電波快。
(d) 微波帶有能量而無線電波沒有。
(e) 無線電波能繞過大廈但微波不能。

MC9 選擇題 9

Which of the following statements about a magnetic mono-pole is correct?

(a) It must have a south pole.
(b) It must have a north pole.
(c) It has a south pole or a north pole, but not both.
(d) It must have a north pole and a south pole.
(e) It must have more than one pairs of north poles and south poles.

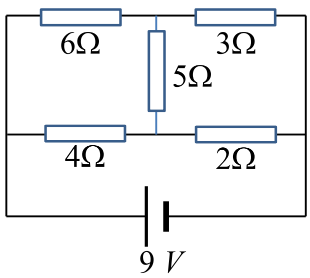
以下哪一項關於磁單極子的說明是正確的？

(a) 它只含有南極。
(b) 它只含有北極。
(c) 它有一個南極或一個北極，但不能同時含有兩極。
(d) 它必須有一個北極和一個南極。
(e) 它必須有一對以上的南、北極。

<u>MC10</u> 選擇題 10

As shown, the electric currenet through the 3Ω resistor is _____ Amperes.
如圖所示，流經 3Ω電阻的電流爲_____安培。

(a) 0 (b) 1.0 (c) 1.75 (d) 2 (e) 2.25



《End of MC's 選擇題完》

Open Problems 開放題

Total 6 problems 共 6 題

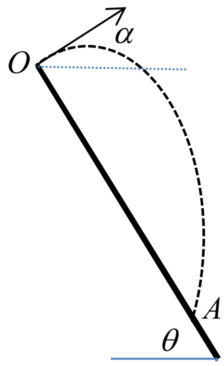
Q1* (10 points)　　題 1（10 分）

Estimate the force of heavy rain on an umbrella, and compared it with that of wind with a modest wind speed of 36 km/hour. Please provide the details on how you reach the answer.

估計下大雨時雨傘受到的衝擊力，并與中度風（風速 36 公里/小時）對傘的力作比較。請給出導出答案所需的細節。

Q2 (10 points)　　題 2（10 分）

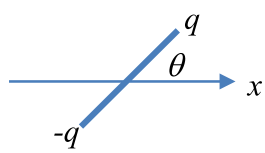
In a ski jumping competition on a slope with inclination angle $\theta = 60^0$, an althlet jumps at point-O with initial speed $v_0 = 25$ m/s and lands at point-A. Find the optimum jumping angle α so that the distance OA is maximum, and the maximum distance. (You may need to use $\sin x \cos y = \dfrac{\sin(x+y)+\sin(x-y)}{2}$.)



在傾角$\theta = 60^0$的雪坡上舉行跳臺滑雪比賽。運動員在起跳點 O 以速率 $v_0 = 25$ m/s、仰角α 起跳，最後落在斜坡上 A 點。取 O、A 兩點的距離 L 爲比賽成績。求運動員可以跳得最遠距離的仰角α和這個最遠距離 L。(你或許會用到：$\sin x \cos y = \dfrac{\sin(x+y)+\sin(x-y)}{2}$.)

Q3 (15 points)　　題 3（15 分）

A rigid uniform stick of length L is restricted to move on the horizontal X-Y plane. A point charge q is fixed on one end of the stick and a point charge $-q$ is fixed on the other end. At a particular moment the velocity of the stick center is $\vec{v} = v_x\hat{x} + v_y\hat{y}$, the stick is at an angle θ to the X-axis, and is spinning around its center at angular velocity ω. A uniform magnetic field B is applied along the Z-axis. Find the net force on the stick and the net torque relative to the center of the stick.



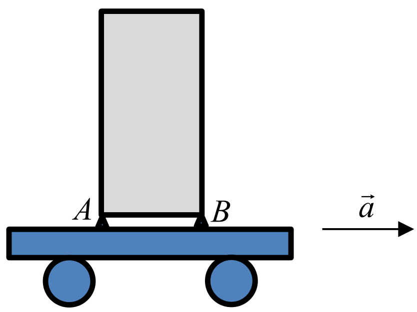
一長度爲 L 的均勻杆，只能在平面 X-Y 上運動。杆的一端帶有點電荷 q，另一端帶有點電荷$-q$。在某一時刻，杆中心的速度爲$\vec{v} = v_x\hat{x} + v_y\hat{y}$，杆與 X-軸成角度$\theta$，杆繞其中心旋轉的角速率爲$\omega$。外加磁場 B 沿 Z-軸方向。求杆受到的合力，以及相對於其中心的力矩。

Q4 (15 points)　　題 4（15 分）

A 50 kg refrigerator, which can be regarded as a uniform block, is resting on two feet A and B on a cart. The static friction coefficient between the feet and the cart floor is 0.4. The height of the refrigerator is 3 times of its width.

(a) Find the maximum horizontal acceleration below which the refrigerator can still rest on the cart. (5 points)

(b) For acceleration $a = 0.1g$, where g is the gravity acceleration, find the total force acting on each foot by the cart. (10 points)



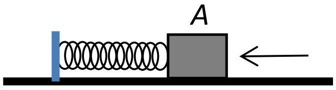
一個質量爲 50 kg 的雪櫃，可當作一個均勻的長方體，其高度是寬度的 3 倍，放在貨車上，雪櫃的前、後腳 A、B 與貨車地板之間的摩擦係數爲 0.4。

(a) 若貨車加速時，雪櫃仍不會相對貨車而動，求最大加速度。（5 分）

(b) 若貨車加速度爲 $a = 0.1g$，g 爲重力加速度，求貨車對雪櫃每個腳的作用力。（10 分）

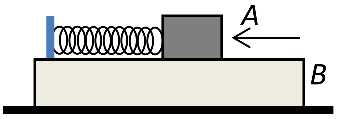
Q5 (10 points)　　題 5（10 分）

(a) A block of mass M is connected to a weight-less spring with force constant k and on a horizontal plane. The friction coefficient between the block and the floor is μ. Initially the spring is at its natural length. Now give the block an initial velocity v to the left. If the block stops oscillating completely before it moves to the right, find the displacement of the block and the initial velovity in tems of M, k, μ, and gravity acceleration g. (4 points)



一質量爲 M 的方塊，連在一力常數爲 k 的輕彈簧上。物塊與水平面之間的摩擦係數爲μ。初始時彈簧沒有被拉開或壓縮。現給物塊一個向左的初始速度 v。若物塊在向右運動前已不再振動，求它的位移和初始速度。答案需以 M、k、μ、重力加速度 g 表達。（4 分）

(b) The block-spring assembly in (a) is now fixed on a plateform B with the same mass M as the block, which is on a smooth floor, as shown in the figure. The friction coefficient between the block and the platform is μ. Initially the spring is at its natural length. Now give the block an initial velocity v to the left. If the block stops oscillating completely before it moves to the right relative to the platform, find the displacement of the block relative to the platform and the initial velovity in tems of M, k, μ, and gravity acceleration g. (6 points)



將物塊和彈簧放在質量同樣爲 M 的底座 B 上，物塊 A 與底座 B 之間的摩擦係數爲μ。底座放在平滑的平面上。初始時彈簧沒有被拉開或壓縮。現給物塊一個向左的初始速度 v。若物塊相對於底座向右運動前已不再振動，求它相對於底座的位移和初始速度。答案需以 M、k、μ、重力加速度 g 表達。（6 分）

Q6 (20 points)　　題 6（20 分）

A number of gas cylinders, each containing n moles of ideal gas with pressure P_0 and volume V_0, are used to fill up an empty tank with volume $V = 10V_0$, until the gas pressure in it reaches $P = P_0/2$. The temperature remains the same throughout the process.

(a) If a gas pump is used to pump all the gas inside a cylinder to the tank, how many cylinders of gas is needed? (5 points)

(b) If the pump is not used, then the gas transfer can only be done by natural flow of gas. Suppose a number of cylinders can be connected together and to the tank. A valve on the inlet of the tank is then opened to let gas flow until no gas is flowing in or out of the tank. How many cylinders of gas is needed? (5 points)

(c) Suppose each time only one cylinder can be connected to the inlet of the tank and no pump is used. After the gas stops flowing, the valve is closed, the cylinder is removed, and another cylinder full of gas is connected to the inlet,…., find the number of cylinders needed. (10 points)

若干容積爲 V_0 的氣瓶，每瓶裝有氣壓爲 P_0 的 n 摩爾理想氣體，被用來給一個空的氣室充氣，直到氣室的氣壓達到 $P = P_0/2$。氣室的容積爲 $V = 10V_0$。在整個充氣過程中氣體的溫度不變。

(a) 若使用氣泵，把每個氣瓶裡的氣體全部抽進氣室，需要用幾瓶氣？ (5 分)

(b) 若不用氣泵，則充氣只能靠氣體的自然流動來實現。如果可以一次把幾個氣瓶和氣室連接在一起，打開氣室入口的閥門讓氣體流入，氣體停止流動後將閥門關上，需要用幾瓶氣? (5 分)

(c) 若不用氣泵，並且每次只能把一個氣瓶連接到氣室的入口，气体停止流動後將閥門關上，将气瓶移走，换上另一瓶满的气瓶，….，需要用幾瓶氣? (10 分)

《END 完》

Hong Kong Physics Olympiad 2012
2012 年香港物理奧林匹克競賽

(Senior Level 高級組)

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學院
The Education Bureau of HKSAR
香港特區政府教育局
The Hong Kong Physical Society
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

March 17, 2012
2012 年 3 月 17 日

MC1-5 (b) (c) (d) (c) (e)
MC6-10 (a) (a) (e) (c) (b)

Q1 (10 points)
Heavy rain ~ 100 mm/hour = 100/3600 ≈ 0.03 kg/s = Φ (1 point)
Area of umbrella A ~ 1 m^2 (1 point)
Rain drop speed ~ 10 m/s (1 point)

$$ft = mv, \Rightarrow f_{rain} = \frac{mv}{t} = A \cdot \Phi \cdot v = 1 \cdot 0.03 \cdot 10 = 0.3N \quad \text{(2 points)}$$

Wing:
Wing speed is 10m/s, (1 point) $\Phi = 1.2 \cdot 10 = 12$ kg/s (1 point)

$f_{wind} = A \cdot \Phi \cdot v = 1 \cdot 12 \cdot 10 = 120N$. (3 points)

Any answers within a factor of 10 are fine.

Q2 (10 points)

$$L\cos\theta = (v_0 \cos\alpha)t \quad \text{(horizontally)},$$
$$L\sin\theta = (-v_0 \sin\alpha)t + \frac{1}{2}gt^2 \quad \text{(vertically)}.$$
(2 points)

Other forms are fine as long as the expression for L below is correct.

Then we have,

$$L = f(\alpha) = \frac{2v_0^2}{g\cos^2\theta}(\sin\theta\cos^2\alpha + \cos\theta\sin\alpha\cos\alpha)$$
$$= \frac{2v_0^2}{g\cos^2\theta}[\sin\theta\frac{1}{2}(1+\cos 2\alpha) + \cos\theta\frac{1}{2}\sin 2\alpha]$$
$$= \frac{v_0^2}{g\cos^2\theta}[\sin\theta + (\sin\theta\cos 2\alpha + \cos\theta\sin 2\alpha)]$$

$$= \frac{v_0^2}{g\cos^2\theta}[\sin\theta + \sin(\theta + 2\alpha)]. \quad \text{(6 points)}$$

$L_{\max} = 476m$ while $\alpha = 15^0$. (2 points)

Q3 (15 points). Point Dipole

$$\overrightarrow{v_a} = \overrightarrow{v_c} + \overrightarrow{\varpi} \times \overrightarrow{r_{ca}} = (v_x + \frac{1}{2}L\varpi\sin\theta)x + (v_y - \frac{1}{2}L\varpi\cos\theta)y,$$
$$\overrightarrow{v_b} = \overrightarrow{v_c} + \overrightarrow{\varpi} \times \overrightarrow{r_{cb}} = (v_x - \frac{1}{2}L\varpi\sin\theta)x + (v_y + \frac{1}{2}L\varpi\cos\theta)y.$$

Lorentz force $\vec{f} = q\vec{v} \times \vec{B}$.

$$\overrightarrow{f_x} = qB(v_y - \frac{1}{2}L\varpi\sin\theta)x + (-q)B(v_y + \frac{1}{2}L\varpi\cos\theta)x = -qBL\varpi\cos\theta x,$$

$$\overrightarrow{f_y} = qB(-v_x - \frac{1}{2}L\varpi\sin\theta)y + (-q)B(-v_x + \frac{1}{2}L\varpi\sin\theta)y = -qBL\varpi\sin\theta y$$

Net force $\vec{f} = \overrightarrow{f_x} + \overrightarrow{f_y} = -qBL\varpi(\cos\theta x + \sin\theta y)$. (net force is along the stick)

Net force only depends on rotation.

To caculate torques, only consider Lorentz forces perpendicular to the stick,

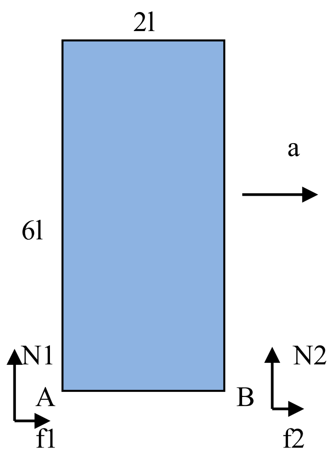
$\overrightarrow{M_a} = q(v_x \cos\theta \widehat{x + v_y \sin\theta y}) \times \vec{B} \times \overrightarrow{r_{ca}}$,

$\overrightarrow{M_b} = -q(v_x \cos\theta \widehat{x + v_y \sin\theta y}) \times \vec{B} \times \overrightarrow{r_{cb}}$.

Net torque: $\vec{M} = \overrightarrow{M_a} + \overrightarrow{M_b} = qBL(v_x\cos\theta + v_y\sin\theta)\hat{z}$.

Net torque only depends on translation.

Q4 (15 points). Refrigerator



(a) Rotation: $mgl = ma_{\max}(3l) \to a_{\max} = \frac{1}{3}g$

Translation: $\mu mg = ma_{\max} \to a_{\max} = \mu g \rangle \frac{1}{3}g$

Hence $a_{\max} = \frac{1}{3}g = 3.27m/s^2$

(b) $f_1 + f_2 = ma \quad N_1 + N_2 = mg$

Point A: $ma(3l) + N_2(2l) = mgl$

Then $N_1 = 0.65mg = 318.5N$, $N_2 = 0.35mg = 171.5N$

$f_1 : f_2 = N_1 : N_2 \quad f_1 = 0.65ma = 31.85N \quad f_2 = 0.35ma = 17.15N$

Net forces: $\overrightarrow{F_A} = 318.5N\widehat{y + 31.85N}\widehat{x =} 320.8N(\tan\theta = 10)$

$\overrightarrow{F_B} = 171.5N\widehat{y + 17.15N}\widehat{x =} 172.4N(\tan\theta = 10)$

Q5 (15 points)

Block-Spring System

Force balance (a) $kx = \mu Mg$, so $x = \frac{\mu Mg}{k}$, (2 points)

Energy conservation $\frac{1}{2}Mv_0^2 = \frac{1}{2}kx^2 + \mu Mgx$, so $v_0 = \mu g\sqrt{\frac{3M}{k}}$.(2 points)

(b) $kx = \mu Mg$, so $x = \frac{\mu Mg}{k}$, (2 points)

Momentum Conservation $(M + M)v_f = Mv_0$ (1 point)

Energy Conservation $\frac{1}{2}Mv_0^2 = \frac{1}{2}(M + M)v_f^2 + \frac{1}{2}kx^2 + \mu Mgx, \Rightarrow v_0 = \mu g\sqrt{\frac{6M}{k}}$. (3 points)

Q6 (20 points)

(a)

At the final stage, the number of moles of gas in the tank is $n_f = \frac{PV}{RT} = \frac{0.5P_0 \cdot 10V_0}{RT} = 5n$. So N= 5 cylinders are needed. (5 points)

(b)

$P_0 NV_0 = P(NV_0 + V) \Rightarrow N = 0.5(N+10) \Rightarrow N = 10$. (5 points)

(c)

First one, $P_1 = \frac{P_0 V_0}{V+V_0} = \frac{1}{11}P_0$, $n_1 = \frac{Vn}{V+V_0} = \frac{10}{11}n$, (1 point)

Second, $P_2 = \frac{(n+n_1)RT}{V+V_0} = \frac{(1+10/11)n}{11V_0}\frac{P_0 V_0}{n} = \frac{1}{11}(1+\frac{10}{11})P_0$, $n_2 = \frac{V(n+n_1)}{V+V_0} = \frac{10}{11}(1+\frac{10}{11})n$ (1 point)

Third, $P_3 = \frac{(n+n_2)RT}{V+V_0} = \frac{1}{11}(1+\frac{10}{11}+\frac{10\cdot 10}{11\cdot 11})P_0$, $n_3 = \frac{V(n+n_2)}{V+V_0} = \frac{10}{11}(1+\frac{10}{11}+\frac{10\cdot 10}{11\cdot 11})n$ (1 point)

$$P_2 - P_1 = \left(\frac{21}{11\cdot 11} - \frac{1}{11}\right)P_0 = \frac{10}{11\cdot 11}, \quad n_2 - n_1 = \frac{10\cdot 21}{11\cdot 11}n - \frac{10}{11}n = \frac{10}{11}\cdot\frac{10}{11}n$$

$$P_3 - P_2 = \frac{(11\cdot 11 + 10\cdot 21)}{11\cdot 11\cdot 11}P_0 - \frac{21}{11\cdot 11}P_0 = \frac{10\cdot 10}{11\cdot 11\cdot 11}P_0, \quad n_3 - n_2 = \frac{10\cdot 21}{11\cdot 11}n - \frac{10}{11}n = \frac{10\cdot 10\cdot 10}{11\cdot 11\cdot 11}n$$

$$n_N = \frac{(n+n_{N-1})V}{V+V_0} = \frac{10}{11}\left[1+\frac{10}{11}+\frac{10\cdot 10}{11\cdot 11}+\ldots+\left(\frac{10}{11}\right)^{N-1}\right]n = \frac{10}{11}\left[\frac{1-(10/11)^N}{1-10/11}\right]n = 10n[1-(10/11)^N],$$

(5 points)

$P_N = \frac{(n+n_{N-1})RT}{V+V_0} = [1-(10/11)^N]P_0$, $P_N = \frac{1}{2}P_0 \Rightarrow N = \frac{\ln 2}{\ln(11/10)} = 7.3$. (2 points)

So 8 cylinders are needed. (1 point)

Hong Kong Physics Olympiad 2013
2013 年香港物理奧林匹克競賽

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學苑
The Education Bureau of the HKSAR Government
香港特別行政區政府教育局
The Physical Society of Hong Kong
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

19 May, 2013
2013 年 5 月 19 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID Number in the field of “I. D. No.”, and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of “Course & Section No.”

   請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 “I. D. No.” 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格填上你的英文姓名，最後於 “Course & Section No. ” 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant I. D. Number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別注明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	$9.8\ \mathrm{m/s^2}$
Gravitational constant 萬有引力常數	G	$6.67 \times 10^{-11}\ \mathrm{N\ m^2/kg^2}$
Radius of Earth 地球半徑	R_E	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	r_E	1.5×10^{11} m
Mass of Sun 太陽質量	M_{Sun}	1.99×10^{30} kg
Mass of Earth 地球質量	M_E	5.98×10^{24} kg
Air Density 空氣密度	ρ_0	$1.2\ \mathrm{kg/m^3}$
Water Density 水密度	ρ_{w}	$1.0 \times 10^3\ \mathrm{kg/m^3}$

Trigonometric identities:
三角學恆等式:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \qquad \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sin 2x = 2\sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right] \qquad \cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

Multiple Choice Questions

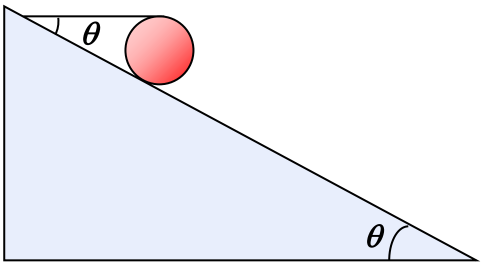
(Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.25 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.)

選擇題（每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.25 分，但全部選擇題最低為 0 分。）

1. A massive rope of mass m and length L, as shown in the figure, rests on a horizontal table. If the coefficient of static friction between the table and the rope is μ_s, what fraction of the rope can hang over the edge of the table without the rope sliding?
   如圖所示，一根質量為 m、長度為 L 的繩纜，靜放在水平桌面上。若繩纜與桌面的靜止摩擦係數為μ_s，則懸垂於桌緣外的繩纜，可以達到全長的什麼分數，而繩纜仍然不至滑下？

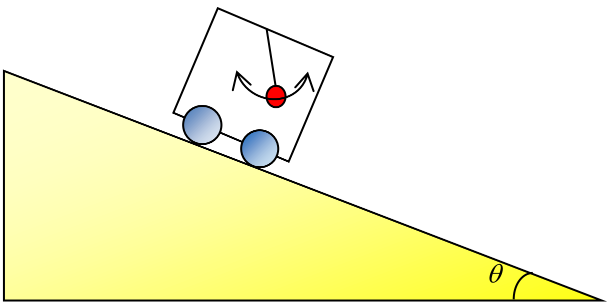
   A. $\dfrac{\mu_s}{1+\mu_s}$　　B. $\dfrac{\mu_s}{1+2\mu_s}$　　C. $1 - \mu_s$　　D. $\sqrt{1+\mu_s}$　　E. $\dfrac{2}{3}\mu_s$

2. As shown in the figure, a rigid sphere of mass m and radius R is held at rest by a horizontal string on an inclined plane with an inclination θ. If the sphere does not move, what is the minimum coefficient of static friction μ_s, between the sphere and the incline?
   如圖所示，有質量為 m、半徑為 R 的球體，被一平行方向的繩索固定於傾角為θ的斜面上。若球體不動，則球體與斜面之間的靜止摩擦係數，最小值應是什麼？



   A. $\tan\theta$　　B. $\dfrac{1-\cos\theta}{1+\cos\theta}$　　C. $\dfrac{\cos\theta}{1+\cos\theta}$　　D. $\dfrac{\sin\theta}{1+\cos\theta}$　　E. $1-\cos\theta$

3. A simple pendulum of length L is mounted in a massive cart that slides down a plane inclined at an angle θ with the horizontal. Find the period T of small oscillations of this pendulum if the cart moves down the plane with acceleration $a = g\sin\theta$.
   一部具相當質量的車上懸有一長度為 L 的簡單鐘擺，車子滑下傾角為θ的斜面。當車子以加速度 $a = g\sin\theta$在斜面向下運動時，求鐘擺以小幅度振動的周期 T。



A. $2\pi\sqrt{\dfrac{L}{g\cos\theta}}$ B. $2\pi\sqrt{\dfrac{L}{g\sqrt{2(1+\sin\theta)}}}$ C. $2\pi\sqrt{\dfrac{L\sin\theta}{g\cos\theta}}$

D. $2\pi\sqrt{\dfrac{L}{g\sqrt{1+3\sin^2\theta}}}$ E. $2\pi\sqrt{\dfrac{L}{g(1-\sin\theta)}}$

4. A stone of mass M is tied to a string. It is whirled in a vertical circle of radius L. At the highest point of the circle, the speed of the stone is v. Find the tension of the string.
一塊質量為 M 的石子繫於一根繩子上。石子在空中以半徑為 L 的垂直圓圈旋轉。在圓圈的最高點，石子的速率是 v。求繩子的張力。

A. $M\left(g+\dfrac{v^2}{L}\right)$ B. $M\left(g+\dfrac{v^2}{2L}\right)$ C. $M\left(g-\dfrac{v^2}{L}\right)$ D. $M\dfrac{v^2}{L}$ E. $M\left(\dfrac{v^2}{L}-g\right)$

5. A ring with mass m is hung vertically at the lower end of a uniform chain of total mass m and length L. Its upper end A is fixed, as shown in figure (a). The lower end B is raised until it is at the same position as A, and the ring slides to the midpoint of the string, as shown in figure (b). What is the minimum work required in this process?
有質量為 m 的小環，繫於一質量為 m、長度為 L 的均勻繩索的下端。繩索的上端固定，如圖(a)所示。繩索的下端被提到上端同樣高度，而小環滑到繩索的中點，如圖(b)所示。這過程所需的功，最小是多少？

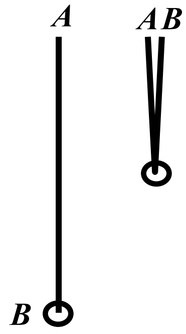
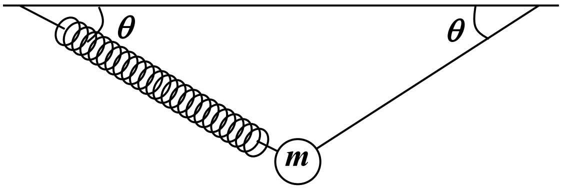


Fig. (a) Fig. (b)

A. mgL B. $\dfrac{3}{4}mgL$ C. $\dfrac{1}{2}mgL$ D. $\dfrac{1}{4}mgL$ E. $\dfrac{3}{2}mgL$

6. In a spacecraft orbiting around the Earth, an astronaut has a feeling of weightlessness. In Earth's reference frame, the explanation is
A. the weight of the astronaut becomes zero.
B. the gravitational field inside the spacecraft becomes zero.
C. the net force acting on the astronaut becomes zero.
D. the astronaut is falling freely.
E. there is no change in the momentum of the astronaut.
太空船環繞地球運動，其中的太空人感到沒有重量。在地球的參照系內，這現象的解釋是
A. 太空人的重量變為零。
B. 太空船內的引力場變為零。
C. 作用於太空人身上的力，總和為零。
D. 太空人在自由下墜。
E. 太空人的動量沒有改變。

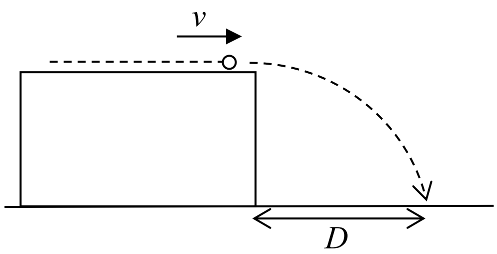
7. In the figure, a mass m is hung by a light spring and a light string at the ceiling. Both the spring and the string make an angle θ with the horizontal at equilibrium. If the string is suddenly cut, what is the instantaneous acceleration of the mass m?



如圖所示，有質量為 m 的物體，被輕量的彈簧和輕量的繩索繫於天花板上。在平衡狀態下，彈簧和繩索與水平方向都成角度 θ。若繩索突然斷了，物體的瞬時加速度是多少？

A. $\dfrac{2g}{\sin\theta}$ B. $\dfrac{g}{\sin\theta}$ C. $\dfrac{g}{2\sin\theta}$ D. $\dfrac{g}{\cos\theta}$ E. $\dfrac{g}{2\cos\theta}$

8. A particle is projected horizontally from the edge of a smooth table with initial speed v. The particle hits the ground at a horizontal distance D from the table. Different values of v are used and the corresponding values of D are recorded. Which of the following pair of quantities will give a straight-line curve?
有粒子從平滑桌子的邊緣以初速 v 拋射向地面，粒子在水平距離 D 處著地。不同的 v 值得到不同的 D 值。下面哪一對數量具有線性關係？



A. v and D　　B. v^2 and D　　C. v and D^2
D. v and $\frac{1}{D}$　　E. v and $\frac{1}{\sqrt{D}}$

9. Which of the following facts is/are direct evidence(s) supporting Newton's first law of motion?
(1) A feather and a coin spend equal time to reach the ground when dropped from the same height on the Moon surface.
(2) A satellite orbits around the Earth with uniform speed without supply of fuel.
(3) A man is thrown forward on a bus which stops suddenly.
下面哪些事實可以作為牛頓第一定律的直接證據？
(1) 一根羽毛和一枚硬幣從相同高度掉到月球表面，所需的時間相同。
(2) 衛星以均速環繞地球運動，不用消耗燃料。
(3) 當巴士突然停下來，乘客被拋向前。
A. (3) only　B. (1) and (2) only　C. (1) and (3) only　D. (2) and (3) only　E. (1), (2) and (3)

10. Two masses, m_A and m_B ($m_B > m_A$) are put on a smooth horizontal table as shown. The maximum static friction between the two masses is f. A gradually increasing horizontal force acts on m_A and the two masses accelerate together. The masses start to slip over each other when the force attains F_1. (figure (a)) If initially, the force acts on m_B instead, the masses start to slip over each other when the force attains F_2. (figure (b)) Compare F_1 and F_2.
如圖所示，兩物體，m_A 和 m_B ($m_B > m_A$)，被置於平滑的水平桌子上。兩物體之間的最大摩擦力為 f。有一沿水平方向作用於 m_A 的外力逐漸增加，使兩物體一起加速。當外力達到 F_1 時，兩物體之間產生滑動（圖(a)）。若從起首考慮，外力作用於 m_B 而非 m_A，則當外力達到 F_2 時，兩物體之間產生滑動（圖(b)）。試比較 F_1 和 F_2。
A. $F_1 > 2F_2$　B. $F_1 > F_2$　C. $F_1 = F_2$　D. $F_1 < F_2$　E. $F_1 < F_2/2$

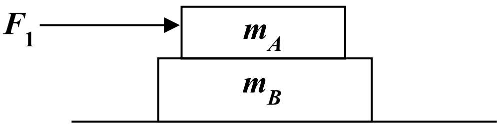


figure (a)

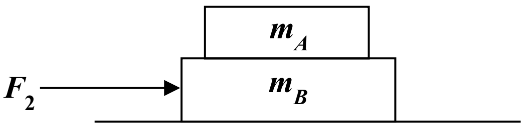
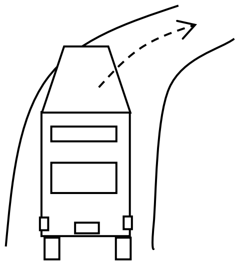


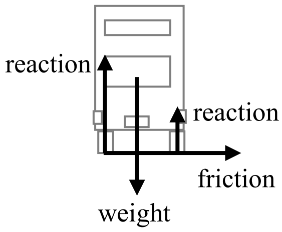
figure (b)

11. Planet P is moving in a circular orbit around a star X, while in another stellar system, planet Q is moving in a circular orbit around a star Y. The orbital radius of P is twice that of Q and the orbital period of P is also twice that of Q. Find the ratio of mass of X to that of Y.
行星 P 以圓形軌道環繞恆星 X，而在另一恆星系，行星 Q 以圓形軌道環繞恆星 Y。P 的軌道半徑為 Q 的兩倍，P 的軌道周期也為 Q 的兩倍。求 X 與 Y 的質量比。
A. 8:1　B. 4:1　C. 2:1　D. 1:1　E. 1:2

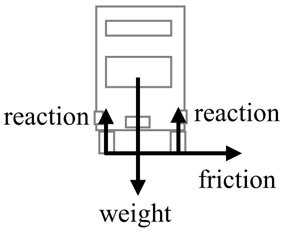
12. A bus is turning around a corner on a horizontal road. The diagram shows the rear view of the bus, which is turning to its right. Which of the following diagram best shows the force diagram of the bus?
有巴士在平路上拐彎，圖中所示為巴士右轉時，從後看的示意圖。下面哪張作用力圖最為正確？



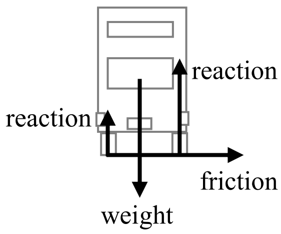
A.



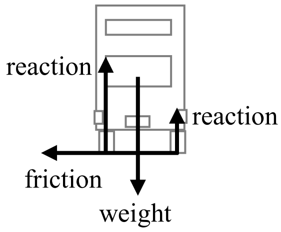
B.



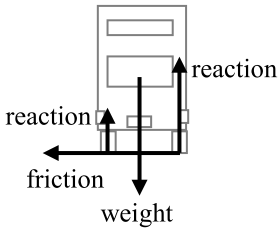
C.



D.

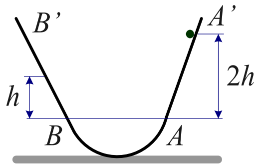


E.



13. A cylindrical trough is placed on a horizontal plane. The two edges of the trough are connected with smooth inclined planes *AA*' and *BB*' at locations *A* and *B* respectively. The trough edges and the inclined planes are tangential at the connections, and *A*, *B* are located at the same horizontal level, as shown in the figure. A small mass slides freely down the slope *A*'*A* from a height of 2*h* above the horizontal level *AB*. It enters the trough at position *A*, reaches *B* and then slides upward along slope *BB*' until it reaches a height of *h* above the horizontal level *AB*. It then slides downwards along slope *B*'*B* and enters the trough again. Neglecting air resistance, the small mass
一個圓弧形槽的底部放在水平地面上，槽的兩側與光滑斜坡 *AA*'、*BB*'相切，相切處 *A*、*B* 位於同一水平面內，槽與斜坡在豎直平面內的截面如圖所示。一小物塊從斜坡 *A*'*A* 上距水平面 *AB* 的高度為 2*h* 處沿斜坡自由滑下，並自 *A* 處入槽內，到達 *B* 後沿斜坡 *BB*'向上滑行，到達距水平面 *AB* 高度為 *h* 的位置；接著小物塊沿斜坡 *B'B* 滑下並從 *B* 處進入槽內。若不考慮空氣阻力，則小物塊

A. reaches a height lower than *A*	A. 到達低於 *A* 的位置
B. just reaches the location *A*	B. 恰好到達 *A* 處
C. passes *A* and reaches a height less than *h*	C. 過 *A* 處後上升高度小於 *h*
D. passes *A* and reaches the height *h*	D. 過 *A* 處後上升高度等於 *h*
E. passes *A* and reaches the height 2*h*	E. 過 *A* 處後上升高度等於 2*h*



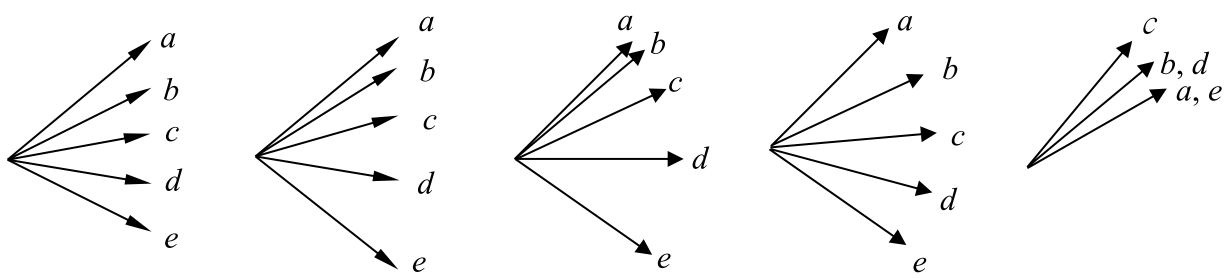
14. Consider a satellite of mass *m* orbiting around the Earth in a circular orbit of radius R_S. Given the radius of the Earth is R_E, and the gravitational acceleration on Earth's surface is *g*, and atmospheric resistance can be neglected, then the energy required to launch the satellite from Earth's surface is
一顆質量為 *m* 的人造衛星以圓形軌道環繞地球，軌道半徑為 R_S。設地球半徑為 R_E，地面重力加速度為 *g*，大氣層對衛星的阻力忽略不計，則從地面發射該人造衛星所需能量為

A. $mgR_E\left(1-\frac{R_E}{2R_S}\right)$ B. $mgR_E\left(1+\frac{R_E}{2R_S}\right)$ C. $mgR_E\left(\frac{1}{2}-\frac{R_E}{2R_S}\right)$

D. $mgR_E\left(\frac{1}{2}+\frac{R_E}{2R_S}\right)$ E. $mgR_E\left(\frac{R_S}{R_E}-1\right)$

15. The velocity vectors of an object performing projectile motion are drawn from time instants a to e at fixed time intervals. Which of the following gives a possible drawing?
把一物體在拋射過程中不同時間的速度矢量畫出來，其中時刻 a 到 e 的間隔相同。下面哪張圖是可能的？

A. B. C. D. E.

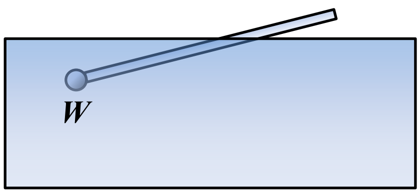


16. It is known that the gravitational acceleration is $g = GM_E / R_S^2$ and is directed vertically towards Earth's centre. Due to Earth's rotation, the acceleration **f** of a freely falling object in Hong Kong has a magnitude f different from g, and **f** is no longer pointing vertically downwards. In fact,
已知重力加速度為 $g = GM_E / R_S^2$ 並指向地球中心。因著地球轉動，在香港的自由落體加速度則為 **f**。它的數值 f 與 g 不同，而 **f** 的方向也不是完全垂直。其實，

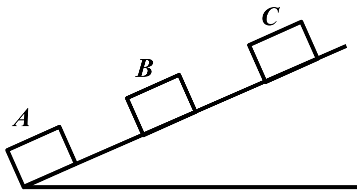
A. $f < g$ and **f** has a Northward component A. $f < g$ 而 **f** 偏向北
B. $f < g$ and **f** has a Southward component B. $f < g$ 而 **f** 偏向南
C. $f > g$ and **f** has a Northward component C. $f > g$ 而 **f** 偏向北
D. $f > g$ and **f** has a Southward component D. $f > g$ 而 **f** 偏向南
E. $f > g$ and **f** has a Eastward component E. $f > g$ 而 **f** 偏向東

17. A uniform rod floats in water. A ball with weight W is attached to one end of the rod, and the volume of the ball is negligible. This structure causes the rod to float at an inclined position, with the other end remaining above the water surface, as shown in the figure. If the part of the rod above the water surface is $1/n$ of the total length, calculate the weight of the rod.
一根均勻桿浮在水中，一端附著重量為 W 而體積可忽略不計的小球。這結構令桿子傾斜地浮著，致另一端保持在水面上。若水面上的桿長為全長的 $1/n$，求桿的重量。

A. $W(n+1)$ B. Wn C. $W(n-1)$ D. $W/(n+1)$ E. $W/(n-1)$

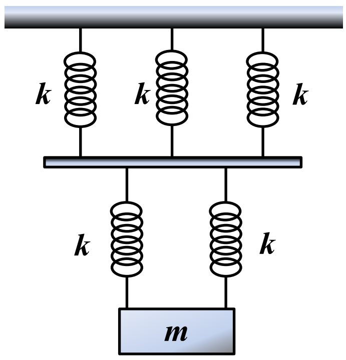


18. A block has an initial kinetic energy of 128 J. It slides up from point A at the bottom of the inclined plane with uniform deceleration. When it passes point B, its kinetic energy is reduced by 80 J, and its mechanical energy is reduced by 35 J. Calculate the work done against friction when the block moves from A to the highest point C on the inclined plane.
一滑塊以 128 J 的初動能，從斜面底端的 A 點沿斜面向上作勻減速直線運動，它經過 B 點時，動能減少了 80 J，機械能減少了 35 J，求滑塊從 A 到最高點 C 對摩擦力所作的功。
A. 42 J B. 48 J C. 56 J D. 72 J E. 128 J



19. A block of mass m is placed on a smooth horizontal surface and is attached to a spring of force constant k. When the block is pulled sideways and released, it undergoes simple harmonic motion. At the equilibrium position, its velocity is 3 m/s. Calculate its velocity when it moves to the position at two-third of the amplitude from the equilibrium position.
一質量為 m 的方塊置於平滑水平面，並繫著彈性係數為 k 的彈簧。當方塊被拉向一旁再放鬆時，方塊即進行簡諧運動。方塊在平衡位置時的速度為 3 m/s，求在振幅三分之二處的速度。
A. $\sqrt{2}$ m/s B. $\sqrt{3}$ m/s C. 2 m/s D. $\sqrt{5}$ m/s E. $\sqrt{6}$ m/s

20. As shown in the figure, a block of mass m is hung from the ceiling by the system of springs consisting of two layers. The upper layer consists of 3 springs in parallel, and the lower layer consists of 2 springs in parallel. The force constants of all springs are k. Calculate the frequency of the vertical oscillations of the block.
如圖所示，天花板下一組兩層的彈簧懸吊著質量為 m 的方塊。上層有 3 個並排的彈簧，下層則有 2 個並排的彈簧。所有彈簧的彈力常數都是 k。求方塊上下振動時的頻率。
A. $\frac{1}{2\pi}\sqrt{\frac{k}{5m}}$ B. $\frac{1}{2\pi}\sqrt{\frac{4k}{5m}}$ C. $\frac{1}{2\pi}\sqrt{\frac{5k}{6m}}$ D. $\frac{1}{2\pi}\sqrt{\frac{6k}{5m}}$ E. $\frac{1}{2\pi}\sqrt{\frac{5k}{2m}}$



《END OF MC's 選擇題完》

Open Problems 開放題

Total 5 problems 共 5 題

The Open Problem(s) with the '*' sign may require information on page 2.
帶 * 的開放題可能需要用到第二頁上的資料。

1*. James Bond Ski Chased by a Killer 殺手追殺占士邦 (10 marks)

In James Bond movie "The Spy Who Loved Me" (鐵金鋼勇破海底城), James skiied down a snowy slope in an attempt to escape from the killer. Unfortunately, the killer had a higher skiing speed than James's; they are 45 m/s and 40 m/s respectively. Now consider an alternative version of the movie.
James noted that he and the killer carried the same kind of rifle and estimated that their masses M (including body mass, skis, weapon, backpack, etc) were about the same. Recalling high school physics knowledge, James realised that each time he fired his weapon back at the killer, his momentum would change; whereas when the killer fired, the killer's momentum would also change. Every time James fired a bullet, the killer would fire back accordingly.

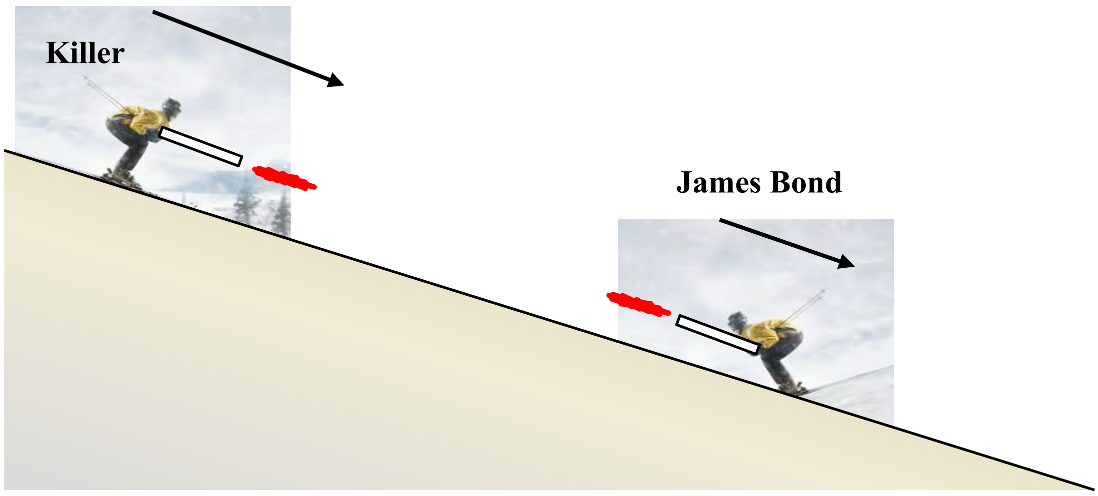
(a) How many bullets James had to fire in order to assure that the killer couldn't catch up with him? Assume that all bullets missed their targets (otherwise this exercise would terminate). Given M = 100 kg, mass and the muzzle velocity of a bullet are m = 0.02 kg and v = 500 m/s respectively.

(b) Traveling with the final velocity obtained in part (a), James Bond escaped by sliding down a cliff at an inclination of 20^o. After 20 seconds, he opened his parachute and landed safely. The average air drag during his fall is 600 N in both the vertical and horizontal directions, as long as the velocity components are nonzero. Calculate the landing position of James Bond and the height of the cliff. (Neglect the distance he traveled with a parachute.)

在占士邦電影"鐵金剛勇破海底城"裡，占士邦滑下雪坡，企圖逃離殺手的追殺。不幸殺手的滑速比占士邦高，分別為 45 m/s 和 40 m/s。現考慮電影的另一版本。
占士邦留意到殺手和他採用同樣的手槍，又推測兩人的質量 M（包括身體、滑雪裝備、武器、背包等）相若。他想起中學學到的物理學，理解到每當他向對方開火，他的動量就會改變；同樣當對方開火，對方的動量也會改變。每次占士邦發射一顆子彈，對方也同樣回射一顆。

(a) 占士邦需要發射多少顆子彈，才能保證殺手不會追上他？設 M = 100 kg，子彈的質量為 m = 0.02 kg，手槍射擊速度為 v = 500 m/s。

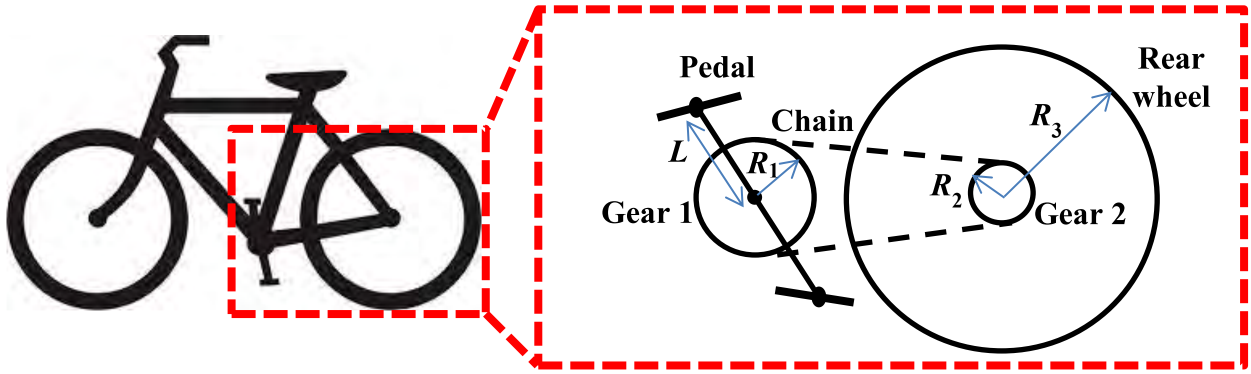
(b) 占士邦以(a)部的最終速度，滑下傾角為 20^o 的懸崖，成功逃脫。20 秒後，他打開降傘，安全著陸。設他在下墜過程中，垂直方向和水平方向的空氣阻力平均值均為 600 N(只要該速度的分量為非零)，計算占士邦著陸的位置和懸崖的高度。（可忽略降傘滑翔的距離。）



2. The Bicycle 自行車 (10 marks)

A student rides a bicycle on a slope of inclination θ. Due to air drag, he found that the bicycle can barely move down the slope without his pedaling. He would like to estimate the power he needs to drive the bicycle up the same slope at a uniform velocity.
To achieve this, he measured that during the up-slope drive, one of his feet pedaled N cycles in a time interval T (assuming that the pedaling is continuous and at a uniform rate). He also obtained the following data: the total mass of the bicycle and the rider m, length of pedal crank L, radius of gear 1 R_1, radius of gear 2 R_2, radius of rear wheel R_3, as shown in the figure.
It is given that the air drags during the up-slope and down-slope drives have the same magnitude, and there are no slippings between the wheels and the slope during both the up-slope and down-slope drives. The energy loss due to the relative motion of the bicycle components is negligible.
(a) Derive an expression for the force needed to drive the bicycle up-slope at uniform velocity.
(b) Derive an expression for the power needed to drive the bicycle up-slope at uniform velocity.

某同學選了一個傾角為 θ 的斜坡，因有風阻，他騎在自行車上剛好能在不踩踏板的情況下讓自行車沿斜坡勻速向下行駛。現在他想估測沿此斜坡向上勻速行駛時的功率。
為此，他數出在上坡過程中某一隻腳蹬踩踏板的圈數 N(設不間斷的勻速蹬踏)，並測得所用的時間 T，再測得下列相關資料：自行車和人的總質量 m、踏板桿的長度 L、輪盤半徑 R_1、飛輪半徑 R_2、車後輪半徑 R_3，如圖所示。
已知上、下坡過程中的風阻大小相等，不論是在上坡還是下坡過程中，車輪與坡面接觸處都無滑動。不計自行車內部各部件之間因相對運動而消耗的能量。
(a) 試導出駕駛自行車勻速上坡所需作用力的表達式。
(b) 試導出估測功率的表達式。

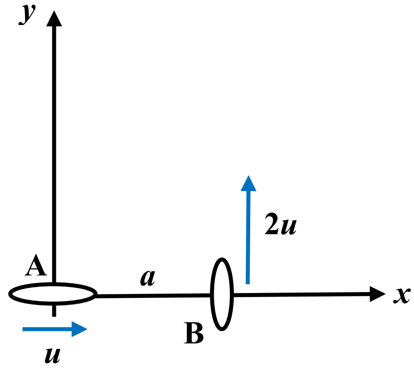


3. The Ships 輪船 (10 marks)

Consider two ships on the sea as shown in the figure. Ship A moves with velocity u directed to East. Ship B moves with velocity $2u$ directed to North. At time $t = 0$, ship B crosses the path of ship A at a distance a in front of ship A.
(a) Find the shortest distance between the ships, and the time they reaches this position.
(b) Suppose at $t = 0$, the speed of ship A remains at u, but its direction can be adjusted. What should be the direction of ship A such that the shortest distance between the two ships is minimum, and how much is this minimum shorter than the result obtained in part (a)?

如圖所示，海面上有兩艘輪船，船 A 以速度 u 向正東方向航行，船 B 以速度 $2u$ 向正北方向航行。在時刻 $t = 0$ 時，船 B 恰好同時經過船 A 的航線並位於船 A 的前方，船 B 到船 A 的距離為 a。

(a) 求兩船最接近的距離，和到達這距離的時刻。
(b) 假如在時刻 $t = 0$，船 A 的速率仍是 u，但方向可以自由調節，那麼船 A 應走什麼方向，兩船最接近的距離才是最短，這最短距離比(a)部得出的距離短多少?



4*. STEP (15 marks)

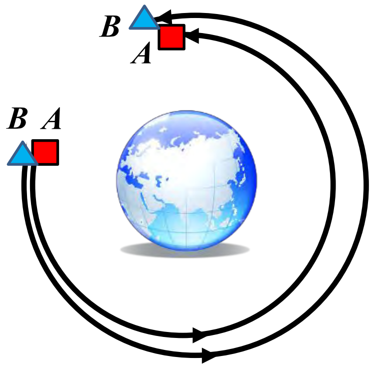
According to Newton's second law of motion, $F = m_I a$, where m_I is the *inertial mass*. According to Newton's law of universal gravitation, the gravitational force between Earth and an object is $F = GM_E m_G / R^2$, where m_G is the *gravitational mass* of the object (here R is the distance between Earth's center and the object). Presently, it is widely accepted that $m_G = m_I$, but some physicists would like to test the validity of this assumption. If there is a difference between m_I and m_G, even as small as one part in 10^{18}, our present understanding about gravity has to be revised. Hence they proposed a satellite experiment called STEP to measure the mass ratio $r = m_G/m_I$. (STEP represents Satellite Test of the Equivalence Principle.)

In the proposed experiment, several test bodies are enclosed in a vacuum box in a satellite that orbits around the Earth. The box protects the test bodies from outside disturbances and all forces from the satellite acting on the test bodies have been carefully eliminated, so that each test body can be considered as a mini-satellite orbiting around the Earth.

(a) The proposed satellite has a circular orbit with a period of 24 hours. Calculate its orbital radius R. Express your answer in multiples of R_E, the radius of Earth.
(b) Consider test bodies A and B with mass ratios r_A and r_B respectively, as shown in the figure. Suppose the two bodies have the same position at a point on the orbit. When body A completes one orbit, what is the displacement of body B relative to body A?
(c) Simplify your result in part (b) using the approximation $(1 + x)^n \approx 1 + nx$ when $|x| << 1$. Suppose the position sensors in the satellite can detect position changes as little as 10^{-15} m. What is the duration of the satellite flight before differences in the mass ratio of the order 10^{-18} can be detected? Express your answer in hours.

根據牛頓運動第二定律，$F = m_I a$，其中 m_I是*慣性質量*。根據牛頓萬有引力定律，地球作用於物體的引力是 $F = GM_E m_G / R^2$，其中 m_G 是*引力質量*（這裡 R 是地球中心與物體的距離）。現時普遍接納 $m_G = m_I$，但有物理學家希望驗證這假設的真確性。如果 m_I與 m_G 有差別，即使是 10^{18} 分之一那麼微小，我們對萬有引力的理解也要改寫。所以他們提出名為 STEP 的衛星實驗以量度質量比 $r = m_G/m_I$。（STEP 代表 Satellite Test of the Equivalence Principle。）
在提出的實驗中，環繞地球的衛星中有一真空箱，數個實驗物封在其中。箱子保護實驗物免受外力干繞，衛星作用在實驗物上的所有作用力都被仔細隔絕，使每一實驗物都可考慮成環繞地球的袖珍衛星。

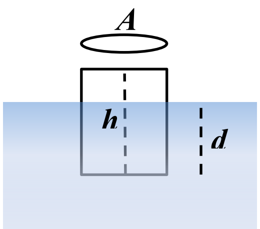
(a) 提出的衛星軌道為圓形，周期為 24 小時。試計算其軌道半徑 R。答案應以地球半徑 R_E的倍數為單位。
(b) 考慮實驗物 A 和 B，質量比分別為 r_A 和 r_B，如圖所示。設兩物在軌道某點位置相同。當物體 A 完成一周，物體 B 相對於物體 A 的位移是多少?
(c) 試用$|x| << 1$時的近似$(1 + x)^n \approx 1 + nx$簡化(b)部的結果。設衛星上的位置感應器可檢測到小至 10^{-15} m 的位移。衛星航行多久，才能檢測到 10^{-18} 量階的質量比？答案應以小時為單位。

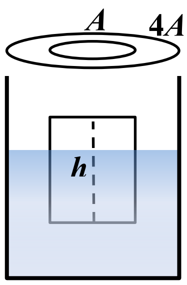


5. The Floating Ice 浮冰 (15 marks)

As shown in the left figure, a cylindrical piece of ice floats in water. Its cross sectional area is A and its height is h. The density of ice and water are ρ_I and ρ_W respectively.
(a) Find d, the depth of ice immersed in water.
(b) Suppose the ice is pushed slightly in the vertical direction. Find the frequency of oscillations. You may assume that the motion of water is negligible.
(c) Suppose the ice floats in water in a container of cross-section area $4A$, as shown in the right figure. The ice is displaced from equilibrium by z in the vertical direction. (i) Calculate the change in the total potential energy of the system, up to order z^2. (ii) Calculate the kinetic energy of the system during the push when the ice moves at velocity v. You may assume that the water below the bottom level of the ice does not move, and the water above the bottom level of the ice moves with the same velocity. (iii) Hence find the frequency of oscillations.

如左圖所示，一圓柱形的冰塊浮在水里。它的橫切面積是 A，高度是 h。冰和水的密度分別為 ρ_I和ρ_W。
(a) 求冰塊浸在水里的深度 d。
(b) 設冰塊在垂直方向被輕推，求它的振動頻率。你可假設水的流動可以忽略。
(c) 如右圖所示，設冰塊浮在容器的水裡，容器的橫切面積為 $4A$。冰塊在垂直方向被推，相對於平衡態的位移為 z。(i) 試計算系統總位能的改變，算至 z^2 階。(ii) 試計算當在冰塊被推過程中，冰塊速率為 v 時的系統總動能。你可假設低於冰塊底部水平的水不動，而高於冰塊底部水平的水以同一速度流動。(iii) 由此找出系統的振動頻率。





《END 完》

Hong Kong Physics Olympiad 2013
2013 年香港物理奧林匹克競賽

Jointly Organized by

The Hong Kong Academy for Gifted Education
香港資優教育學苑
The Education Bureau of the HKSAR Government
香港特別行政區政府教育局
The Physical Society of Hong Kong
香港物理學會
The Hong Kong University of Science and Technology
香港科技大學

共同舉辦

19 May, 2013
2013 年 5 月 19 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English.

   所有題目均爲中英對照。你可選擇以中文或英文作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你亦可開始作答開放式題目，而無須等候任何宣佈。

3. Please follow the instructions on the multiple-choice answer sheet, and use a HB pencil to write your 8-dight Participant ID Number in the field of “I. D. No.”, and fill out the appropriate circles **fully**. After that, write your English name in the space provided and your Hong Kong ID number in the field of “Course & Section No.”

   請依照選擇題答題紙的指示，用 HB 鉛筆在選擇題答題紙的 “I. D. No.” 欄上首先寫上你的 8 位數字參賽號碼，並把相應寫有數字的圓圈完全塗黑，然後在適當的空格填上你的英文姓名，最後於 “Course & Section No. ” 欄內填上你的身分證號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. On the cover of the answer book, please write your Hong Kong ID number in the field of "Course Title", and write your English name in the field of "Student Name" and your 8-dight Participant I. D. Number in the field of "Student Number". You can write your answers on both sides of the sheets in the answer book.

   在答題簿封面上，請於 Course Title 欄中填上你的身分證號碼；請於 Student Name 欄中填上你的英文姓名；請於 Student Number 欄中填上你的 8 位數字參賽號碼。 答題簿可雙面使用。

6. The information provided in the text and in the figure of a question should be put to use together.

   解題時要將文字和簡圖提供的條件一起考慮。

7. Some open problems are quite long. Read the entire problem before attempting to solve them. If you cannot solve the whole problem, try to solve some parts of it. You can even use the answers in some unsolved parts as inputs to solve the others parts of a problem.

   開放題較長，最好將整題閱讀完後才著手解題。若某些部分不會做，也可把它們的答案當作已知來做其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別注明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m/s^2
Gravitational constant 萬有引力常數	G	6.67×10^{-11} N m^2/kg^2
Radius of Earth 地球半徑	R_E	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	r_E	1.5×10^{11} m
Mass of Sun 太陽質量	M_{Sun}	1.99×10^{30} kg
Mass of Earth 地球質量	M_E	5.98×10^{24} kg
Air Density 空氣密度	ρ_0	1.2 kg/m^3
Water Density 水密度	ρ_w	1.0×10^3 kg/m^3

Trigonometric identities:
三角學恆等式:

$$\sin(x+y) = \sin x \cos y + \cos x \sin y \qquad \cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sin 2x = 2\sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right] \qquad \cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

$$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$$

Multiple Choice Questions

(Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.25 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.)

選擇題（每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.25 分，但全部選擇題最低為 0 分。）

1. A massive rope of mass m and length L, as shown in the figure, rests on a horizontal table. If the coefficient of static friction between the table and the rope is μ_s, what fraction of the rope can hang over the edge of the table without the rope sliding?
如圖所示，一根質量為 m、長度為 L 的繩纜，靜放在水平桌面上。若繩纜與桌面的靜止摩擦係數為μ_s，則懸垂於桌緣外的繩纜，可以達到全長的什麼分數，而繩纜仍然不至滑下？

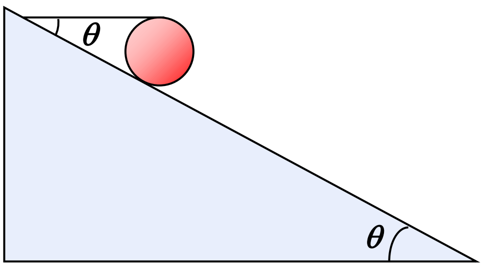
A. $\dfrac{\mu_s}{1+\mu_s}$ B. $\dfrac{\mu_s}{1+2\mu_s}$ C. $1 - \mu_s$ D. $\sqrt{1+\mu_s}$ E. $\dfrac{2}{3}\mu_s$

Solution:
Assume that a length rL is on the table, so the length $(1 – r)L$ is the part of the rope which hangs over the edge of the table.
The tension in the rope at the edge of the table is then $(1 – r)mg$, and the friction force on the part of the rope on the table is $f = \mu_s (rmg)$. This must be the same as the tension in the rope at the edge of the table, so $(1 – r)mg = \mu_s(rmg)$ and $r = 1/(1+ \mu_s)$.
The fraction that hangs over the edge is

$$\frac{(1-r)L}{L} = \frac{\mu_s}{1+\mu_s}$$

Note: the result is independent of L and mg of the rope.
Answer: A.

2. As shown in the figure, a rigid sphere of mass m and radius R is held at rest by a horizontal string on an inclined plane with an inclination θ. If the sphere does not move, what is the minimum coefficient of static friction μ_s, between the sphere and the incline?
如圖所示，有質量為 m、半徑為 R 的球體，被一平行方向的繩索固定於傾角為θ的斜面上。若球體不動，則球體與斜面之間的靜止摩擦係數，最小值應是什麼？



A. $\tan\theta$ B. $\dfrac{1-\cos\theta}{1+\cos\theta}$ C. $\dfrac{\cos\theta}{1+\cos\theta}$ D. $\dfrac{\sin\theta}{1+\cos\theta}$ E. $1 - \cos\theta$

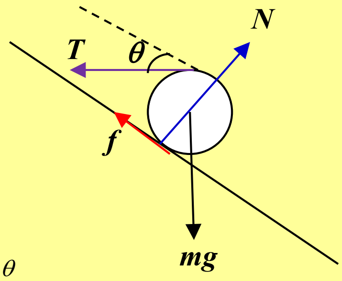
Solution:
Consider the torques about an axis through the center of the sphere:
$TR – fR = 0 \;\Rightarrow\; T = f$

Apply $\Sigma F_x = 0$ to the sphere:

$f + T\cos\theta - mg\sin\theta = 0$

$f(1+\cos\theta) = mg\sin\theta$

$$f = \frac{mg\sin\theta}{1+\cos\theta}$$



Similarly, apply $\Sigma F_y = 0$ to the sphere:

$N - mg\cos\theta - T\sin\theta = 0$

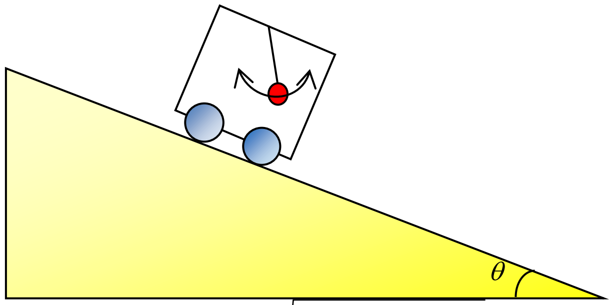
$$N = mg\cos\theta + f\sin\theta = mg\cos\theta + \left(\frac{mg\sin^2\theta}{1+\cos\theta}\right) = mg\cdot\frac{\cos\theta+\cos^2\theta+\sin^2\theta}{1+\cos\theta}$$

$$= mg\cdot\frac{\cos\theta+1}{1+\cos\theta} = mg$$

$$\therefore\ f \le \mu_s N \quad \Rightarrow \mu_s \ge \frac{f}{N} = \frac{mg\sin\theta}{1+\cos\theta} \div mg = \frac{\sin\theta}{1+\cos\theta}$$

Answer: D.

3. A simple pendulum of length L is mounted in a massive cart that slides down a plane inclined at an angle θ with the horizontal. Find the period T of small oscillations of this pendulum if the cart moves down the plane with acceleration $a = g\sin\theta$.
一部具相當質量的車上懸有一長度為 L 的簡單鐘擺，車子滑下傾角為θ的斜面。當車子以加速度 $a = g\sin\theta$在斜面向下運動時，求鐘擺以小幅度振動的周期 T。

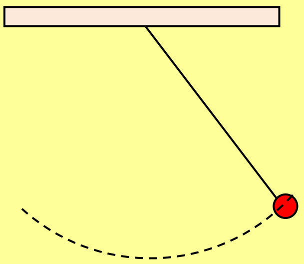


A. $2\pi\sqrt{\dfrac{L}{g\cos\theta}}$ B. $2\pi\sqrt{\dfrac{L}{g\sqrt{2(1+\sin\theta)}}}$ C. $2\pi\sqrt{\dfrac{L\sin\theta}{g\cos\theta}}$

D. $2\pi\sqrt{\dfrac{L}{g\sqrt{1+3\sin^2\theta}}}$ E. $2\pi\sqrt{\dfrac{L}{g(1-\sin\theta)}}$

Solution:

The cart accelerates down the plane with a constant acceleration of $g\sin\theta$. This happens because the cart is much more massive than the bob, so the motion of the cart is unaffected by the motion of the bob oscillating back and forth. The path of the bob is quite complex in the reference frame of the inclined plane, but in the reference frame moving with the cart the path of the bob is much simpler—in this frame the bob moves back and forth along a circular arc.



To find the magnitude of effective gravity acting on the bob, g_{eff}, first we draw the vector addition diagram so as to enable us to have a clear picture of the motion of the bob with respect to the cart.

Using the cosine rule, g_{eff} is given by

$$g_{eff}^2 = g^2 + g^2\sin^2\theta - 2(g)(g\sin\theta)\cos\left(\frac{\pi}{2}-\theta\right)$$

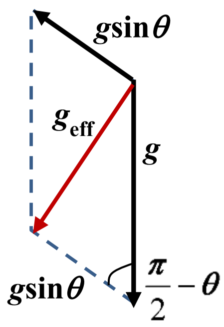
$$= g^2 + g^2\sin^2\theta - 2g^2\sin^2\theta = g^2\cos^2\theta$$

Thus, $g_{\text{eff}} = g\cos\theta$

The period of this motion is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g_{eff}}}$

Substituting $g\cos\theta$ for g_{eff} in the equation, we have

$$T = 2\pi\sqrt{\frac{L}{g\cos\theta}}$$



Answer: A.

4. A stone of mass M is tied to a string. It is whirled in a vertical circle of radius L. At the highest point of the circle, the speed of the stone is v. Find the tension of the string.
一塊質量為 M 的石子繫於一根繩子上。石子在空中以半徑為 L 的垂直圓圈旋轉。在圓圈的最高點，石子的速率是 v。求繩子的張力。

A. $M\left(g+\frac{v^2}{L}\right)$ B. $M\left(g+\frac{v^2}{2L}\right)$ C. $M\left(g-\frac{v^2}{L}\right)$ D. $M\frac{v^2}{L}$ E. $M\left(\frac{v^2}{L}-g\right)$

Solution:
Consider forces acting on the stone. Using Newton's second law,

$$T + Mg = M\frac{v^2}{L} \Rightarrow T = M\left(\frac{v^2}{L}-g\right)$$

Answer: E.

5. A ring with mass m is hung vertically at the lower end of a uniform chain of total mass m and length L. Its upper end A is fixed, as shown in figure (a). The lower end B is raised until it is at the same position as A, and the ring slides to the midpoint of the string, as shown in figure (b). What is the minimum work required in this process?
有質量為 m 的小環，繫於一質量為 m、長度為 L 的均勻繩索的下端。繩索的上端固定，如圖(a)所示。繩索的下端被提到上端同樣高度，而小環滑到繩索的中點，如圖(b)所示。這過程所需的功，最小是多少？

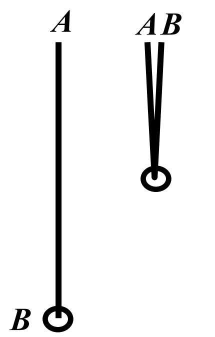


Fig. (a) Fig. (b)

A. mgL B. $\frac{3}{4}mgL$ C. $\frac{1}{2}mgL$ D. $\frac{1}{4}mgL$ E. $\frac{3}{2}mgL$

Solution:
Consider the center of mass of the chain. Its height increases by $L/2 - L/4$.
Consider the center of mass of the bob. Its height increases by $L - L/2$.
Total work done = increase in potential energy

$$= mg\left(\frac{L}{2}-\frac{L}{4}\right) + mg\left(L-\frac{L}{2}\right) = \frac{3}{4}mgL$$

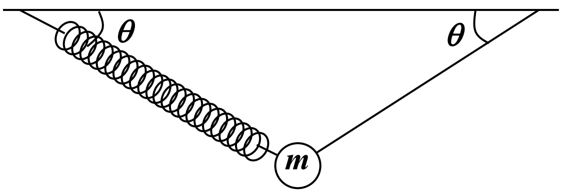
Answer: B.

6. In a spacecraft orbiting around the Earth, an astronaut has a feeling of weightlessness. In Earth's reference frame, the explanation is
   A. the weight of the astronaut becomes zero.
   B. the gravitational field inside the spacecraft becomes zero.
   C. the net force acting on the astronaut becomes zero.

D. the astronaut is falling freely.
E. there is no change in the momentum of the astronaut.
太空船環繞地球運動，其中的太空人感到沒有重量。在地球的參照系內，這現象的解釋是
A. 太空人的重量變為零。
B. 太空船內的引力場變為零。
C. 作用於太空人身上的力，總和為零。
D. 太空人在自由下墜。
E. 太空人的動量沒有改變。

Answer: D.

7. In the figure, a mass m is hung by a light spring and a light string at the ceiling. Both the spring and the string make an angle θ with the horizontal at equilibrium. If the string is suddenly cut, what is the instantaneous acceleration of the mass m?
如圖所示，有質量為 m 的物體，被輕量的彈簧和輕量的繩索繫於天花板上。在平衡狀態下，彈簧和繩索與水平方向都成角度 θ。若繩索突然斷了，物體的瞬時加速度是多少？



A. $\frac{2g}{\sin\theta}$ B. $\frac{g}{\sin\theta}$ C. $\frac{g}{2\sin\theta}$ D. $\frac{g}{\cos\theta}$ E. $\frac{g}{2\cos\theta}$

Solution:
Consider the balance of the vertical forces.

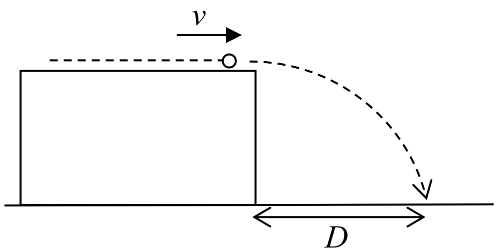
$$2T\sin\theta = mg \quad \Rightarrow \quad T = \frac{mg}{2\sin\theta}$$

When the string is suddenly cut, the mass accelerates due to the gravitational force and the tension in the spring. When these two forces are added, the result is minus the tension of the string. Using Newton's second law,

$$\frac{mg}{2\sin\theta} = ma \quad \Rightarrow \quad a = \frac{g}{2\sin\theta}$$

Answer: C.

8. A particle is projected horizontally from the edge of a smooth table with initial speed v. The particle hits the ground at a horizontal distance D from the table. Different values of v are used and the corresponding values of D are recorded. Which of the following pair of quantities will give a straight-line curve?
有粒子從平滑桌子的邊緣以初速 v 拋射向地面，粒子在水平距離 D 處著地。不同的 v 值得到不同的 D 值。下面哪一對數量具有線性關係？



A. v and D B. v^2 and D C. v and D^2
D. v and $\frac{1}{D}$ E. v and $\frac{1}{\sqrt{D}}$

Solution:

The time of flight of the projectile is given by $h = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}}$

Hence $D = vt = v\sqrt{\frac{2h}{g}}$

Answer: A.

9. Which of the following facts is/are direct evidence(s) supporting Newton's first law of motion?
   (1) A feather and a coin spend equal time to reach the ground when dropped from the same height on the Moon surface.
   (2) A satellite orbits around the Earth with uniform speed without supply of fuel.
   (3) A man is thrown forward on a bus which stops suddenly.
   下面哪些事實可以作為牛頓第一定律的直接證據？
   (1) 一根羽毛和一枚硬幣從相同高度掉到月球表面，所需的時間相同。
   (2) 衛星以均速環繞地球運動，不用消耗燃料。
   (3) 當巴士突然停下來，乘客被拋向前。
   A. (3) only B. (1) and (2) only C. (1) and (3) only D. (2) and (3) only E. (1), (2) and (3)

Answer: A.

10. Two masses, m_A and m_B ($m_B > m_A$) are put on a smooth horizontal table as shown. The maximum static friction between the two masses is f. A gradually increasing horizontal force acts on m_A and the two masses accelerate together. The masses start to slip over each other when the force attains F_1. (figure (a)) If initially, the force acts on m_B instead, the masses start to slip over each other when the force attains F_2. (figure (b)) Compare F_1 and F_2.
    如圖所示，兩物體，m_A和 m_B ($m_B > m_A$)，被置於平滑的水平桌子上。兩物體之間的最大摩擦力為 f。有一沿水平方向作用於 m_A的外力逐漸增加，使兩物體一起加速。當外力達到 F_1時，兩物體之間產生滑動（圖(a)）。若從起首考慮，外力作用於 m_B而非 m_A，則當外力達到 F_2時，兩物體之間產生滑動（圖(b)）。試比較 F_1和 F_2。
    A. $F_1 > 2F_2$ B. $F_1 > F_2$ C. $F_1 = F_2$ D. $F_1 < F_2$ E. $F_1 < F_2/2$

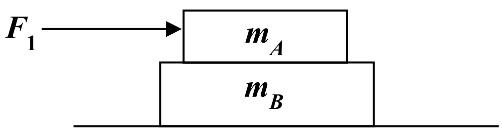


figure (a)

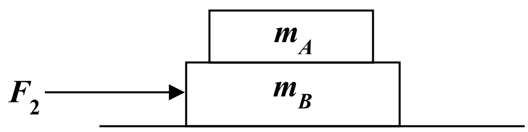


figure (b)

Solution:

In figure (a), $F_1 = (m_A + m_B)a$ and $f = m_B a$ $\Rightarrow$ $f = \frac{m_B}{m_A + m_B}F_1$ $\Rightarrow$ $F_1 = \mu m_A g\left(\frac{m_A + m_B}{m_B}\right)$.

In figure (b), $F_2 = (m_A + m_B)a$ and $f = m_A a$ $\Rightarrow$ $f = \frac{m_A}{m_A + m_B}F_2$ $\Rightarrow$ $F_2 = \mu m_A g\left(\frac{m_A + m_B}{m_A}\right)$.

$\Rightarrow \frac{F_1}{F_2} = \frac{m_A}{m_B} < 1$

Answer: D.

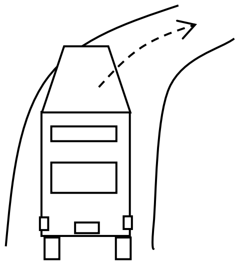
11. Planet P is moving in a circular orbit around a star X, while in another stellar system, planet Q is moving in a circular orbit around a star Y. The orbital radius of P is twice that of Q and the orbital period of P is also twice that of Q. Find the ratio of mass of X to that of Y.
    行星 P 以圓形軌道環繞恆星 X，而在另一恆星系，行星 Q 以圓形軌道環繞恆星 Y。P 的軌道半徑為 Q 的兩倍，P 的軌道周期也為 Q 的兩倍。求 X 與 Y 的質量比。
    A. 8:1 B. 4:1 C. 2:1 D. 1:1 E. 1:2

Solution:
Using Newton's second law ad Newton's law of univerasal gravitation, we obtain

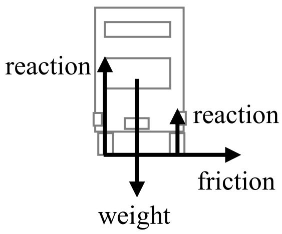
$$T^2 = \frac{4\pi^2}{GM}R^3 \quad \Rightarrow \quad M = \frac{4\pi^2}{G}\frac{R^3}{T^2} \quad \Rightarrow \quad \frac{M_X}{M_Y} = \left(\frac{R_P^3}{R_Q^3}\right)\left(\frac{T_Q^2}{T_P^2}\right) = \left(\frac{2^3}{1^3}\right)\left(\frac{1}{2^2}\right) = 2$$

Answer: C.

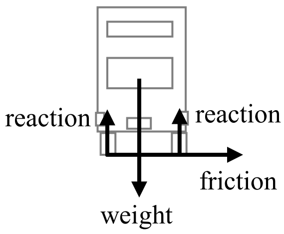
12. A bus is turning around a corner on a horizontal road. The diagram shows the rear view of the bus, which is turning to its right. Which of the following diagram best shows the force diagram of the bus?
有巴士在平路上拐彎，圖中所示為巴士右轉時，從後看的示意圖。下面哪張作用力圖最為正確？



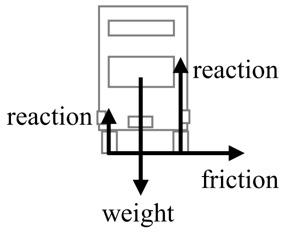
A.



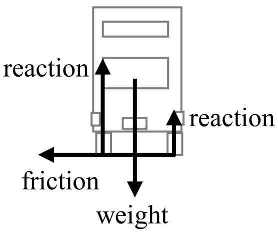
B.



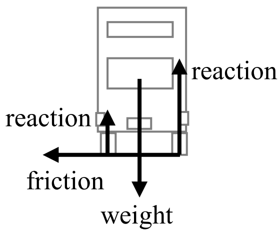
C.



D.



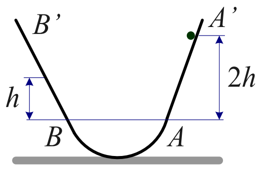
E.



Answer: A.

13. A cylindrical trough is placed on a horizontal plane. The two edges of the trough are connected with smooth inclined planes *AA*' and *BB*' at locations *A* and *B* respectively. The trough edges and the inclined planes are tangential at the connections, and *A*, *B* are located at the same horizontal level, as shown in the figure. A small mass slides freely down the slope *A*'*A* from a height of 2*h* above the horizontal level *AB*. It enters the trough at position *A*, reaches *B* and then slides upward along slope *BB*' until it reaches a height of *h* above the horizontal level *AB*. It then slides downwards along slope *B*'*B* and enters the trough again. Neglecting air resistance, the small mass
一個圓弧形槽的底部放在水平地面上，槽的兩側與光滑斜坡 *AA*'、*BB*'相切，相切處 *A*、*B* 位於同一水平面內，槽與斜坡在豎直平面內的截面如圖所示。一小物塊從斜坡 *A*'*A* 上距水平面 *AB* 的高度為 2*h* 處沿斜坡自由滑下，並自 *A* 處入槽內，到達 *B* 後沿斜坡 *BB*'向上滑行，到達距水平面 *AB* 高度為 *h* 的位置；接著小物塊沿斜坡 *B*'*B* 滑下並從 *B* 處進入槽內。若不考慮空氣阻力，則小物塊

A. reaches a height lower than *A* | A. 到達低於 *A* 的位置
B. just reaches the location *A* | B. 恰好到達 *A* 處
C. passes *A* and reaches a height less than *h* | C. 過 *A* 處後上升高度小於 *h*
D. passes *A* and reaches the height *h* | D. 過 *A* 處後上升高度等於 *h*
E. passes *A* and reaches the height 2*h* | E. 過 *A* 處後上升高度等於 2*h*



Solution:
When the mass slides down from AA', it loses potential energy mgh and reaches a height of h above AB. If the frictional force remains the same when it slides backwards, then it should lose the same amount of potential energy and hence should reach a height of 0 above AB. However, during the slides, the frictional force is given by

$$f = \mu N = \mu\left(mg + m\frac{v^2}{R}\right)$$

Since the velocity during the second slide is reduced compared with the first slide, the frictional force is reduced, and the potential energy loss is reduced. This enables the small mass to reach a height above A, but the height cannot exceed h.
Answer: C.

14. Consider a satellite of mass m orbiting around the Earth in a circular orbit of radius R_S. Given the radius of the Earth is R_E, and the gravitational acceleration on Earth's surface is g, and atmospheric resistance can be neglected, then the energy required to launch the satellite from Earth's surface is
一顆質量為 m 的人造衛星以圓形軌道環繞地球，軌道半徑為 R_S。設地球半徑為 R_E，地面重力加速度為 g，大氣層對衛星的阻力忽略不計，則從地面發射該人造衛星所需能量為

A. $mgR_E\left(1-\frac{R_E}{2R_S}\right)$ B. $mgR_E\left(1+\frac{R_E}{2R_S}\right)$ C. $mgR_E\left(\frac{1}{2}-\frac{R_E}{2R_S}\right)$

D. $mgR_E\left(\frac{1}{2}+\frac{R_E}{2R_S}\right)$ E. $mgR_E\left(\frac{R_S}{R_E}-1\right)$

Solution:
Orbital velocity of the satellite $u=\sqrt{\frac{GM}{R_S}}$

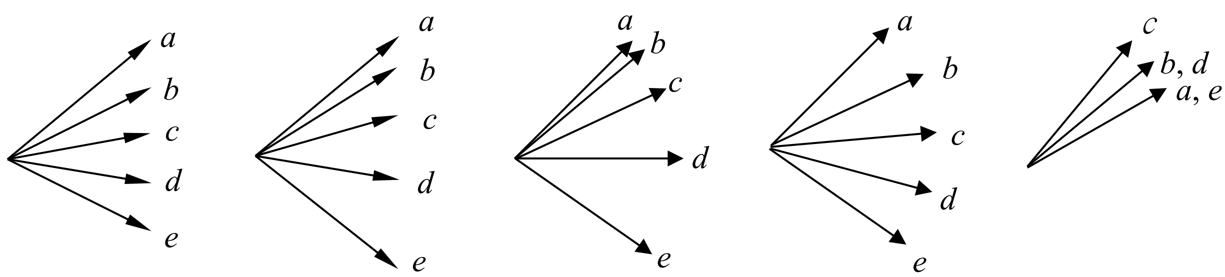
Using the conservation of energy,

$$E - G\frac{Mm}{R_E} = \frac{1}{2}mu^2 - G\frac{Mm}{R_S} \quad \Rightarrow \quad E = G\frac{Mm}{R_E} - G\frac{Mm}{2R_S} = G\frac{Mm}{R_E}\left(1-\frac{R_E}{2R_S}\right)$$

Since $G\frac{Mm}{R_E^2}=mg$, we have $E = mgR_E\left(1-\frac{R_E}{2R_S}\right)$
Answer: A.

15. The velocity vectors of an object performing projectile motion are drawn from time instants a to e at fixed time intervals. Which of the following gives a possible drawing?
把一物體在拋射過程中不同時間的速度矢量畫出來，其中時刻 a 到 e 的間隔相同。下面哪張圖是可能的？

A. B. C. D. E.

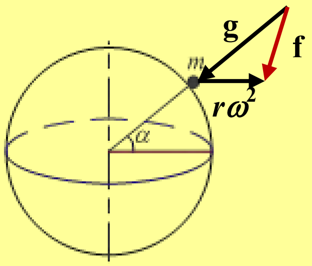


Answer: A.

16. It is known that the gravitational acceleration is $g = GM_E / R_S^2$ and is directed vertically towards Earth's centre. Due to Earth's rotation, the acceleration **f** of a freely falling object in Hong Kong has a magnitude f different from g, and **f** is no longer pointing vertically downwards. In fact,
已知重力加速度為 $g = GM_E / R_S^2$ 並指向地球中心。因著地球轉動，在香港的自由落體加速度則為 **f**。它的數值 f 與 g 不同，而 **f** 的方向也不是完全垂直。其實，

A. $f < g$ and **f** has a Northward component　　A. $f < g$ 而 **f** 偏向北
B. $f < g$ and **f** has a Southward component　　B. $f < g$ 而 **f** 偏向南
C. $f > g$ and **f** has a Northward component　　C. $f > g$ 而 **f** 偏向北
D. $f > g$ and **f** has a Southward component　　D. $f > g$ 而 **f** 偏向南
E. $f > g$ and **f** has a Eastward component　　E. $f > g$ 而 **f** 偏向東

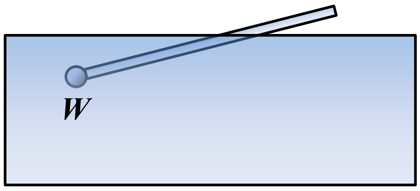
Solution:



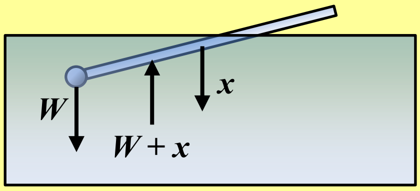
In the reference frame of the rotationg Earth, the centripetal acceleration has to be subtracted from the gravitational acceleration to obtain the free-fall acceleration. From the diagram, we see that $f < g$ and **f** has a Southward component.
Answer: B.

17. A uniform rod floats in water. A ball with weight W is attached to one end of the rod, and the volume of the ball is negligible. This structure causes the rod to float at an inclined position, with the other end remaining above the water surface, as shown in the figure. If the part of the rod above the water surface is $1/n$ of the total length, calculate the weight of the rod.
一根均勻桿浮在水中，一端附著重量為 W 而體積可忽略不計的小球。這結構令桿子傾斜地浮著，致另一端保持在水面上。若水面上的桿長為全長的 $1/n$，求桿的重量。

A. $W(n + 1)$　　B. Wn　　C. $W(n – 1)$　　D. $W/(n + 1)$　　E. $W/(n – 1)$



Solution:

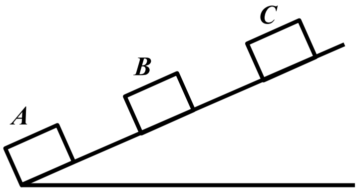


Let x be the weight of the rod, and L the length of the rod.
Since there are only 3 forces acting on the rod, the buoyancy is $W + x$.
The centre of mass is $L/2$ from the lower end of the rod.
The center of buoyancy is halfway of submegered rod. Hence its distance from the lower end of the rod is $L(1 – 1/n)/2$.
As shown in the figure, considering the torques about the lower end of the rod,

$$(W+x)\frac{L}{2}\left(1-\frac{1}{n}\right)=x\frac{L}{2} \quad \Rightarrow \quad x=W(n-1).$$

Answer: C.

18. A block has an initial kinetic energy of 128 J. It slides up from point A at the bottom of the inclined plane with uniform deceleration. When it passes point B, its kinetic energy is reduced by 80 J, and its mechanical energy is reduced by 35 J. Calculate the work done against friction when the block moves from A to the highest point C on the inclined plane.
一滑塊以 128 J 的初動能，從斜面底端的 A 點沿斜面向上作勻減速直線運動，它經過 B 點時，動能減少了 80 J，機械能減少了 35 J，求滑塊從 A 到最高點 C 對摩擦力所作的功。
A. 42 J B. 48 J C. 56 J D. 72 J E. 128 J



Solution:
Let $AB = b$. Then the kinetic energy is losing at a rate of $80/b$ Joules per unit length.
At the highest point, the kinetic energy is 0. Hence the distance is $128/(80/b) = 8b/5$.
At the same time, the mechanical energy is losing at a rate of $35/b$ Joules per unit length.
Hence at the highest point, the total mechanical energy loss is $(35/b)(8b/5) = 56$ J.
Answer: C.

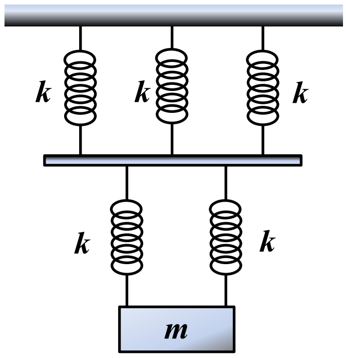
19. A block of mass m is placed on a smooth horizontal surface and is attached to a spring of force constant k. When the block is pulled sideways and released, it undergoes simple harmonic motion. At the equilibrium position, its velocity is 3 m/s. Calculate its velocity when it moves to the position at two-third of the amplitude from the equilibrium position.
一質量為 m 的方塊置於平滑水平面，並繫著彈性係數為 k 的彈簧。當方塊被拉向一旁再放鬆時，方塊即進行簡諧運動。方塊在平衡位置時的速度為 3 m/s，求在振幅三分之二處的速度。
A. $\sqrt{2}$ m/s B. $\sqrt{3}$ m/s C. 2 m/s D. $\sqrt{5}$ m/s E. $\sqrt{6}$ m/s

Solution:
At the equilibrium position, the kinetic energy is $K=\frac{1}{2}(m)(3^2)=\frac{9m}{2}$.
Using the conservation of energy, the potential energy at the maximum displaced position is $E=\frac{9m}{2}=\frac{1}{2}kA^2$.
At two-third of the amplitude from the equilibrium position, the potential energy is $U=\frac{1}{2}k\left(\frac{2A}{3}\right)^2=\left(\frac{4}{9}\right)\left(\frac{1}{2}kA^2\right)=\left(\frac{4}{9}\right)\left(\frac{9}{2}m\right)=2m$.
Hence the kinetic energy is $K=E-U=\frac{9m}{2}-2m=\frac{5m}{2} \quad \Rightarrow \quad \frac{1}{2}mv^2=\frac{5m}{2} \quad \Rightarrow \quad v=\sqrt{5}$.
Answer: D.

20. As shown in the figure, a block of mass m is hung from the ceiling by the system of springs consisting of two layers. The upper layer consists of 3 springs in parallel, and the lower layer consists of 2 springs in parallel. The force constants of all springs are k. Calculate the frequency of the vertical oscillations of the block.

如圖所示，天花板下一組兩層的彈簧懸吊著質量為 m 的方塊。上層有 3 個並排的彈簧，下層則有 2 個並排的彈簧。所有彈簧的彈力常數都是 k。求方塊上下振動時的頻率。

A. $\frac{1}{2\pi}\sqrt{\frac{k}{5m}}$ B. $\frac{1}{2\pi}\sqrt{\frac{4k}{5m}}$ C. $\frac{1}{2\pi}\sqrt{\frac{5k}{6m}}$ D. $\frac{1}{2\pi}\sqrt{\frac{6k}{5m}}$ E. $\frac{1}{2\pi}\sqrt{\frac{5k}{2m}}$



Solution:
Let x_1 = extensions of the springs in the upper layer
x_2 = extensions of the springs in the lower layer
x = displacement of the block
Consider the forces acting on the interface between the upper and lower layers: $3x_1 = 2x_2$

Hence $x_1 = \frac{2}{3}x_2$, $x = x_1 + x_2 = \frac{5}{3}x_2$, $x_2 = \frac{3}{5}x$.

Consider forces acting on the block: $ma = -2kx_2 = -\frac{6}{5}kx \quad \Rightarrow \quad a = -\frac{6k}{5m}x$

Hence $f = \frac{1}{2\pi}\sqrt{\frac{6k}{5m}}$.

Answer: D.

《END OF MC's 選擇題完》

Open Problems 開放題

Total 5 problems 共 5 題

The Open Problem(s) with the '*' sign may require information on page 2.
帶 * 的開放題可能需要用到第二頁上的資料。

1*. James Bond Ski Chased by a Killer 殺手追殺占士邦 (10 marks)

In James Bond movie "The Spy Who Loved Me" (鐵金鋼勇破海底城), James skiied down a snowy slope in an attempt to escape from the killer. Unfortunately, the killer had a higher skiing speed than James's; they are 45 m/s and 40 m/s respectively. Now consider an alternative version of the movie.
James noted that he and the killer carried the same kind of rifle and estimated that their masses M (including body mass, skis, weapon, backpack, etc) were about the same. Recalling high school physics knowledge, James realised that each time he fired his weapon back at the killer, his momentum would change; whereas when the killer fired, the killer's momentum would also change. Every time James fired a bullet, the killer would fire back accordingly.

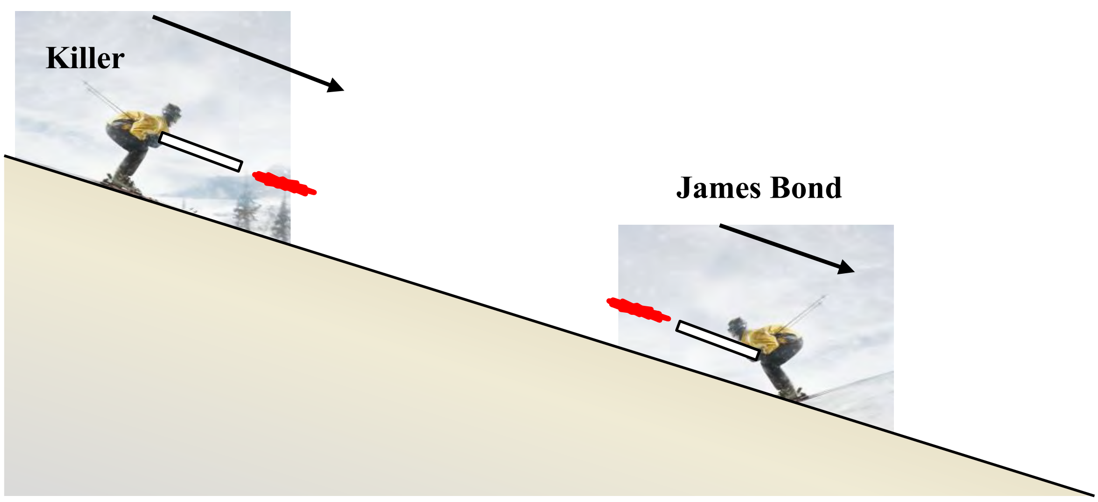
(a) How many bullets James had to fire in order to assure that the killer couldn't catch up with him? Assume that all bullets missed their targets (otherwise this exercise would terminate). Given M = 100 kg, mass and the muzzle velocity of a bullet are m = 0.02 kg and v = 500 m/s respectively.

(b) Traveling with the final velocity obtained in part (a), James Bond escaped by sliding down a cliff at an inclination of 20^o. After 20 seconds, he opened his parachute and landed safely. The average air drag during his fall is 600 N in both the vertical and horizontal directions, as long as the velocity components are nonzero. Calculate the landing position of James Bond and the height of the cliff. (Neglect the distance he traveled with a parachute.)

在占士邦電影"鐵金剛勇破海底城"裡，占士邦滑下雪坡，企圖逃離殺手的追殺。不幸殺手的滑速比占士邦高，分別為 45 m/s 和 40 m/s。現考慮電影的另一版本。
占士邦留意到殺手和他採用同樣的手槍，又推測兩人的質量 M（包括身體、滑雪裝備、武器、背包等）相若。他想起中學學到的物理學，理解到每當他向對方開火，他的動量就會改變；同樣當對方開火，對方的動量也會改變。每次占士邦發射一顆子彈，對方也同樣回射一顆。

(a) 占士邦需要發射多少顆子彈，才能保證殺手不會追上他？設 M = 100 kg，子彈的質量為 m = 0.02 kg，手槍射擊速度為 v = 500 m/s。

(b) 占士邦以(a)部的最終速度，滑下傾角為 20^o 的懸崖，成功逃脫。20 秒後，他打開降傘，安全著陸。設他在下墜過程中，垂直方向和水平方向的空氣阻力平均值均為 600 N(只要該速度的分量為非零)，計算占士邦著陸的位置和懸崖的高度。（可忽略降傘滑翔的距離。）



Solution:
(a) Using the conservation of linear momentum, the change in velocity after shooting a bullet in the backward direction by Jame Bond is given by

$$Mv_i = (M - m)v_f + m(-v + v_i)$$

$$v_f - v_i = \frac{mv}{M - m} \approx \frac{mv}{M} = \frac{(0.02)(500)}{100} = 0.1 \text{ m/s}$$

Similarly, using the conservation of linear momentum, the change in velocity after shooting a bullet in the forward direction by the killer is given by

$$Mv_i = (M - m)v_f + m(v + v_i)$$

$$v_f - v_i = -\frac{mv}{M - m} \approx -\frac{mv}{M} = -\frac{(0.02)(500)}{100} = -0.1 \text{ m/s}$$

Hence after both James Bond and the killer both shoot a bullet, the relative velocity reduces by 0.2 m/s. Since the relative velocity is 45 – 40 = 5 m/s, the number of bullets shot by James Bond is 5/0.2 = 25.
(b) Final velocity of James Bond $= 40 + (0.1)(25) = 42.5 \text{ m/s}$.

Horizontal direction: Acceleration during James Bond's fall: $a = -\frac{600}{100} = -6 \text{ m/s}^2$

Horizontal distance from the cliff: $x = 42.5\cos 20^\circ - (6)(20) = -80.1 \text{ m} < 0$

Hence instead, we have to calculate the horizontal distance up to the point of zero velocity.

$$0^2 - (42.5\cos 20^\circ)^2 = 2(-6)x \quad \Rightarrow \quad x = -\frac{(42.5\cos 20^\circ)^2}{2(-6)} = 133 \text{ m}$$

Vertical direction: Acceleration during James Bond's free fall: $a = \frac{(100)(9.8) - 600}{100} = 3.8 \text{ m/s}^2$

Distance of free fall: $y = 42.5\sin 20^\circ (20) + \frac{1}{2}(3.8)(20^2) = 1{,}051 \text{ m}$

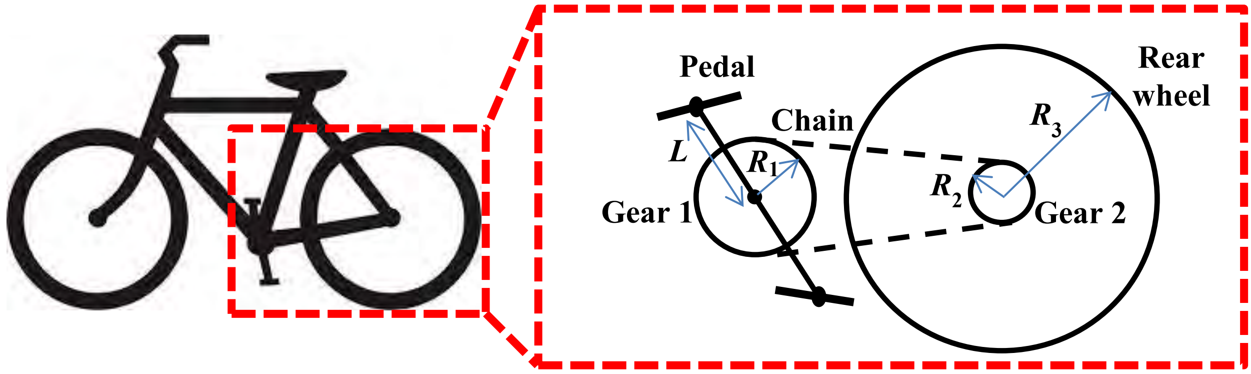
James Bond is 133 m horizontally from the edge of the cliff, and 1,051 m below the cliff.

2. The Bicycle 自行車 (10 marks)

A student rides a bicycle on a slope of inclination θ. Due to air drag, he found that the bicycle can barely move down the slope without his pedaling. He would like to estimate the power he needs to drive the bicycle up the same slope at a uniform velocity.
To achieve this, he measured that during the up-slope drive, one of his feet pedaled N cycles in a time interval T (assuming that the pedaling is continuous and at a uniform rate). He also obtained the following data: the total mass of the bicycle and the rider m, length of pedal crank L, radius of gear 1 R_1, radius of gear 2 R_2, radius of rear wheel R_3, as shown in the figure.
It is given that the air drags during the up-slope and down-slope drives have the same magnitude, and there are no slippings between the wheels and the slope during both the up-slope and down-slope drives. The energy loss due to the relative motion of the bicycle components is negligible.
(a) Derive an expression for the force needed to drive the bicycle up-slope at uniform velocity.
(b) Derive an expression for the power needed to drive the bicycle up-slope at uniform velocity.

某同學選了一個傾角為 θ 的斜坡，因有風阻，他騎在自行車上剛好能在不踩踏板的情況下讓自行車沿斜坡勻速向下行駛。現在他想估測沿此斜坡向上勻速行駛時的功率。
為此，他數出在上坡過程中某一隻腳蹬踩踏板的圈數 N(設不間斷的勻速蹬踏)，並測得所用的時間 T，再測得下列相關資料：自行車和人的總質量 m、踏板桿的長度 L、輪盤半徑 R_1、飛輪半徑 R_2、車後輪半徑 R_3，如圖所示。
已知上、下坡過程中的風阻大小相等，不論是在上坡還是下坡過程中，車輪與坡面接觸處都無滑動。不計自行車內部各部件之間因相對運動而消耗的能量。

(a) 試導出駕駛自行車勻速上坡所需作用力的表達式。
(b) 試導出估測功率的表達式。



Solution:

Down-slope: $f = mg\sin\theta$.

Up-slope: $F = f + mg\sin\theta = 2mg\sin\theta$.

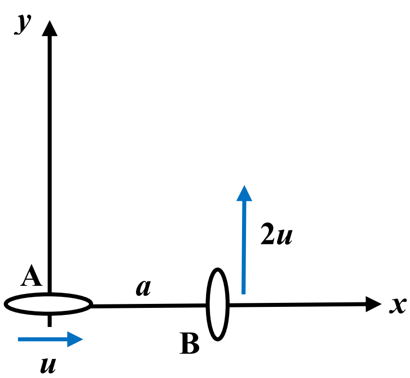
$$\omega = \frac{2\pi N}{t},\ v_1 = R_1\omega = v_2,\ \frac{v_3}{R_3} = \frac{v_2}{R_2},\ v_3 = \frac{v_2 R_3}{R_2} = \omega\frac{R_1 R_3}{R_2} = \frac{2\pi N}{t}\frac{R_1 R_3}{R_2}$$

$$\therefore\quad P = Fv_3 = \frac{2\pi N}{T}\frac{R_1 R_3}{R_2}\cdot 2mg\sin\theta = \frac{4\pi N R_1 R_3 mg\sin\theta}{TR_2}.$$

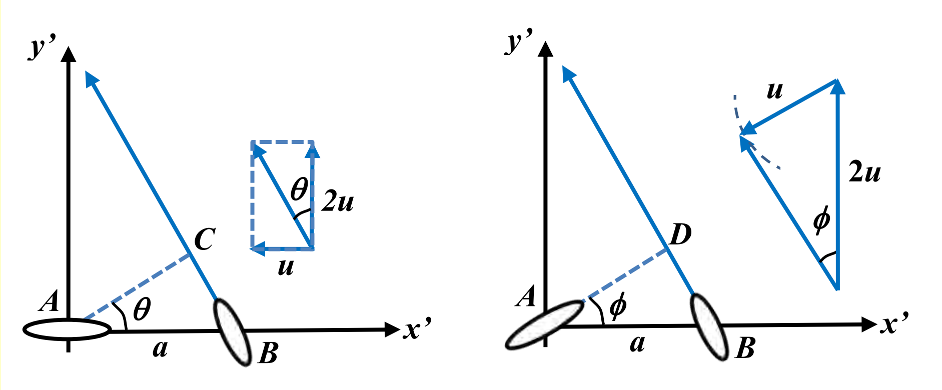
3. The Ships 輪船 (10 marks)

Consider two ships on the sea as shown in the figure. Ship A moves with velocity u directed to East. Ship B moves with velocity $2u$ directed to North. At time $t = 0$, ship B crosses the path of ship A at a distance a in front of ship A.
(a) Find the shortest distance between the ships, and the time they reaches this position.
(b) Suppose at $t = 0$, the speed of ship A remains at u, but its direction can be adjusted. What should be the direction of ship A such that the shortest distance between the two ships is minimum, and how much is this minimum shorter than the result obtained in part (a)?

如圖所示，海面上有兩艘輪船，船 A 以速度 u 向正東方向航行，船 B 以速度 $2u$ 向正北方向航行。在時刻 $t = 0$ 時，船 B 恰好同時經過船 A 的航線並位於船 A 的前方，船 B 到船 A 的距離為 a。
(a) 求兩船最接近的距離，和到達這距離的時刻。
(b) 假如在時刻 $t = 0$，船 A 的速率仍是 u，但方向可以自由調節，那麼船 A 應走什麼方向，兩船最接近的距離才是最短，這最短距離比(a)部得出的距離短多少?



Solution:



(a) As shown in the left figure, the path of ship B is given by the line BC in the reference frame of ship A. The shortest distance is given by the length AC.

Shortest distance $= a\cos\theta = \dfrac{2a}{\sqrt{5}}$

Velocity of ship B relative to ship A $= \sqrt{(2u)^2 + u^2} = \sqrt{5}u$

Time to reach the shortest distance $= \dfrac{a\sin\theta}{\sqrt{5}u} = \dfrac{a}{\sqrt{5}u}\left(\dfrac{1}{\sqrt{5}}\right) = \dfrac{a}{5u}$

(b) As shown in the right figure, to minimize the shortest distance, the velocity of ship B relative to ship A should make an angle with AB as small as possible. Considering variable directions of the velocity of ship A, the tip of this velocity vector generates a circle as shown in the inset of the figure. The best angle of the relative velocity is given by the tangent to the circle. Hence $\phi = \sin^{-1}(1/2) = 30^{\circ}$.

Shortest distance $= a\cos\phi = \dfrac{\sqrt{3}}{2}a$

Hence to minimize the shortest distance, ship A should move at an angle of 30° North of the East direction. Compared with the result obtained in part (a), the minimum shortest distance is shorter by

$\dfrac{2a}{\sqrt{5}} - \dfrac{\sqrt{3}}{2}a = \dfrac{4-\sqrt{15}}{2\sqrt{5}} = 0.028\ a$

4*. STEP (15 marks)

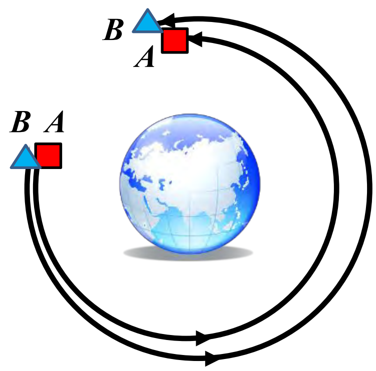
According to Newton's second law of motion, $F = m_I a$, where m_I is the *inertial mass*. According to Newton's law of universal gravitation, the gravitational force between Earth and an object is $F = GM_E m_G / R^2$, where m_G is the *gravitational mass* of the object (here R is the distance between Earth's center and the object). Presently, it is widely accepted that $m_G = m_I$, but some physicists would like to test the validity of this assumption. If there is a difference between m_I and m_G, even as small as one part in 10^{18}, our present understanding about gravity has to be revised. Hence they proposed a satellite experiment called STEP to measure the mass ratio $r = m_G/m_I$. (STEP represents Satellite Test of the Equivalence Principle.)

In the proposed experiment, several test bodies are enclosed in a vacuum box in a satellite that orbits around the Earth. The box protects the test bodies from outside disturbances and all forces from the satellite acting on the test bodies have been carefully eliminated, so that each test body can be considered as a mini-satellite orbiting around the Earth.

(a) The proposed satellite has a circular orbit with a period of 24 hours. Calculate its orbital radius R. Express your answer in multiples of R_E, the radius of Earth.

(b) Consider test bodies A and B with mass ratios r_A and r_B respectively, as shown in the figure. Suppose the two bodies have the same position at a point on the orbit. When body A completes one orbit, what is the displacement of body B relative to body A?
(c) Simplify your result in part (b) using the approximation $(1 + x)^n \approx 1 + nx$ when $|x| << 1$. Suppose the position sensors in the satellite can detect position changes as little as 10^{-15} m. What is the duration of the satellite flight before differences in the mass ratio of the order 10^{-18} can be detected? Express your answer in hours.

根據牛頓運動第二定律，$F = m_I a$，其中 m_I是*慣性質量*。根據牛頓萬有引力定律，地球作用於物體的引力是$F = GM_E m_G / R^2$，其中 m_G是*引力質量*（這裡 R 是地球中心與物體的距離）。現時普遍接納 $m_G = m_I$，但有物理學家希望驗證這假設的真確性。如果 m_I與 m_G有差別，即使是 10^{18} 分之一那麼微小，我們對萬有引力的理解也要改寫。所以他們提出名為 STEP 的衛星實驗以量度質量比 $r = m_G/m_I$。（STEP 代表 Satellite Test of the Equivalence Principle。）
在提出的實驗中，環繞地球的衛星中有一真空箱，數個實驗物封在其中。箱子保護實驗物免受外力干繞，衛星作用在實驗物上的所有作用力都被仔細隔絕，使每一實驗物都可考慮成環繞地球的袖珍衛星。
(a) 提出的衛星軌道為圓形，周期為 24 小時。試計算其軌道半徑 R。答案應以地球半徑 R_E的倍數為單位。
(b) 考慮實驗物 A 和 B，質量比分別為 r_A和 r_B，如圖所示。設兩物在軌道某點位置相同。當物體 A 完成一周，物體 B 相對於物體 A 的位移是多少?
(c) 試用$|x| << 1$ 時的近似$(1 + x)^n \approx 1 + nx$ 簡化(b)部的結果。設衛星上的位置感應器可檢測到小至 10^{-15} m 的位移。衛星航行多久，才能檢測到 10^{-18}量階的質量比？答案應以小時為單位。



Solution:
(a) Using Newton's second law of motion and Newton's law of universal gravitation,

$$M\frac{v^2}{R} = \frac{GM_E M}{R^2} \Rightarrow v = \sqrt{\frac{GM_E}{R}} \Rightarrow T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{R^3}{GM_E}} \Rightarrow T^2 = \frac{4\pi^2}{GM_E}R^3 \Rightarrow$$

$$R = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times 10^{-11})(5.98\times 10^{24})(24\times 3600)^2}{4\pi^2}} = 4.2250\times 10^7 \text{ m} = 6.62 R_E.$$

(b) Using Newton's second law of motion and Newton's law of universal gravitation,

$$m_I\frac{v^2}{R} = \frac{GM_E m_G}{R^2} \Rightarrow v = \sqrt{\frac{GM_E r}{R}} \Rightarrow T_A = \frac{2\pi R}{v_A} = 2\pi\sqrt{\frac{R^3}{GM_E r_A}}.$$

Similarly, $v_B = \sqrt{\frac{GM_E r_B}{R}}$.

Hence the distance moved by body B is $v_B T_A = \left(\sqrt{\frac{GM_E r_B}{R}}\right)\left(2\pi\sqrt{\frac{R^3}{GM_E r_A}}\right) = 2\pi R\sqrt{\frac{r_B}{r_A}}$.

Displacement of body B from body A $= 2\pi R\left(\sqrt{\frac{r_B}{r_A}} - 1\right)$.

Since $r_A \approx r_B \approx 1$, displacement $= \frac{2\pi R}{\sqrt{r_A}}\left(\sqrt{r_B} - \sqrt{r_A}\right) \approx 2\pi R\left\{[1 + (r_B - 1)]^{\frac{1}{2}} - [1 + (r_A - 1)]^{\frac{1}{2}}\right\}$

$$\approx 2\pi R\left\{\left[1 + \frac{1}{2}(r_B - 1)\right] - \left[1 + \frac{1}{2}(r_A - 1)\right]\right\} = \pi R(r_B - r_A)$$

(c) For differences in the mass ratio of the order 10^{-18}, displacement in one orbit

$$= \pi R(r_B - r_A) = \pi(4.2250 \times 10^{7})(10^{-18}) = 1.3273 \times 10^{-10} \text{ m}$$

Duration of satellite flight $= 24\left(\frac{10^{-15}}{1.3273 \times 10^{-10}}\right) = 0.000181 \text{ h} = 0.65 \text{ s}$

Remark: Hence in principle, the sensor should be sensitive enough to detect this effect. In fact, the main challenge of the experiment is that the sensor is also sensitive to other disturbances, and a lot of effort has to be made to screen out the other disturbances.

5. The Floating Ice 浮冰 (15 marks)

As shown in the left figure, a cylindrical piece of ice floats in water. Its cross sectional area is A and its height is h. The density of ice and water are ρ_I and ρ_W respectively.

(a) Find d, the depth of ice immersed in water.

(b) Suppose the ice is pushed slightly in the vertical direction. Find the frequency of oscillations. You may assume that the motion of water is negligible.

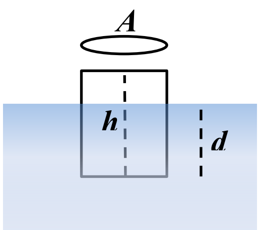
(c) Suppose the ice floats in water in a container of cross-section area $4A$, as shown in the right figure. The ice is displaced from equilibrium by z in the vertical direction. (i) Calculate the change in the total potential energy of the system, up to order z^2. (ii) Calculate the kinetic energy of the system during the push when the ice moves at velocity v. You may assume that the water below the bottom level of the ice does not move, and the water above the bottom level of the ice moves with the same velocity. (iii) Hence find the frequency of oscillations.

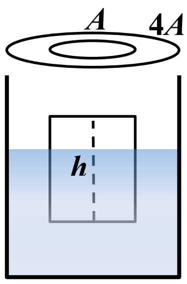
如左圖所示，一圓柱形的冰塊浮在水里。它的橫切面積是 A，高度是 h。冰和水的密度分別為 ρ_I和ρ_W。

(a) 求冰塊浸在水里的深度 d。

(b) 設冰塊在垂直方向被輕推，求它的振動頻率。你可假設水的流動可以忽略。

(c) 如右圖所示，設冰塊浮在容器的水裡，容器的橫切面積為 $4A$。冰塊在垂直方向被推，相對於平衡態的位移為 z。(i) 試計算系統總位能的改變，算至 z^2 階。(ii) 試計算當在冰塊被推過程中，冰塊速率為 v 時的系統總動能。你可假設低於冰塊底部水平的水不動，而高於冰塊底部水平的水以同一速度流動。(iii) 由此找出系統的振動頻率。





Solution:

(a) Using Archimedes' principle, weight of ice = buoyancy = weight of water displaced

$$\rho_I Ahg = \rho_W Adg \quad \Rightarrow \quad d = \frac{\rho_I h}{\rho_W}$$

(b) Since the motion of water is negligible, we only have to consider weight and buoyancy acting on the ice. Using Newton's second law,

$$ma = mg - \rho_W A(d+z)g = -\rho_W Agz \quad \Rightarrow \quad a = -\frac{\rho_W Ag}{m}z = -\frac{\rho_W Ag}{\rho_I Ah}z = -\frac{\rho_W g}{\rho_I h}z = -\frac{g}{d}z$$

Frequency of oscillation: $f = \frac{1}{2\pi}\sqrt{\frac{g}{d}}$

(c) (i) The potential energy change of ice $= -mgz = -\rho_I Ahgz$

Rise in water level $= \frac{Az}{3A} = \frac{z}{3}$

The potential energy change of water is the work done in moving the water from the bottom of the ice to the water surface. The initial center of mass is $d + z/2$ below the equilibrium water surface. The final center of mass is $z/6$ above the water surface. Hence the potential energy change of water

$$= (\rho_W Az)g\left(d + \frac{z}{2} + \frac{z}{6}\right) = \rho_W Ag\left(dz + \frac{2}{3}z^2\right)$$

Total potential energy change of the system $= -\rho_I Aghz + \rho_W Ag\left(dz + \frac{2}{3}z^2\right) = \frac{2}{3}\rho_W Agz^2$

(ii) Kinetic energy of ice $= \frac{1}{2}mv^2 = \frac{1}{2}\rho_I Ahv^2$

The velocity of water $= \frac{Av}{3A} = \frac{v}{3}$

Kinetic energy of water $= \frac{1}{2}(\rho_W 3Ad)\left(\frac{v}{3}\right)^2 = \frac{1}{6}\rho_W Adv^2$

Total kinetic energy of the system $= \frac{1}{2}\rho_I Ahv^2 + \frac{1}{6}\rho_W Adv^2 = \frac{2}{3}\rho_W Adv^2$

(iii) Total energy change of the system $= \frac{2}{3}\rho_W Adv^2 + \frac{2}{3}\rho_W Agz^2$

This is equivalent to the energy of a harmonic oscillator with an effective mass of $m_{\text{effective}} = \frac{4}{3}\rho_W Ad$ and a spring constant of $k_{\text{effective}} = \frac{4}{3}\rho_W Ag$.

Frequency of oscillation: $f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{effective}}}{m_{\text{effective}}}} = \frac{1}{2\pi}\sqrt{\frac{g}{d}}$.

The result is the same as part (b). Hence in this approximation, the frequency of oscillation is independent of the cross-sction area of the container.

《END 完》

Hong Kong Physics Olympiad 2014
2014 香港物理奧林匹克

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11 May, 2014
2014 年 5 月 11 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

   所有題目均爲中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你即可開始作答開放式題目，而無須等候任何宣佈。

3. On the cover of the answer book and the multiple-choice answer sheet, please write your HKID Number in the "Seat Number" box, your 8-digit Contestant number in the "Student Number" box and your English Name in the "Name" box.

   在答題簿封面及選擇題答題紙上，請於 Seat Number 欄中填上你的身份證號碼，於 Student Number 欄中填上你的 8 位數字參賽者號碼及於 Name 欄上填上你的英文姓名。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. The open problems are long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the following parts.

   開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來解答其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別註明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m/s^2
Gravitational constant 萬有引力常數	G	6.67×10^{-11} N m^2/kg^2
Radius of Earth 地球半徑	R_E	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	r_E	1.5×10^{11} m
Earth-Moon distance 地球-月球距離	r_m	384400 km
Mass of Sun 太陽質量	M_{Sun}	1.99×10^{30} kg
Mass of Earth 地球質量	M_E	5.98×10^{24} kg
Density of Air 空氣密度	ρ_0	1.2 kg/m^3
Density of Water 水密度	ρ_w	1000 kg/m^3
Sea Water Density 海水密度	ρ_{sea}	1022 kg/m^3

Trigonometric Identities:
三角學恆等式：

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

Multiple Choice Questions

Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.25 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.

選擇題

每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.25 分，但全部選擇題最低為 0 分。

1. An accelerometer is a device to measure acceleration. At the earth sea level, the acceleration due to gravity is g = 9.8 ms^{-2}. Most of the smartphones such as Android and Apple iPhone consist of a built-in accelerometer. If a smartphone is moving vertically upward at sea level with constant velocity 9.8 ms^{-1}, the instantaneous scalar readings of the x, y, and z components from the accelerometer are:

   加速度計是一種用來測量加速度的裝置。在地球海平面的重力加速度是 g = 9.8 ms^{-2}。 大部分的智能手機如 Android 和蘋果 iPhone 都有一個內置的加速度計。 如果智能手機正在 以均速 9.8 ms^{-1} 在地球海平面垂直向上移動，加速度計 x, y, and z 的瞬時讀數是：

   A. x = 0 g, y = 0 g, z = 0 g.
   B. x = 1 g, y = 0 g, z = 0 g.
   C. x = 0 g, y = 1 g, z = 0 g.
   D. x = 0 g, y = 0 g, z = 1 g.
   E. x = 1 g, y = 1 g, z = 1 g.

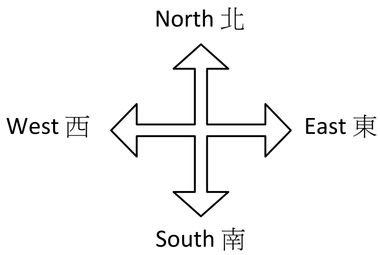
2. A toy rocket is launched upward from rest with a net acceleration of 2.5 g, where g is the acceleration due to gravity. The engine is turned off after 5 seconds while the rocket continues to move upward with negligible air resistance. Assume that the mass of the rocket is kept constant, what maximum elevation does the rocket reach in terms of g?

   一個玩具火箭從靜止被向上發射，其淨加速度為 2.5 g，而 g 是重力加速度。沒有空氣阻力下，發動機 5 秒後被關閉，而火箭繼續向上爬升。假設火箭質量保持不變，以 g 來推導火 箭可達的最大高度。

   A. (31 g) m
   B. (50 g) m
   C. (78 g) m
   D. (109 g) m
   E. (159 g) m

3. A car (mass 200 kg) is travelling clockwisely around a flat roundabout (diameter 10 m) at a constant speed 5 m/s, as shown in the figure. What is the acceleration of the car?

   如圖所示，一輛汽車 (質量 200 kg) 正在以 5 m/s 的恆定速度圍繞著一個平坦的迴旋處順時針方向行駛 (直徑 10 m)。這輛汽車的加速度是什麼？



A. Zero 零
B. 5 m/s^2 East 東
C. 5 m/s^2 West 西
D. 10 m/s^2 East 東
E. 10 m/s^2 West 西

4. A machine gun fires bullets (10 g each) at a rate of 200 bullets per minute to hit a target. The speed just before hitting the target is 600 m/s, and the bullets are stopped in the target. The cross sectional area of the target is 2 m^2. Find the average force exerted on the target.

一挺機關槍每分鐘發 200 顆子彈（每顆子彈為 10 g）， 並以 600 m/s 的速度擊中目標， 目標的橫切面積為 2 m^2。 假設子彈停留在目標中，求子彈打在目標上的平均力。

A. 10 N.
B. 20 N.
C. 40 N.
D. 80 N.
E. 100 N.

5. Tides are mainly resulted from the difference in the Moon's gravitational field between the Earth's near side and its far side. What is the percentage gravitational force difference?

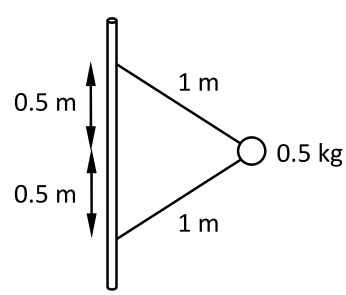
潮汐的產生主要是由於月球對地球的近端和遠端的引力場差異所成。這個引力場的差異百分比是多少?

A. 1%.
B. 3%.
C. 5%.
D. 7%.
E. 9%.

6. As shown in the following figure, a mass (0.5 kg) is tied to two wires (1 m) and revolves in a horizontal circle at a constant speed of 10 m/s. Calculate the tension of the upper wire.

如圖所示，一個質量物 (0.5 kg) 被連接到兩條線 (1 m) 並以 10 m/s 的恆定速度圍繞在水平轉圈。計算上方線的拉力。

A. 28 N
B. 38 N
C. 66 N
D. 105 N
E. 115 N



7. A person is pulling a heavy box with a 50 N force on a horizontal floor at constant velocity for 5 m. He then feels exhausted and the force exerted by him decreases linearly from 50 N to near 0 N for the next 5 m. What is the total work done by the person on the box?

一個人以 50 N 的力拉著一個放在水平地板的沉重箱子並以等速拉了 5 m。然後他感到筋疲力盡，而他的拉力以線性函數的方式從 50 N 到 0 N 慢慢地減少。計算此人在盒子上所做的總功。

A. 250 J.
B. 300 J.
C. 325 J.
D. 350 J.
E. 375 J.

8. A mass m_1 at one end of a spring establishes simple harmonic motion with a period T_1. If the mass is replaced with a different mass m_2, what is the period of this oscillation?

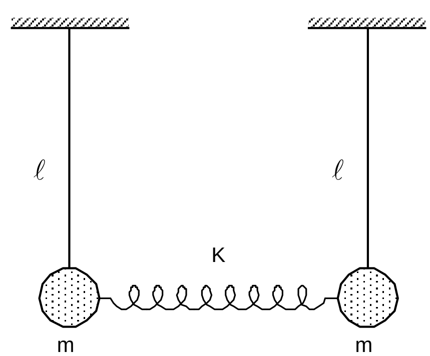
一個週期 T_1 的簡諧運動由一個質量物 m_1 和一個彈簧組立。 假設那個質量物被替換為另一 質量物 m_2，問該振盪的週期是什麼？

A. $2\pi\sqrt{m_2/m_1}$
B. $2\pi\sqrt{m_1/m_2}$
C. $2\pi\sqrt{T_1}$
D. $T_1\sqrt{m_1/m_2}$
E. $T_1\sqrt{m_2/m_1}$

9. As shown in the following figure, two pendulums are constructed with identical massless springs and mass objects (mass *m*). The initial distance between the masses is the equilibrium length of the spring (spring constant K). What is the frequency of the system if the two balls are oscillating out of phase?

如圖所示，兩個鐘擺由相同且無質量的彈簧和質量物 (質量 m) 所構成。質量物之間的初始 距離是彈簧 (彈簧常數 K) 的平衡長度。如果這兩個球以異相的方式振盪，這個系統的頻率 是什麼？

A. $\sqrt{\dfrac{g}{l}}$
B. $\sqrt{\dfrac{K}{2m}}$
C. $\sqrt{\dfrac{2K}{m}}$
D. $\sqrt{\dfrac{g}{l}+\dfrac{2K}{m}}$
E. $\sqrt{\dfrac{2g}{l}+\dfrac{K}{2m}}$



10. What is the minimum rotation period of a spherical neutron star with uniform mass density ρ such that material will not fly off from the equator? (G = Universal gravitational constant)

一個均勻質量密度 ρ 的球形中子星，它的自轉週期最小值應是什麼才能使得其物質不會從它的赤道飛出去? (G = 萬有引力常數)

A. $\left(\dfrac{3\pi}{\rho G}\right)^{1/2}$

B. $\left(\dfrac{\pi}{\rho G}\right)^{1/2}$

C. $\dfrac{3}{4\pi G}$

D. $\dfrac{4}{3\pi G}$

E. $\left(\dfrac{3}{8\pi\rho G}\right)^{1/2}$

11. A solid ball weighs 10 N in air, but 6 N when it is submerged in water. If the ball weighs 2 N when it is submerged in an unknown liquid, find the specific gravity of the unknown liquid.

一個實心球在空氣中的重量為 10 N，但當它被淹沒在水中時為 6 N。如果那個球被淹沒在一種不明液體時為 2N，求不明液體的比重。

A. 1.0.
B. 1.5.
C. 2.0.
D. 2.5.
E. 3.0.

12. A bullet (mass 10 g) is shot vertically and upward into a brick (2 kg). The bullet is embedded in the brick, and the brick is lifted upward 5 mm in a time interval of 1.2 ms. Assume that the force on the bullet is constant during penetration and air resistance is negligible. Find the initial kinetic energy of the bullet.

一顆子彈 (質量 10 g) 垂直向上射向一塊磚 (2 kg)。子彈藏在磚塊中，並用了 1.2 ms 把磚頭 向上抬起 5 mm。 假設在穿過過程中的力是恆定的而且空氣阻力的因素可忽略， 計算子彈的 初始動能。

A. 5 J.
B. 10 J.
C. 15 J.
D. 20 J.
E. 25 J.

13. A ball is hit from an inclined plane making an angle θ with the horizontal direction towards the downward direction. The ball has an initial velocity v_o and flies at an initial angle θ above the horizontal direction. Assume that the air resistance is negligible, at what horizontal distance from the initial point of hitting does the ball touch the inclined plane again?

一個球從一斜坡向下被擊出，斜坡與水平方向的角度為 θ。 球的初始速度為 v_o， 初始方向 高於水平方向的角度為 θ。 假設空氣阻力可忽略，當球擊中斜坡水平時， 從起初點計的距離 是多少？

A. $\dfrac{4v_o^{\ 2}\tan\theta}{g}$

B. $\dfrac{4v_o^{\ 2}\sin\theta\cos\theta}{g}$

C. $\dfrac{4v_o^{\ 2}\sin\theta}{g}$

D. $\dfrac{4v_o^{\ 2}\cos\theta}{g}$

E. $\dfrac{4v_o^{\ 2}\cot\theta}{g}$

14. Three identical blocks of mass m are stacked vertically. Block 1 is on top and rests on Block 2, which rests on Block 3, which rests on a table. What is the net force acting on Block 1? The weight of the 3 Blocks is measured to be 3 N.

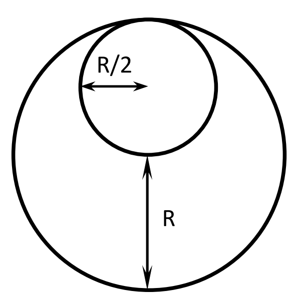
三個相同的方塊 (質量 m) 垂直堆疊在一起。 方塊 1 位於且靜止在方塊 2 的上方， 而它們同時靜止且位於方塊 3 的上方， 這三個方塊都靜止在桌子上。 問方塊 1 的淨作用力 是什麼？ 假設這三個方塊的重量被測量為 3 N。

A. Zero.
B. 1/3 N.
C. 1 N.
D. 2 N.
E. 3 N.

15. The following figure is a spherical styrofoam of radius R. A cavity of radius R/2 is made in the sphere. If the cavity is filled with a solid material of density 5 times of styrofoam and with the same thickness, where is the new center of mass?

圖中是一個半徑 R 的球形發泡膠。一個半徑 R/2 的球形空穴在裡面形成。如果這個空穴被填滿了一種固體材料，其密度為發泡膠的 5 倍而厚度與發泡膠相同，問新的質心在哪？

A. R / 2 upward from the center of the styrofoam sphere.
B. R / 3 upward from the center of the styrofoam sphere.
C. R / 4 upward from the center of the styrofoam sphere.
D. R / 5 upward from the center of the styrofoam sphere.
E. R / 6 upward from the center of the styrofoam sphere.



A. 從發泡膠球的中心向上起計 R / 2 的位置。
B. 從發泡膠球的中心向上起計 R / 3 的位置。
C. 從發泡膠球的中心向上起計 R / 4 的位置。
D. 從發泡膠球的中心向上起計 R / 5 的位置。
E. 從發泡膠球的中心向上起計 R / 6 的位置。

16. What is the potential energy for a tide cycle per square meter of ocean surface, if the sea water level difference between a high tide and a low tide is Δh (in meter)?

假設漲潮和退潮時的海水水位差是 Δh（以米作單位），以每平方米海面計算，一個潮週期的位能是多少？

A. 1.0 Δh^2 Watt-hour.
B. 1.2 Δh^2 Watt-hour.
C. 1.4 Δh^2 Watt-hour.
D. 1.6 Δh^2 Watt-hour.
E. 1.8 Δh^2 Watt-hour.

17. The weight of an object on the Moon surface is 1/6 of that on the Earth surface. A pendulum clock on the Earth environment ticks once per second. On the Moon environment, the clock would tick once every

一個物體在月球表面上的重量是其在地球表面的 1/6 倍。一個擺鐘在地球上每秒擺動一次。在月球上，這個時鐘擺動一次之時間為

A. $1/6$ s.
B. $1/\sqrt{6}$ s.
C. 1 s.
D. $\sqrt{6}$ s.
E. 6 s.

18. Geostationary satellite is its orbital period the same as the Earth's rotation period. Calculate the height of a geostationary satellite above the Earth's surface in terms of G (Gravitational constant), M_{Earth} (mass of the Earth), and R_{Earth} (radius of the Earth), where G, M_{Earth} and R_{Earth} are in SI units.

地球同步衛星是指它的軌道週期和地球自轉週期時間一樣。 以 G (萬有引力常數)， M_{Earth} (地球質量)，和 R_{Earth} (地球半徑) 來推導地球同步衛星在地球表面上的高度。其中，G， M_{Earth}，和 R_{Earth} 是以國際單位制為基礎。

A. $\dfrac{1231\times(G\cdot M_{\text{Earth}})}{\pi}$

B. $\dfrac{1231\times(G\cdot M_{\text{Earth}})^{1/3}}{\pi^{2/3}} - R_{\text{Earth}}$

C. $\dfrac{1231\times(G\cdot M_{\text{Earth}})^{2/3}}{\pi^{1/3}} - R_{\text{Earth}}$

D. $\dfrac{1231\times(G\cdot M_{\text{Earth}})^{1/3}}{\pi^{2/3}} + R_{\text{Earth}}$

E. $\dfrac{1231\times(G\cdot M_{\text{Earth}})^{2/3}}{\pi^{1/3}} + R_{\text{Earth}}$

19. In comparison of a pendulum with large-angle oscillations (period T_L and magnitude A_L, maximum tension in the string F_L) to a pendulum with small-angle oscillations (period T_S and amplitude A_S, maximum tension in the string F_S), which of the following is true?

大角度 (週期 T_L， 幅度 A_L， 繩子最大張力 F_L) 與小角度鐘擺 (週期 T_S， 幅度 A_S， 繩子最大張力 F_S) 作比較，下列哪項是正確的？

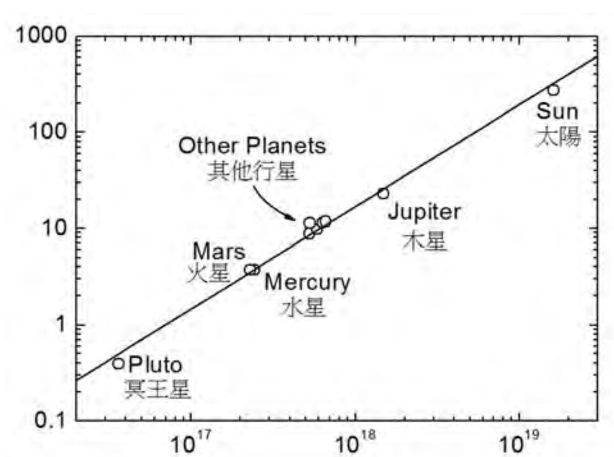
A. $T_L > T_S$ and $F_L < F_S$.
B. $T_L < T_S$ and $F_L > F_S$.
C. $T_L > T_S$ and $F_L > F_S$.
D. $T_L < T_S$ and $F_L < F_S$.
E. $T_L = T_S$ and $F_L = F_S$.

20. The following figure is most probably a plot of

A. square of the orbital period of a planet against the cube of the semi-major axis of its orbit.
B. cube of the orbital period of a planet against the square of the semi-major axis of its orbit.
C. surface gravity of a planet against (M/R^2), where M is the mass and R is the equatorial radius of a planet.
D. surface gravity of a planet against (M^2/R), where M is the mass and R is the equatorial radius of a planet.
E. distance of a planet from the Galactic Center in Astronomical Unit against the mass of a planet.

以下的圖表最有可能是表示

A. 行星軌道週期之平方與其軌道半長軸之立方的關係。
B. 行星軌道週期之立方與其軌道半長軸之平方的關係。
C. 行星表面引力與 (M/R^2) 之關係，M 和 R 分別是行星之質量和赤道半徑。
D. 行星表面引力與 (M^2/R) 之關係，M 和 R 分別是行星之質量和赤道半徑。
E. 行星從銀河系中心的距離 (以天文單位計) 與其質量之關係。



--- END OF MULTIPLE CHOICE SESSION 選擇題完 ---

Open Problems 開放題
Total 5 Problems 共 5 題

1. Venus Transit (15 Marks)

Venus transit is an astronomical phenomenon when the planet Venus passes directly between the Sun and Earth. As illustrated in Fig. 1, for two different observation points A and B on the Earth, Venus appears as two separate black dots (A' and B') on the Sun surface.

(a) Suppose Venus takes 225 days to orbit the sun, calculate the ratio of $a_{\mathrm{Earth}}/a_{\mathrm{Venus}}$, where a_{Earth} and a_{Venus} are the mean distances of the planets from the Sun, respectively.

(b) On a Venus transit day, two observations were made in Hong Kong and Bangkok. They are geographically separated by 1800 km. Bangkok is 37° South of West of Hong Kong. Calculate the distance A'B'.

(c) Another observation finds that the Sun diameter is 290 times the distance of the two separate black dots. Calculate the diameter of the Sun.

(d) Calculate the time difference between the exit of Venus transit as observed in Hong Kong and Bangkok. Give your answer in minutes. (Hint: calculate the velocity of Earth relative to Sun, then the velocity of Sun and Venus relative to Earth, then the velocity of the shadow of Venus projected on Sun's surface, then the velocity of the shadow of Venus sweeping on Sun's surface.)

金星凌日 (15 分)

金星凌日是一種天文現象，當地球、金星、太陽成一直線時，便會出現金星凌於太陽表面的天象。 如圖 1 所示， 對於兩個不同在地球表面的觀測點 A 和 B， 金星會顯示在太陽表面兩個 獨立的黑點 (A' 和 B') 。

(a) 假設金星需要 225 天圍繞太陽，計算 $a_{\mathrm{Earth}}/a_{\mathrm{Venus}}$ 之值。 其中 a_{Earth} 和 a_{Venus} 分別是行星與太陽之平均距離。

(b) 在金星凌日當天，兩個觀察分別在香港和曼谷同時進行了。 在地理上， 這 2 個 城市的距離為 1800 km，而曼谷位於香港西方向南 37°。 計算 A' 與 B' 之距離。

(c) 假設太陽的直徑是兩個單獨黑點距離的 290 倍。計算太陽的直徑。

(d) 試計算香港和曼谷兩地觀測到金星凌日結束的時差，答案請以分鐘表達。
(提示：計算地球相對於太陽的速度，然後計算金星和太陽相對於地球的速度、 金星在太陽表面影子的速度、金星影子掃過太陽表面的速度。)

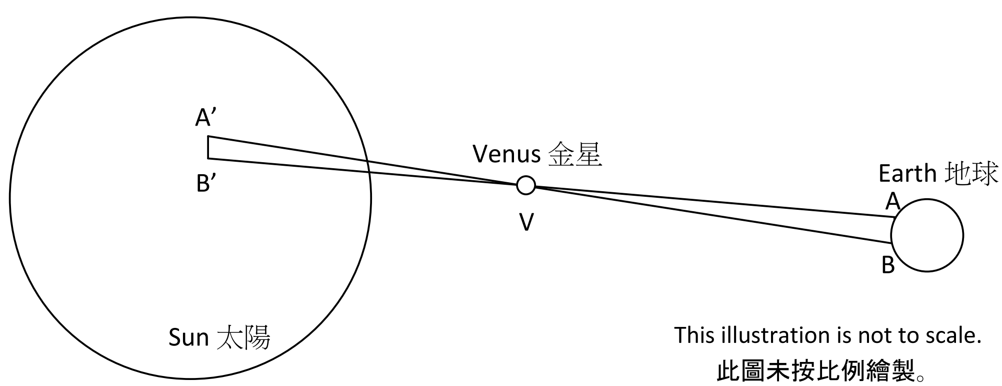


Fig. 1 – Venus Transit
圖 1 – 金星凌日

2. Terminal Velocity of Free Falling Object (10 Marks)

A spherical styrofoam of mass 2 g and radius 2 cm is released for free falling. The only forces acting on the styrofoam is the gravitational force and a drag force. The drag force is due to air resistance and is velocity (v) dependent. At low velocities, air resistance is negligible; but at high velocities, constant falling velocity is expected.

(a) Suppose the drag force is linear proportional to its falling velocity, write down the equation of motion. State the velocities in the limits of both low and high velocities. Sketch the graph of falling velocity against time with the indications of both the velocity limits.

(b) Suppose that the drag force is given by $F_d = -\frac{1}{2}C_d \cdot \rho \cdot A \cdot v^2$; where C_d is the drag coefficient, which is ~ 0.5 for a spherical object; ρ is the air density; A is the total cross-sectional area. Estimate the terminal velocity.

(c) Estimate the time taken by the falling styrofoam sphere to reach a velocity comparable to the terminal velocity.

自由下落物的終極速度（10 分）

一個質量 2 g 半徑 2 cm 的球形發泡膠正在自由下落。 作用於發泡膠的力只有引力和 後曳力。後曳力是由於空氣阻力並且是速度 (v) 的函數。 在低速度下，空氣阻力可忽略; 但在高的速度下，以勻速下降是可預期的。

(a) 假設後曳力是以線性正比於它的降落速度，寫下運動方程式。闡明在極低和極高速度 狀況下的速度值。草繪下降速度與時間的關係圖，在圖中標明極低和極高速度的狀況。

(b) 假設該後曳力的計算公式是 $F_d = -\frac{1}{2}C_d \cdot \rho \cdot A \cdot v^2$; 而 C_d 是阻力係數，對於一個球形物體大約等於 0.5 ; ρ 為空氣密度; A 是總橫截面面積。估計終端速度。

(c) 估算發泡膠球從靜止至達到相當於終極速度所需的時間。

3. Total Energy in a Surface Wave (15 Marks)

Consider a surface wave traveling on the sea (Fig. 2). At an instant, its surface profile can be approximated by $z = A\sin\left(\frac{2\pi x}{\lambda}\right)$, where A is the amplitude and λ is the wavelength.

(a) Find the potential energy per unit wavelength per unit width of the surface wave in terms of ρ (sea water density), g (acceleration due to gravitation), and A.

Hint: $\int \sin^2\left(\frac{2\pi x}{\lambda}\right)dx = \frac{x}{2} - \frac{\lambda\sin(4\pi x/\lambda)}{8\pi}$.

(b) Assume equipartition of energy, that is, the average potential energy equals the average kinetic energy of surface wave. Find the total energy over a whole wavelength.

(c) Given that the wave velocity $v_g = \frac{\lambda}{2T}$ and the relationship $\lambda = \frac{gT^2}{2\pi}$, derive the power of a wave period per unit width in terms of ρ, g, A, T (period of a wave), and H (wave height).

表面波的總能量（15 分）

在海面上有一表面波（圖 2），其表面輪廓在某時刻可表示為 $z = A\sin\left(\frac{2\pi x}{\lambda}\right)$。其中，$A$ 是幅度而 λ 是波長。

(a) 計算表面波每波長及每單位寬度的勢能。答案請以 ρ (海水密度)， g (重力加速度)，和 A 表達。

提示：$\int \sin^2\left(\frac{2\pi x}{\lambda}\right)dx = \frac{x}{2} - \frac{\lambda\sin(4\pi x/\lambda)}{8\pi}$.

(b) 假設能量是均分的，即表面波的平均勢能等於其平均動能。 計算一整體波長每單位寬度的總能量。

(c) 已知波速為 $v_g = \frac{\lambda}{2T}$，及關係 $\lambda = \frac{gT^2}{2\pi}$， 推導出一個週期表面波每單位寬度的功率。 答案請以 ρ， g， A， T（波週期），與及 H（波浪高度）表達。

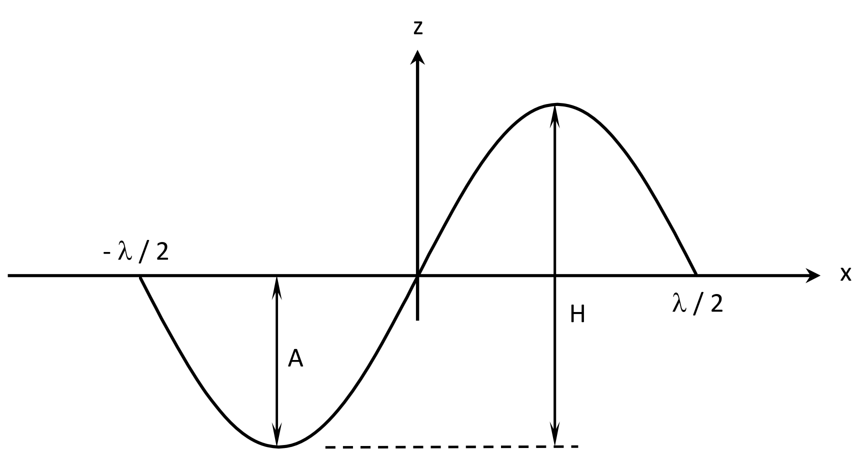


Fig. 2 – Surface Wave
圖 2 – 表面波

4. A Sliding Block up a Slope Platform (10 Marks)

A solid cube (length L each) of mass m starts to slide up a stationary slope platform from the bottom. The slope platform has a mass M, an inclination angle θ and the slope is smooth (Fig. 3). The slope platform is free to slide along the horizontal surface without friction.

(a) Calculate the initial velocity v_0 of the sliding block in terms of h, θ, m, M, L, and g, such that the cube just reaches the upper end of the slope.

(b) The cube then slides down the slope platform. What is the final kinetic energy of the cube and the slope platform when the cube reaches the lower end of the slope platform?

一個依住斜坡平台向上滑動的方塊 (10分)

如圖 3 所示， 一個質量為 m 的立方體 (每邊長度 L)，正從靜止的斜坡台的底部開始向上滑動。斜坡台的質量 為 M，傾斜角為 θ ，表面是光滑的。斜坡台可以自由地沿水平表面滑動。

(a) 已知立方體剛剛能到達斜坡的上端，試計算滑動立方體的初始速度 v_0， 答案請以 h, θ, m, M, L 和 g 表達。

(b) 其後立方體在斜坡台上向下滑動，當立方體到達斜坡台下端時，立方體和斜坡台的總動能是什麼?

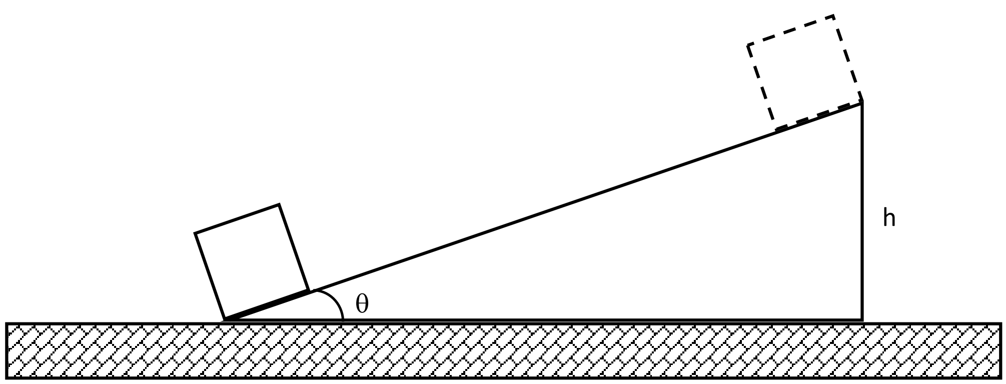


Fig. 3 – Sliding Block up a Slope Platform
圖 3 – 斜坡平台滑動的方塊

5. A Physical Pendulum Motion (10 Marks)

Two identical uniform thin rods (mass m, and length L) are connected at right angle to form a rigid upside-down "T" shape. The center of one rod is connected to one end of another rod. The upside-down "T" shape is suspended and allowed to form a pendulum motion, as shown in Fig. 4.

(a) The kinetic energy of the system is $\frac{17}{24}mv^2$, where v is the velocity of the meeting point of the two rods. Derive the equation of the pendulum motion in terms of m, L, and g.

(b) Calculate the period of the system if θ is small. (Hint: $\sqrt{1-\frac{x^2}{L^2}} \approx 1-\frac{x^2}{2L^2}$ when x << L.)

(c) At t = 0, the meeting point of the two rods is displaced by x_0. What is the time when the displacement becomes $x_0/2$ the first time?

一個物理鐘擺的運動 (10 分)

兩個相同均勻的薄桿 （ 質量 m， 長度 L ）被連接成直角，以形成一個硬性倒掛的 "T" 形。 一個桿的中心被連接到另一個桿的末端。 如圖 4 所示， 這個倒掛的 "T" 形被懸掛並容許成一 個鐘擺運動。

(a) 已知該系統的動能是 $\frac{17}{24}mv^2$， 其中 v 為兩桿交點的速度。以 m，L，和 g 來推導鐘擺運動方程式。

(b) 如果 θ 的數值是很小，求系統的週期。(提示：當 x << L 時，$\sqrt{1-\frac{x^2}{L^2}} \approx 1-\frac{x^2}{2L^2}$)

(c) 當 t = 0 時，兩桿交點的位移為 x_0。問位移第一次為 $x_0/2$ 時，時間是什麼？

Axis of oscillation: perpendicular to paper
振盪軸： 垂直於紙

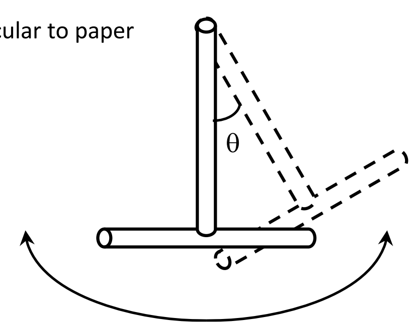


Fig. 4 – Physical Pendulum Motion
圖 4 - 物理鐘擺運動

--- END OF EXAM PAPER 全卷完 ---

Multiple Choices:

1. Answer: A or D.

The phone is moving at constant velocity. The only force acting on the phone is the earth gravity.

Remark: The answer is D. However, since the question asks for "instantaneous scalar readings", it is not unreasonable to also consider A as a valid answer.

2. Answer: D.

Stage 1: h1 = (1/2) (2.5 g) $(5)^2$ = (31.25 g) m

Stage 2: v = (2.5 g) (5) = 12.5 g ms^{-1}; h2 = $(12.5\text{ g})^2$ / 2g = (78.125 g) m

Maximum Height = (31.25 + 78.125) g = (109.375 g) m

3. Answer: C.

a = v^2 / r West = 5^2 / 5 West = 5 ms^{-2} West

4. Answer: B.

F = (0.01 x 200 x 600) / 60 = 20 N.

5. Answer D.

Apply Newton's law of universal gravitation at the Earth's near side and its far side.

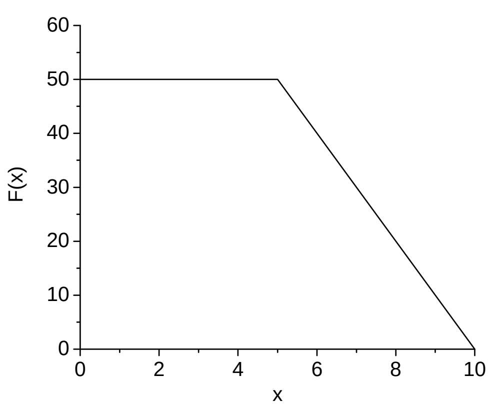
6. Answer B.

Bottom Wire: $T_2 = \dfrac{m}{2\cos\theta}\left(\dfrac{v^2}{r} - g\cot\theta\right) = 28.43\text{ N}$

Bottom Wire: $T_1 = T_2 + \dfrac{mg}{\sin\theta} = 38.23\text{ N}$

7. Answer E.

Work Done = Area under the following curve.



8. Answer E.

Apply $T = 2\pi\sqrt{\dfrac{m}{k}}$ to perform calculation.

9. Answer D.

Two normal modes.

First Mode: $f_1 = \sqrt{\frac{g}{l}}$

Second Mode: Consider the center of mass of the 2 masses being unchanged, the effective spring constant is $K_1 = \frac{2m}{m}K = 2K$

$$\Rightarrow f = \sqrt{\frac{g}{l} + \frac{K_1}{m}} = \sqrt{\frac{g}{l} + \frac{2K}{m}}$$

10. Answer A.

$$\frac{GMm}{r^2} > m\omega^2 r \qquad \Rightarrow \qquad T > \sqrt{\frac{3\pi}{\rho G}}$$

11. Answer C.

$$\frac{\rho_{liquid}}{\rho_{water}} = \frac{F_{liquid}}{F_{liquid}} = 2$$

12. Answer D.

Energy Conservation:

$$\frac{1}{2}(m+M)v^2 = (m+M)gh$$

$$v = 0.313\,\mathrm{ms^{-1}}$$

Momentum Conservation:

$$mv_b = (m+M)v$$

$$v_b = 62.9\,\mathrm{ms^{-1}}$$

Initial Kinetic Energy:

$$KE = \frac{1}{2}m{v_b}^2 = 19.8\,\mathrm{J}$$

13. Answer B or C.

$$x = v_0 t \cos\theta$$

$$y = v_0 t \sin\theta - \frac{1}{2} g t^2$$

When the ball hits the inclined plane again,

$$y = -x \tan\theta$$

Eliminating *x*,

$$-v_0 t \sin\theta = v_0 t \sin\theta - \frac{1}{2} g t^2$$

$$t = \frac{4 v_o \sin\theta}{g}$$

$$x = v_o (\cos\theta)(t) = \frac{4 v_o{}^2 \sin\theta \cos\theta}{g}$$

$$s = \frac{x}{\cos\theta} = \frac{4 v_o{}^2 \sin\theta}{g}$$

Remark: Students reading the English version may choose answer B. Students reading the Chinese version may choose answer C. Due to this discrepancy, both answers are accepted.

14. Answer A.

15. Answer E.

Consider the system with 3 spheres:
a. sphere of density ρ and radius R with $y_1 = 0$ (center of mass).
b. sphere of density -ρ and radius R/2 with $y_2 = R/2$.
c. sphere of density 5ρ and radius R/2 with $y_3 = R/2$.
(b) and (c) are equivalent to a sphere of density 4ρ and radius R/2 with $y_4 = R/2$.

Center of mass of the new sphere:

$$Y = \frac{\sum m_i y_i}{\sum m_i} = \frac{\rho\left(\frac{4}{3}\pi R^3\right)(0) + 4\rho\left[\frac{4}{3}\pi (R/2)^3\right](R/2)}{\rho\left(\frac{4}{3}\pi R^3\right) + 4\rho\left[\frac{4}{3}\pi (R/2)^3\right]}$$

$$= \frac{R}{6}$$

16. Answer: C.
Potential Energy:

$$E = g\rho A \int z\, dz$$
$$= \frac{1}{2} g\rho A h^2$$

$$\frac{E}{A} = \frac{1}{2} g\rho h^2$$
$$= 5007.8 h^2 \qquad [\mathrm{J}]$$
$$= 1.39 h^2 \qquad [\mathrm{Wh}]$$

17. Answer D.

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad \Rightarrow \qquad T_M = \sqrt{\frac{g_E}{g_M}} \cdot T_E = \sqrt{6} \cdot T_E$$

18. Answer B.

Using the equation $\frac{GM_{\mathrm{Earth}} m}{R^2} = \frac{mv^2}{R}$ and period of the Earth T = 86400 s to calculate the height of a geostationary satellite.

19. Answer A.

For large angle oscillation, the equation of motion becomes nonlinear, and the approximation $\sin\theta \approx \theta$ is now no longer hold. The restoring force is proportional to $\sin\theta$, rather than θ; therefore its magnitude is less than in the case of SHM. A weaker restoring force also results in slower oscillation, that is, the period becomes longer.

20. Answer C.

It is a plot of $g = GM/R^2$, that is, g against (M/R^2).

Open-ended Questions

1. Venus Transit

(a) Kepler Law: $\frac{P^2}{a^3} = $ Constant, P = Orbit Period, a = Orbit Radius

$$\left(\frac{a_{Earth}}{a_{Venus}}\right)^3 = \left(\frac{365}{225}\right)^2$$

$$\frac{a_{Earth}}{a_{Venus}} = 1.3806$$

(b) $$\frac{A'B'}{AB} = \frac{A'V}{AV} = \frac{A'V}{AA' - A'V} = \frac{1}{a_{Earth}/a_{Venus} - 1} = 2.6273$$

$$A'B' = 2.6273 \times 1800 \text{ km}$$
$$= 4729 \text{ km}$$

(c) Diameter of the Sun = 4729 km x 290 = 1.37x10^6 km

(d) Let v_E = velocity of Earth relative to Sun

Since planetary velocity is given by $v = \sqrt{\frac{GM_{\text{Sun}}}{r}}$, velocity of Venus relative to Sun = $v_E\sqrt{\frac{a_E}{a_V}} = 1.1750 v_E$.

As observed from Earth, velocity of Venus = $v_E\left(\sqrt{\frac{a_E}{a_V}} - 1\right) = 0.1750 v_E$, and velocity of Sun = $-v_E$.

Projected on to the surface of Sun, velocity of the shadow of Venus = $v_E\left(\sqrt{\frac{a_E}{a_V}} - 1\right)\frac{a_E}{a_V} = 0.2416 v_E$.

Hence the velocity of the shadow of Venus sweeping on the surface of Sun =

$$v_E\left[\left(\sqrt{\frac{a_E}{a_V}} - 1\right)\frac{a_E}{a_V} + 1\right] = 1.2416 v_E$$

$$v_E = \frac{2\pi r_E}{T_E} = \frac{(2\pi)(1.5\times 10^{11})}{(365)(24)(60)(60)} = 29886 \text{ m/s} \quad \text{or}$$

$$v_E = \sqrt{\frac{GM_{Sun}}{r_E}} = \sqrt{\frac{(6.67\times 10^{-11})(1.99\times 10^{30})}{1.5\times 10^{11}}} = 29747 \text{ m/s}$$

Time difference = $\frac{4729}{(1.2416)(29886)} = 127 \text{ s} = 2.12 \text{ min}$ or $\frac{4729}{(1.2416)(29747)} = 128 \text{ s} = 2.13 \text{ min}$

2. Terminal Velocity of Free Falling Object

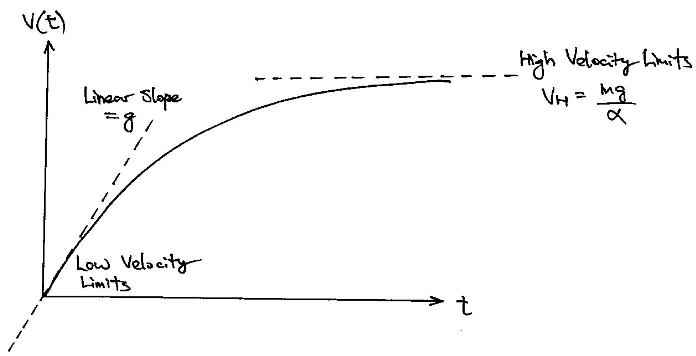
(a) $$m\frac{dv}{dt} = \underbrace{F_g}_{\text{Gravity}} + \underbrace{F_d}_{\text{Drag Force}} = mg - \alpha v$$

At low velocity limits, $F_d \to 0$.

$$m\frac{dv}{dt} \approx F_g \qquad \Rightarrow \qquad v_L \approx gt$$

At high velocity limits, $m\frac{dv}{dt} \to 0$.

$$F_g + F_d \approx mg - \alpha v \qquad \Rightarrow \qquad v_H \approx \frac{mg}{\alpha}$$

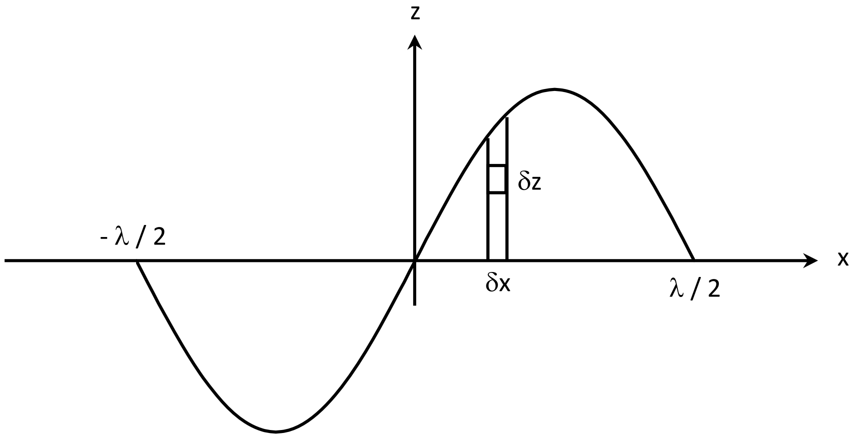


(b) $$mg = \frac{1}{2}C_d A\rho v^2$$

$$v = \sqrt{\frac{2mg}{C_d A\rho}} = \sqrt{\frac{2(2\times10^{-3})(9.8)}{0.5\times\pi\times(0.02)^2\times1.2}}$$
$$= 7.2\ \text{ms}^{-1}$$

(c) $$t \approx \frac{v}{g} = \frac{7.2}{9.8} = 0.7\ \text{s}$$

3. Total Energy in a Surface Wave



(a) Gain in potential energy δV from –z to z of an elemental mass δm = ρ δx δz:

$$\delta V = (\delta m)(g)(2z) = 2\rho g z(\delta x)(\delta z)$$

Total Potential Energy:

$$\begin{aligned} V &= 2\rho g \int_{x=0}^{x=\lambda/2} \int_{z=0}^{z=A\sin(2\pi x/\lambda)} z\, dz\, dx \\ &= 2\rho g \int_{x=0}^{x=\lambda/2} \left[\frac{z^2}{2} \right]_{z=A\sin(2\pi x/\lambda)} dx \\ &= \rho g \int_{x=0}^{x=\lambda/2} A^2 \sin^2(2\pi x/\lambda) dx \end{aligned}$$

Given that $\int \sin^2\left(\frac{2\pi x}{\lambda}\right) dx = \frac{x}{2} - \frac{\lambda \sin(4\pi x/\lambda)}{8\pi}$,

$$\begin{aligned} V &= \rho g A^2 \left[\frac{x}{2} \right]_0^{\lambda/2} \\ &= \frac{1}{4} \rho g A^2 \lambda \end{aligned}$$

Total PE per wavelength:

$$\frac{V}{\lambda} = \frac{1}{4} \rho g A^2$$

(b) Assume equipartition of energy (PE = KE), the total energy over a whole wavelength

$$E = \frac{1}{2} \rho g A^2$$

(c) Power of a Wave Period:

$$P_w = \left(\frac{1}{2}\rho g A^2\right) \times v_g$$

$$= \left(\frac{1}{2}\rho g A^2\right)\left(\frac{\lambda}{2T}\right) \qquad V_g = \frac{\lambda}{2T}$$

λ = Wavelength
T = Period

$$= \frac{\rho g^2 T A^2}{8\pi} \qquad \lambda = \frac{gT^2}{2\pi}$$

$$= \frac{\rho g^2 T H^2}{32\pi}$$

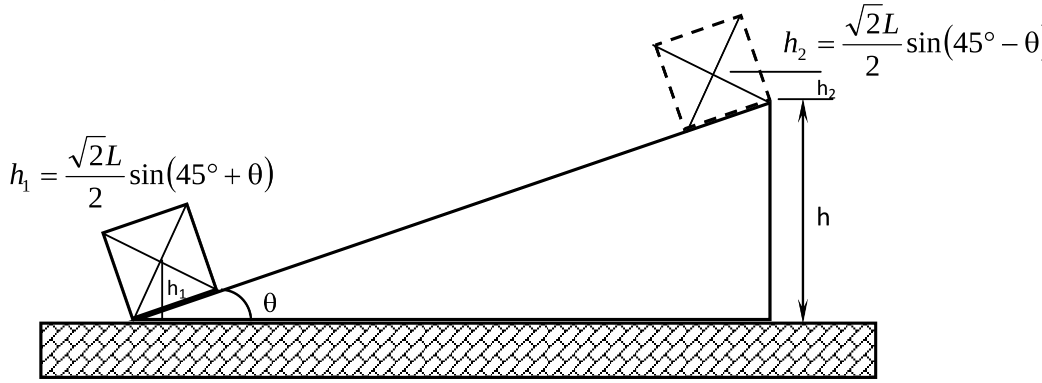
Wave amplitude is half of the wave height

4. A Sliding Block up a Slope Platform

(a) Momentum:

$$mv_o\cos\theta = (m+M)v_{Total} \qquad \Rightarrow \qquad v_{Total} = \frac{mv_o\cos\theta}{m+M}$$

Raise in Center of Mass, h'':



$$h'' = h + h_2 - h_1$$

$$= h + \frac{\sqrt{2}L}{2}\sin(45° - \theta) - \frac{\sqrt{2}L}{2}\sin(45° + \theta)$$

Applying the trigonometric identity: $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$,

$$h'' = h - L\sin\theta$$

Energy:

$$\frac{1}{2}mv_o^{\ 2} = \frac{1}{2}(m+M)v_{Total}^{\ \ 2} + mgh''$$

$$\frac{1}{2}mv_o^{\ 2} = \frac{1}{2}(m+M)\left(\frac{mv_o\cos\theta}{m+M}\right)^2 + mgh''$$

$$\left(m - \frac{m^2\cos^2\theta}{m+M}\right)v_o^{\ 2} = 2mgh''$$

$$v_o^{\ 2} = \frac{2gh''(m+M)}{(m+M) - m\cos^2\theta}$$

$$= \frac{2gh''}{1 - (m\cos^2\theta)/(m+M)}$$

$$v_o = \left[\frac{2gh''}{1 - (m\cos^2\theta)/(m+M)}\right]^{1/2}$$

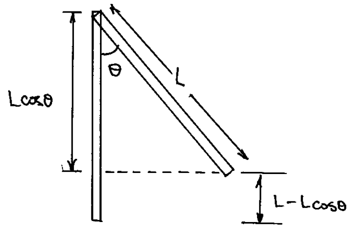
$$= \left[\frac{2g(h - L\sin\theta)}{1 - (m\cos^2\theta)/(m+M)}\right]^{1/2}$$

(b)

$$KE = \frac{1}{2}(m+M)\left(\frac{mv_o\cos\theta}{m+M}\right)^2$$

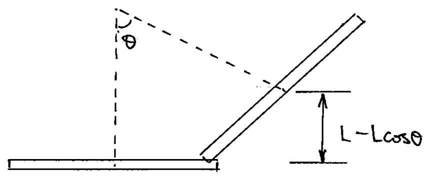
$$= \frac{1}{2}\frac{(mv_o\cos\theta)^2}{m+M}$$

5.



The center of mass is raised by:

$$h_b = \frac{L - L\cos\theta}{2}$$



The center of mass is raised by:

$$h_a = L - L\cos\theta$$

Potential Energy,

$$PE = mg\left[\frac{L - L\cos\theta}{2} + \left(L - L\cos\theta\right)\right]$$

$$= \frac{3}{2}mg\left(L - L\cos\theta\right)$$

$$= \frac{3}{2}mgL\left(1 - \cos\theta\right)$$

$$= \frac{3}{2}mgL\left[1 - \sqrt{1 - \frac{x^2}{L^2}}\right] \approx \frac{3}{2}mgL\left[1 - \left(1 - \frac{x^2}{2L^2}\right)\right] = \frac{3mg}{4L}x^2$$

Kinetic Energy,

$$KE = \frac{17}{24}mv^2$$

PE + KE = Constant

$$\frac{3mg}{4L}x^2 + \frac{17}{24}mv^2 = \text{Constant}$$

(b)

The total energy is equivalent to that of a mass-spring system with an effective mass of $m_{\text{eff}} = \frac{17}{12}m$ and an effective spring constant of $k_{\text{eff}} = \frac{3mg}{4L}$.

Hence

$$\omega = \sqrt{\frac{k_{\text{eff}}}{m_{\text{eff}}}} = \sqrt{\left(\frac{3mg}{2L}\right)\left(\frac{12}{17m}\right)} = \sqrt{\frac{18g}{17L}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{17L}{18g}}$$

(c)

Assume that the initial velocity is 0. When the initial displacement is x_0, the simple harmonic motion is given by

$$x = x_0 \cos(\omega t)$$

When $x = x_0/2$ for the first time,

$$\frac{x_0}{2} = x_0 \cos(\omega t) \quad \Rightarrow \quad \cos(\omega t) = \frac{1}{2} \quad \Rightarrow \quad \omega t = \frac{\pi}{3} \quad \Rightarrow \quad t = \frac{\pi}{3\omega} = \frac{\pi}{3}\sqrt{\frac{17L}{18g}} \text{ or } t = \frac{T}{6}$$

Hong Kong Physics Olympiad 2015
2015 香港物理奧林匹克

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10 May, 2015
2015 年 5 月 10 日

Rules and Regulations 競賽規則

1 All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

所有題目均爲中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你即可開始作答開放式題目，而無須等候任何宣佈。

3. On the cover of the answer book and the multiple-choice answer sheet, please write your 8-digit Contestant number and your English Name in the “Name” box.

在答題簿封面及選擇題答題紙上，請填上你的 8 位數字參賽者號碼及於“Name”欄上填上你的英文姓名。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. The open problems are long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the following parts.

開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來解答其他部分。

The following symbols and constants are used throughout the examination paper unless otherwise specified:
除非特別註明，否則本卷將使用下列符號和常數：

Gravitational acceleration on Earth surface 地球表面重力加速度	g	9.8 m/s^2
Gravitational constant 萬有引力常數	G	6.67×10^{-11} N m^2/kg^2
Radius of Earth 地球半徑	R_E	6378 km
Sun-Earth distance 太陽-地球距離 (= 1 Astronomical Unit (AU)) (= 1 天文單位(AU))	r_E	1.5×10^{11} m
Earth-Moon distance 地球-月球距離	r_m	384400 km
Mass of Sun 太陽質量	M_{Sun}	1.99×10^{30} kg
Mass of Earth 地球質量	M_E	5.98×10^{24} kg
Density of Air 空氣密度	ρ_0	1.2 kg/m^3
Density of Water 水密度	ρ_w	1000 kg/m^3

Trigonometric Identities:
三角學恆等式：

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

Tayler Series:
泰勒級數:

$$\sin(x)\approx x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots$$

$$\cos(x)\approx 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots$$

$$\tan(x)\approx x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$$

Multiple Choice Questions

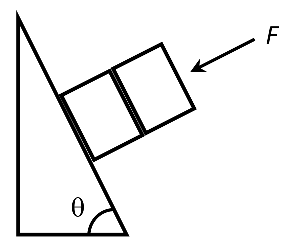
Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.5 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.

選擇題

每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.5 分，但全部選擇題最低為 0 分。

1. A board (mass m) is sandwiched between a stationary incline (angle θ) and another identical board. A force acts on the outer board to keep the boards from slipping. If the coefficient of friction between all the surfaces is μ, what is the component of the minimum compression force, F normal to the board surface?

一塊板（質量 m）被夾在一固定斜台（角度 θ）與另一相同板子之間。一力作用在外層板子以保持它們不會滑下。假設所有表面之間的摩擦係數為 μ，垂直於外層板子的最小壓縮力的分量 F 是多少？



A. $mg(\sin\theta - 2\mu\cos\theta)$

B. $\dfrac{mg(\sin\theta - \mu\cos\theta)}{\mu}$

C. $\dfrac{mg(\sin\theta - 2\mu\cos\theta)}{2\mu}$

D. $\dfrac{mg(\sin\theta - 3\mu\cos\theta)}{2\mu}$

E. $(mg\sin\theta + \mu\cos\theta)$

2. A uniform metal ball (mass m) is tied to the end of a string (massless, length L). The other end of the string is fixed. The ball is moving in a vertical circle centered at the fixed end of the string. Suppose the velocity of the ball at the lowest point is v_o, what is the string tension when it makes an angle θ to the downward vertical position?

一個均勻質量的金屬球（質量 m）被繩子（無質量，長度 L）的一端綁著，另一端則固定。金屬球正繞著繩子的固定點垂直圈打轉。假設金屬球在最低點的速度為 v_o，當它成向下 θ 角度時（從垂直線計），繩子的張力是多少？

A. $\dfrac{mv_o^2}{L} + mg(\cos\theta - 1)$

B. $\dfrac{mv_o^2}{L} + mg(2\cos\theta - 1)$

C. $\dfrac{mv_o^2}{L} + mg(3\cos\theta - 2)$

D. $\dfrac{mv_o^2}{2L} + mg(\cos\theta - 1)$

E. $\dfrac{mv_o^2}{2L} + mg(2\cos\theta - 1)$

3. The orbital properties of a geostationary satellite include i) its orbit is directly over the Earth equator, and ii) its orbital period is the same as the Earth's rotation period. Suppose a lunar stationary satellite (its orbital period is the same as the Moon's rotation period) is placed over the Moon equator, what is the value of the ratio $(R_{\text{Earth}} + H_{\text{Earth}}) / (R_{\text{Moon}} + H_{\text{Moon}})$? R_{Earth} and R_{Moon} are the Earth and the Moon radii, H_{Earth} and H_{Moon} are the satellite heights from the Earth and from the Moon surfaces, respectively. You may assume (Earth mass) / (Moon mass) = 81 and rotation period of the Moon = 27 days.

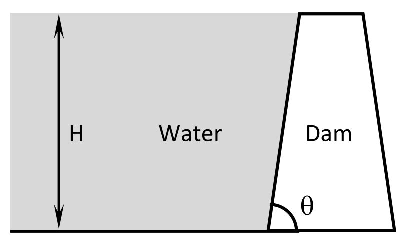
地球同步衛星的軌道有以下特性，包括 一）它的軌道是直接在地球赤道的上空，與及 二）其軌道週期與地球轉動週期是相同的。假設一月球同步衛星（軌道週期與月球轉動週期相同）被放置在月球的赤道上空，$(R_{\text{Earth}} + H_{\text{Earth}}) / (R_{\text{Moon}} + H_{\text{Moon}})$ 的比率值是甚麼？R_{Earth} 和 R_{Moon} 分別是地球和月球之半徑，而 H_{Earth} 和 H_{Moon} 分別是從地球和月球地面上的衛星的高度。你可假設（地球質量）/（月球質量）= 81，而月球的自轉週期為 27 天。

A. (81×27)
B. $(81 \times 27)^2$
C. $(81 / 27)^3$
D. $(1/9)^3$
E. $(1/9)^{1/3}$

4. A dam (height H, width W, one wall inclined at angle θ) is fully filled with water (density ρ). If g is the acceleration due to gravity, the resultant force exerted by the water on the dam wall is given by

一個完全盛載滿了水（密度 ρ）的壩（高度 H，寬度 W，而其中一面之壩壁傾斜角為 θ），假設 g 是重力加速度，水在壩壁所造成的合力是

A. $(\rho g H^2 W)/2$
B. $(\rho g H^2 W)/(2\sin\theta)$
C. $(\rho g H W)/(2\cos\theta)$
D. $(\rho g H^2 W)/(2\cos\theta)$
E. $(\rho g H W)/(\sin\theta + \cos\theta)$

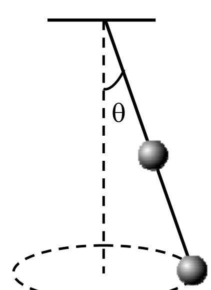


Remark: Question 5 is cancelled.

5. A conical pendulum with 2 identical bobs on a massless rod (length L) makes an angle of θ with the vertical. One bob is mounted at $L/2$ of the wire, whereas the other is mounted at the end of the rod. The period of the rotational motion is:

一個以 2 個完全相同的錘子和一根無質量的棒子（長度 L）所組成的錐形鐘擺，旋轉時與垂直方向成角度 θ。一錘子被安裝在棒子的 $L/2$ 位置，而另一個則被安裝在棒子的末端。該系統的旋轉週期是：

A. $\pi\sqrt{2L\sin\theta/g}$
B. $\pi\sqrt{3L\cos\theta/g}$
C. $2\pi\sqrt{2L\sin\theta/g}$
D. $2\pi\sqrt{3L\cos\theta/g}$
E. $\pi\sqrt{L\tan\theta/g}$



6. Suppose you are standing in an elevator, the elevator is moving upward at a constant speed of 0.5 m/s, the net force on you is

假設你站在電梯中，電梯正以 0.5 m/s 的速度上升，在你身上的淨力是

A. zero.
B. + 0.5 N.
C. - 9.8 N.
D. + 9.8 N.
E. your weight.

7. A pendulum has a length of 1 m. How many swings does the pendulum perform in an hour?

一個 1 米長的鐘擺，一小時可擺動多少次？

A. 60.
B. 300.
C. 900.
D. 1800.
E. 3600.

8. A ball (mass m_2) is hanging at rest from a wire (negligible mass). A head-on elastic collision occurs when it is struck by a stone (mass m_1, horizontal velocity v_1 before collision), the ball swings to a maximum height (h) above the original level. The maximum height (h) is

起初，一個球（質量 m_2）靜止地掛在繩子上（繩子本身的質量可以忽略不計）。當它被一塊石頭（質量 m_1，碰撞前水平速度為 v_1）擊中並發生對正彈性碰撞時，從原高度計，該球擺動至最大高度（h）。最大高度（h）是

A. $\frac{1}{2g}\left(\frac{m_1}{m_1+m_2}\right)v_1^2$
B. $\frac{1}{2g}\left(\frac{m_1}{m_1+m_2}\right)^2 v_1^2$
C. $\frac{2}{g}\left(\frac{m_1}{m_1+m_2}\right)v_1^2$
D. $\frac{2}{g}\left(\frac{m_1}{m_1+m_2}\right)^2 v_1^2$
E. $\frac{2}{g}\left(\frac{m_1}{m_2}\right)^2 v_1^2$

9. An electric water pump draws water from a well of depth 10 m at a rate 0.1 m^3/s. The water is then rejected from the pump with velocity 2 m/s. If the efficiency of the pump is 80%, the power consumption of the pump is

一電動水泵以 0.1 m^3/s 的速率將水從 10 m 深的井泵到地面。水然後以 2 m/s 的速度從泵流出。假設泵的效率是 80%，泵的功率是

A. 8 kW.
B. 10 kW.
C. 12.5 kW.
D. 17.5 kW.
E. 20 kW.

10. A car (mass *m*) undergoes a circular horizontal track (radius *r*) at constant speed *v*. Suppose *h* is the height of the center of mass (CG) above ground, and *a* is the horizontal separations between the left/right wheels and the center of mass. What are the normal forces N_1 and N_2? The direction of friction force *f* is indicated.

一輛汽車（質量 *m*）以恆定速度 *v* 在一圓形且水平的路上行駛（半徑為 r）。假設 *h* 是地面與質心（CG）的高度，*a* 是左/右車輪和質心之間的水平距離。圖中所示是摩擦力 *f* 的方向。問 N_1 和 N_2 的法向力是甚麼？

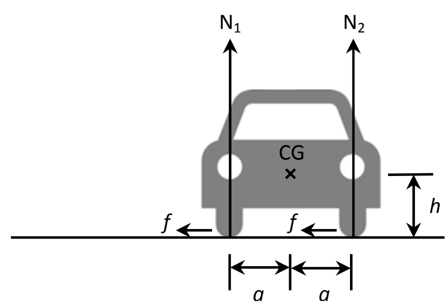
A. $N_1 = \frac{1}{2}\left(mg - \frac{hmv^2}{ar}\right),\ N_2 = \frac{1}{2}\left(mg + \frac{hmv^2}{ar}\right).$

B. $N_1 = \frac{1}{2}\left(mg + \frac{hmv^2}{ar}\right),\ N_2 = \frac{1}{2}\left(mg - \frac{hmv^2}{ar}\right).$

C. $N_1 = \frac{mg}{2} - \frac{hmv^2}{ar}, \quad N_2 = \frac{mg}{2} + \frac{hmv^2}{ar}.$

D. $N_1 = \frac{mg}{2} + \frac{hmv^2}{ar}, \quad N_2 = \frac{mg}{2} - \frac{hmv^2}{ar}.$

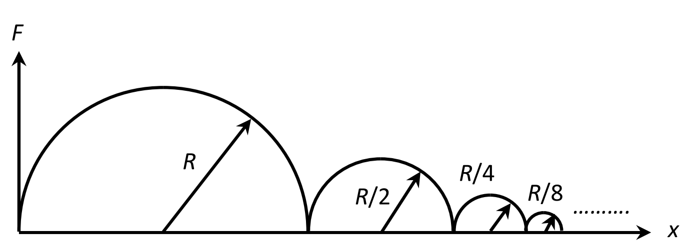
E. $N_1 = \frac{mg}{2} + \frac{hfmv^2}{ar}, \quad N_2 = \frac{mg}{2} - \frac{hfmv^2}{ar}.$



11. How much work is done in the following force (*F*) against distance (*x*) graph? The force is an infinite series of semi circles with radii of R, R/2, R/4, R/8, ……….

以下的力（*F*）與距離（*x*）的關係圖有多少功？該力的性質是由無窮級數的半圓所組成，半徑為 R， R/2， R/4， R/8， ……….。

A. $(1 / 3)\ \pi R^2$.
B. $(2 / 3)\ \pi R^2$.
C. πR^2.
D. $(4 / 3)\ \pi R^2$.
E. $(5 / 3)\ \pi R^2$.



12. A binary star system consists of 2 stars orbiting around their common center of mass. The two stars have identical mass. The orbital speed of each star is 200 km/s and the orbital period of each is 12.2 days. Find the mass of each star.

一雙星體系擁有兩顆星，兩顆星的質量相同，而它們的軌道是圍繞着它們共同的質心。每顆星都以 200 km/s 的軌道速度運行，而它們的軌道週期為 12.2 天。求每顆星的質量。

A. 0.00010 Solar Mass
B. 0.00020 Solar Mass
C. 10.1 Solar Mass
D. 20.2 Solar Mass
E. 40.4 Solar Mass

A. 0.00010 太陽質量
B. 0.00020 太陽質量
C. 10.1 太陽質量
D. 20.2 太陽質量
E. 40.4 太陽質量

13. A block lies on a horizontal and frictionless turntable. The block is connected to a string which is attached to the center of the turntable. The turntable is rotating with slow uniform angular speed ω. An observer sits on the turntable. According to the observer, which of the following is/are true?

i. The block is accelerating.
ii. The block is not accelerating.
iii. The magnitude of the string tension is proportional to ω^2.

一方塊躺在一水平且無摩擦力的轉盤上。該方塊被一條無質量的繩子連接到轉盤的中心位置。轉盤以緩慢均勻角速率 ω 旋轉。一觀察者坐在轉盤上。根據該觀察者，下列那項是正確？

i. 該方塊正在加速。
ii. 該方塊沒有加速。
iii. 繩子的張力與 ω^2 為正比。

A. (i) only.
B. (ii) only.
C. (i) and (iii).
D. (ii) and (iii).
E. (i), (ii), and (iii).

14. A rock is launched vertically on the Earth's surface at 8 km/s. Ignoring air resistance and the Earth rotation, the height of the projectile that can rise is:

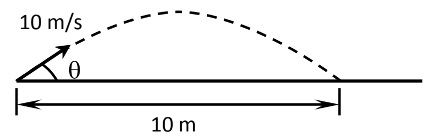
一塊石以 8 km/s 從地球表面垂直發射。在忽略空氣阻力和地球旋轉的情況下，該石塊可達到的高度是：

A. 3,265 km.
B. 6,683 km.
C. 10,597 km.
D. 20,196 km.
E. 40,328 km.

15. A ball is projected at angle(s) θ and at speed 10 m/s, and eventually the range of the projectile is 10 m. The possible value(s) of θ is/are:

一個球以 θ 角度並以 10 m/s 的速度被拋射，最終該球的射程是 10 m。θ 的可能值是：

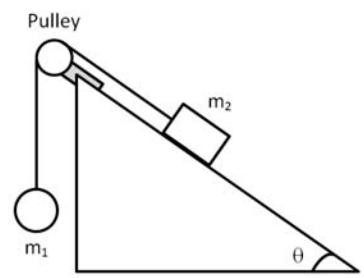
A. 39.25°.
B. 50.75°.
C. 78.5°.
D. 39.25° or 50.75°.
E. 50.75° or 78.5°.



16. As shown in the figure, a ball (mass m_1) and a block (mass m_2) are attached by a cord (massless), which passes over a frictionless pulley. The block lies on a frictionless incline of angle θ. What is the condition that the block will accelerate down the incline?

如圖所示，一個球（質量 m_1）和一方塊（質量 m_2）被連接到一條無質量的線，並繞過一個無摩擦力的滑輪。該方塊躺在一個無摩擦力的斜面上，斜面的角度是 θ。如要方塊向斜面下加速，該斜面角度的條件是甚麼？

A. $\sin\theta > m_1/m_2$.
B. $\cos\theta > m_1/m_2$.
C. $\sin\theta > (m_1/m_2)^2$.
D. $\cos\theta > (m_1/m_2)^2$
E. $(\sin\theta + \cos\theta) > m_1/m_2$.



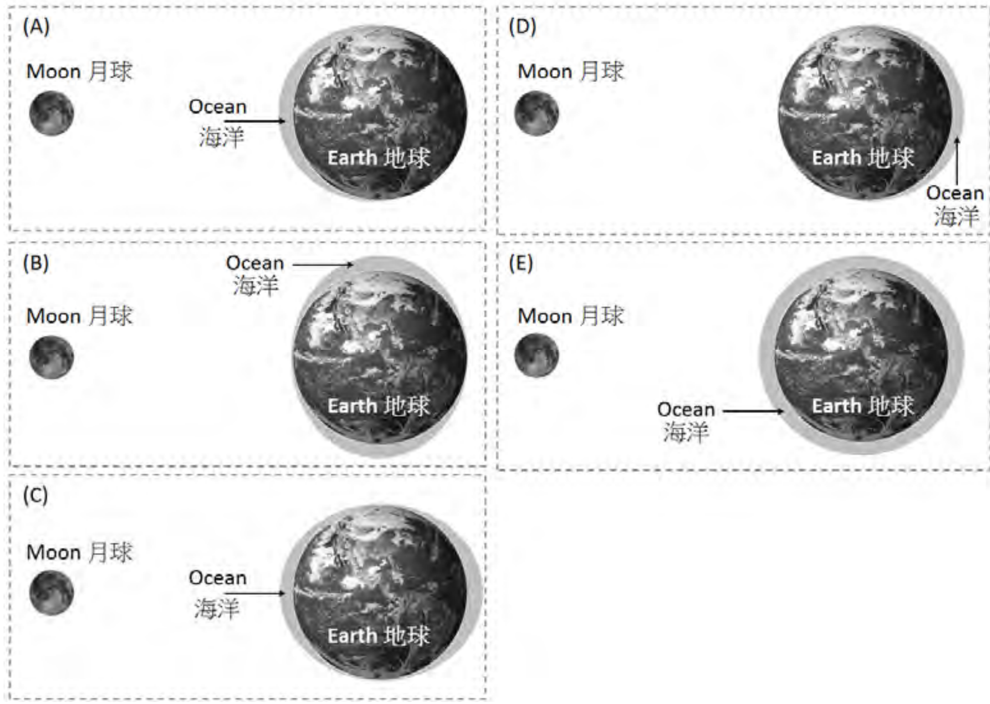
17. A solid sphere is attached at one end of a cord (length R) and is set into motion in a vertical circle about the other end which is fixed. Determine the speed of the sphere when it passes over the top of the circle with zero cord tension.

一實心球被連接在一條繩子的末端（長度為 R），並以繩子的另一端作固定點垂直圈打轉。當它越過圓圈的頂部，同時繩子的張力為零時，求實心球當時的速度。

A. $\sqrt{gR}$
B. $\sqrt{gR\sin\theta}$
C. $\sqrt{gR\cos\theta}$
D. $\sqrt{gR\sin\theta\cos\theta}$
E. $\sqrt{gR\tan\theta}$

18. By considering the gravitational forces of the Moon and the Earth only, which of the following ocean tide is most possible? The diagram is not drawn to scale.

只考慮月球和地球的引力，下列那一個海潮是最有可能發生的？該圖是不按比例繪製。



19. A small block (mass 0.2 kg) is attached to a massless spring and undergoes a simple harmonic motion on a smooth surface (amplitude A = 0.1 m, angular frequency ω = 0.5 radian/s). The displacement of the particle at time t from its equilibrium position is described by

$$x(t) = A\cos(-\omega t),$$

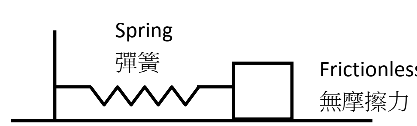
If the potential energy of the spring at the equilibrium position is zero, what is the total energy of the system?

一小方塊（質量 0.2 kg）連接到彈簧並且在光滑水平面上展開簡諧運動（振幅 A = 0.1 m，角頻率 ω = 0.5 radian/s）。其相對於平衡點的位移與時間的關係是

$$x(t) = A\cos(-\omega t)\text{，}$$

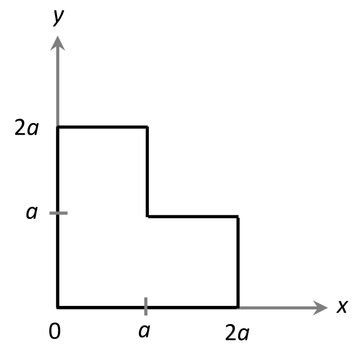
如果該彈簧的平衡點勢能為零，該系統的總能量是多少？

A. 0.05 mJ.
B. 0.10 mJ.
C. 0.15 mJ.
D. 0.20 mJ.
E. 0.25 mJ.



20. The figure shows a "L" shape object of uniform mass per unit area ρ. Where is the center of mass $\vec{r}_G$?

該圖顯示了一個"L"形的均勻物體，每單位面積的質量是 ρ。質心 $\vec{r}_G$ 在何處？



A. $\vec{r}_G = \dfrac{2a}{3}\left(\hat{i} + \hat{j}\right)$.

B. $\vec{r}_G = \dfrac{3a}{4}\left(\hat{i} + \hat{j}\right)$.

C. $\vec{r}_G = \dfrac{4a}{5}\left(\hat{i} + \hat{j}\right)$.

D. $\vec{r}_G = \dfrac{5a}{6}\left(\hat{i} + \hat{j}\right)$.

E. $\vec{r}_G = a\left(\hat{i} + \hat{j}\right)$.

--- END OF MULTIPLE CHOICE SESSION 選擇題完 ---

Open Problems 開放題
Total 5 Problems 共 5 題

1. A bouncing ball (10 Marks)

A ball is allowed to drop freely from rest and at a height H_o above the ground. After hitting the ground, the ball bounces to a height H_1. The bouncing process repeats continuously until the bouncing height is too small to be observed.

(a) Calculate the coefficient of restitution, C_R, in terms of H_o and H_1. C_R is defined as:

C_R = Relative speed after collision / Relative speed before collision.

(b) Determine the total distance that the ball travels. Express your answer in terms of H_o and C_R.

(c) Find the total time duration that the ball travels. Express your answer in terms of H_o and C_R.

Hints: You may assume:

i. C_R is constant throughout all the bouncing,
ii. Time of contact between the ball and the ground is negligible, and
iii. Air resistance is negligible.

反彈球（10 分）

一個球從靜止狀態由 H_o 的高度自由下降到地面。撞落地面後，球反彈到高度 H_1。彈跳過程不斷地重複，直至彈跳高度太小而不能被察覺。

(a) 計算恢復係數，C_R。答案請以 H_o 與及 H_1 表達。C_R 的定義為：

C_R = 碰撞後的相對速度 / 碰撞前相對速度。

(b) 求該球移動了的總距離。答案請以 H_o 與及 C_R 表達。

(c) 求該球移動了的總時間。答案請以 H_o 與及 C_R 表達。

提示：你可以假設：

i. 在所有的反彈，C_R 是常數不變的，
ii. 球和地面之間的接觸時間可以忽略不計，和
iii. 空氣阻力可忽略不計。

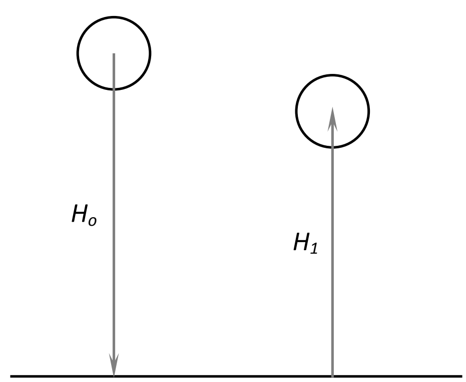


Fig. 1 – Bouncing ball
圖 1 – 反彈球

2. A suspended horizontal platform (15 Marks)

As shown in Fig. 2, a uniform rectangular platform (length L, mass m) is suspended by two ropes making angle θ_1 and θ_2 to the vertical. In order to keep the platform horizontal, a load (mass $2m$) is placed on the platform.

(a) Find the tensions T_1 and T_2 in terms of m, θ_1, θ_2, and/or L.

(b) In terms of m, θ_1, θ_2, and/or L, determine x in order to keep the platform horizontal.

(c) If ($\theta_1 + \theta_2 = 90°$) and $x = L / 8$, find θ_1 and θ_2.

懸吊水平平台（15 分）

如圖 2 所示，一個均勻矩形平台（長度為 L，質量 m）懸掛在兩根繩索上。從垂直計，該角度為 θ_1 和 θ_2。為了保持該平台的水平性，另一負載物（質量 $2m$）被放置在平台上。

(a) 以 m，θ_1，θ_2，和/或 L 作表達，計算 T_1 和 T_2 的張力。

(b) 以 m，θ_1，θ_2，和/或 L 作表達，計算 x 以保持該平台水平性。

(c) 如果 ($\theta_1 + \theta_2 = 90°$) 和 x = $L / 8$，計算 θ_1 和 θ_2。

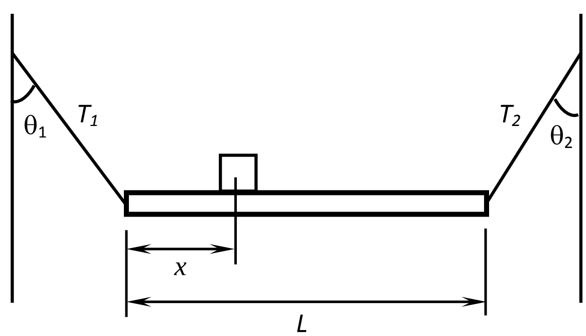


Fig. 2 – Suspended horizontal platform
圖 2 – 懸吊水平平台

3. Oscillating sphere in a cylindrical trough (15 Marks)

A solid sphere (radius R, mass m, and uniform density) rolls without slipping in a cylindrical trough (radius = 5R) with a small angle θ displaced from the equilibrium position. The kinetic energy of the sphere is $KE = \frac{56}{5} mR^2\omega^2$, where ω is the instantaneous angular velocity.

(a) Find the potential energy of the sphere in terms of m, g, R, and θ, where g is the acceleration due to gravity.

(b) Find the total energy of the sphere oscillator. Express your answer in the form of energy transformation in simple harmonic mass-spring system, that is, Total Energy $E = \frac{1}{2} m v_x{}^2 + \frac{1}{2} k x^2$, where you may consider x as an angular displacement.

(c) Determine the effective mass (m_{eff}), effective spring constant (k_{eff}), and period of the oscillation.

在圓筒形槽擺動的球（15 分）

一實心球（半徑 R，質量 m，和均勻密度）在沒有滑動的情況下，以從平衡點計的細小角度 θ 在圓筒形槽（半徑 5R）滾動。已知該球的動能是 $KE = \frac{56}{5} mR^2\omega^2$ ，而 ω 是瞬時角速度。

(a) 以 m，g，R 和/或 θ 作表達，求球體的勢能。

(b) 計算球體擺動系統的總能量。答案請以質量與彈簧的簡諧運動能量轉換形式表達，即是總能量 $E = \frac{1}{2} m v_x{}^2 + \frac{1}{2} k x^2$ ，其中你可考慮 x 為角位移。

(c) 計算該系統的有效質量（m_{eff}），有效彈簧常數（k_{eff}），與及滾動週期。

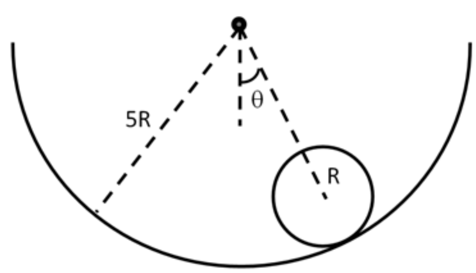


Fig. 3 – Oscillating sphere in a cylindrical trough
圖 3 – 在圓筒形槽擺動的球

4. Maximum gradient that a vehicle can climb up (15 Marks)

An engineer attempts to design a road with an angle of inclination θ to allow a vehicle (mass m) to climb. On one hand the engineer would like to maximize the gradient; on the other hand, the design has to satisfy the specifications that the wheels have to be adhesive to the gradient when the vehicle is a

i. rear wheel drive;

ii. front wheel drive; and

iii. four wheel drive.

The specifications further require a vehicle

iv. to avoid overturning; and

v. to avoid front and rear ends touch the road surface when climbing up a slope from a horizontal road.

A typical vehicle has the following dimensions (Fig. 4). Find the maximum gradients such that the vehicle can climb up at each of the above possibilities (i) to (v). Express your answers in terms of μ_R (the coefficient of static friction between the rear wheels and the road gradient), μ_F (the coefficient of static friction between the front wheels and the gradient), C_R (rear under body clearance), C_F (front under body clearance), H_R (rear over hanging), H_F (front over hanging), h (height between center or mass and the gradient), L_R, L_F, and/or L ($L = L_R + L_F$). You may neglect air resistance and assume that the vehicle is moving at constant speed.

車輛能行駛的最大路面傾斜度（15 分）

一名工程師試圖設計一條傾斜路面（傾斜角 θ）以允許車輛（質量 m）上行。一方面，工程師希望設計一條具有最大傾斜度的道路; 而另一方面，規格要求以下車輪必須緊貼斜路的路面行駛：

i. 後輪驅動 ;

ii. 前輪驅動 ; 與及

iii. 四輪驅動。

規格進一步要求：

iv. 避免車輛翻轉 ;

v. 從水平路面轉上傾斜路面時避免車身前部及後部接觸到路面。

一輛典型的車輛具有如下尺寸（圖 4）。根據上述（i）至（v）所描述的情況，計算每一個情況路面最大的傾斜度以允許車輛行駛。答案請以 μ_R（後輪與傾斜路路面之間的靜摩擦係數），μ_F（前輪與傾斜路路面之間的靜摩擦係數），C_R（車身後方與路面的間隙），C_F（車身前方與路面的間隙），H_R（車身後方懸掛間隙），H_F（車身前方懸掛間隙），h（質心與路面的高度），L_R， L_F，和/或者 L （$L = L_R + L_F$）。空氣阻力可忽略不計，並且假設車輛以恆定速度行駛。

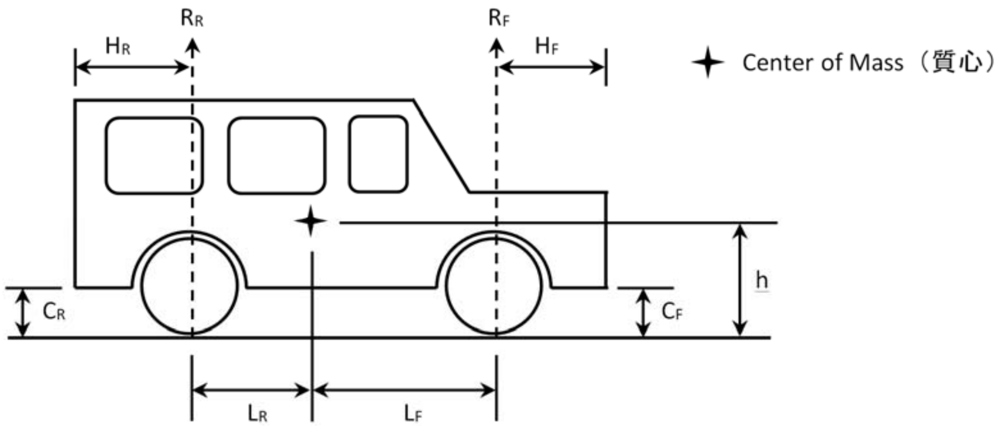


Fig. 4 – Specifications of typical vehicle
圖 4 – 典型車輛的規格

5. Fuel for rocket propulsion (5 Marks)

Consider a rocket in outer space (gravitational forces are negligible). The combustion products are ejected at a constant speed v_e relative to the rocket (total initial mass m_i).

(a) By considering the conservation of momentum only, derive the equation of the rocket velocity change Δv in terms of v_e, total mass change Δm of the rocket, and m_i.
Hint: You may ignore higher order terms.

(b) Given that v_e = 2.5 km/s, and that the rocket generates a constant force 3.5×10^7 N during the fuel burn, calculate the rate of fuel mass change.

火箭推進器之燃料（5 分）

考慮一枚太空中的火箭（引力可忽略不計）。燃料經燃燒後以相對於火箭的恆定速度 v_e 噴出火箭外。火箭的總初始質量為 m_i。

(a) 只考慮動量守恆，試導出火箭速度變化 Δv 的方程。答案請以 v_e，火箭之總質量變化 Δm，與及 m_i 作表達。
提示：你可忽略高階微量項。

(b) 已知 v_e = 2.5 km/s，且火箭在燃料燃燒過程中產生恆力 3.5×10^7 N，試計算燃料質量變化率。

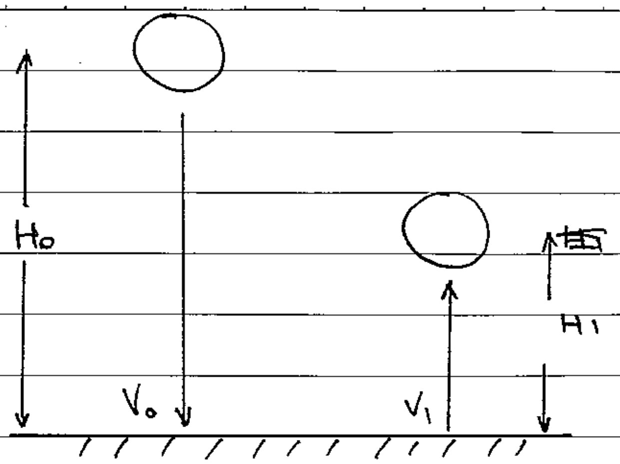
--- END OF EXAM PAPER 全卷完 ---

MC Key

1. D
2. C
3. E
4. B
5. ~~5. B~~ (Cancelled)
6. A
7. D
8. D
9. C
10. A
11. B
12. E
13. D
14. B
15. D
16. A
17. A
18. C
19. E
20. D

Long Questions

1.



$$S = \frac{1}{2} g t^2$$

$$V_0 = g\sqrt{\frac{2H_0}{g}}$$

$$V_1 = g\sqrt{\frac{2H_1}{g}}$$

(a) $$C_R = \frac{g\sqrt{\frac{2H_1}{g}}}{g\sqrt{\frac{2H_0}{g}}} = \sqrt{\frac{H_1}{H_0}}$$

(b) $$H_{Total} = H_0 + \underbrace{2H_1 + 2H_2 + \cdots\cdots}_{\text{include round trip distance}}$$

$$H_1 = C_R^2 H_0$$

$$H_2 = C_R^2 H_1 = C_R^2 C_R^2 H_0 = C_R^4 H_0$$

$$H_3 = C_R^6 H_0$$

$$H_{Total} = H_0 + 2C_R^2 H_0 + 2C_R^4 H_0 + 2C_R^6 H_0 + \cdots\cdots$$

$$= H_0\left[1 + 2C_R^2 + 2C_R^4 + 2C_R^6 + \cdots\right]$$

$$= H_0\left[2\left(1 + C_R^2 + C_R^4 + C_R^6 + \cdots\right) - 1\right]$$

$$= H_0\left[2\left(\frac{1}{1 - C_R^2}\right) - 1\right]$$

(c) $t_{total} = t_0 + t_1 + t_2 + \cdots\cdots$

$$t_0 = \left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

$$t_1 = 2\left(\frac{2H_1}{g}\right)^{\frac{1}{2}}$$

$$= 2\left(\frac{H_1}{H_0}\right)^{\frac{1}{2}}\left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

$$= 2\, C_R \left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

$$t_2 = 2\left(\frac{2H_2}{g}\right)^{\frac{1}{2}}$$

$$= 2\left(\frac{H_2}{H_1}\right)^{\frac{1}{2}}\left(\frac{H_1}{H_0}\right)^{\frac{1}{2}}\left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

$$= 2\, C_R^2 \left(\frac{2H_0}{g}\right)^{\frac{1}{2}}$$

where $t_1, t_2, \cdots$ include round trip time

$$t_{total} = \left(\frac{2H_0}{g}\right)^{\frac{1}{2}}\left[2\left(1 + C_R + C_R^2 + C_R^3 + \cdots\right) - 1\right]$$

$$= \left(\frac{2H_0}{g}\right)^{\frac{1}{2}}\left[2\left(\frac{1}{1 - C_R}\right) - 1\right]$$

2.

(a) x-component: $T_2 \sin\theta_2 - T_1 \sin\theta_1 = 0$

$$\Rightarrow \quad T_2 = T_1 \frac{\sin\theta_1}{\sin\theta_2} \quad \text{———} \ (1)$$

y-component: $T_1 \cos\theta_1 + T_2 \cos\theta_2 - 3\,mg = 0$ ——— (2)

About the position of the load:

$$-x T_1 \cos\theta_1 - \left(\frac{L}{2} - x\right) mg + (L - x) T_2 \cos\theta_2 = 0 \quad \text{———} \ (3)$$

(1) and (2)

$$\Rightarrow \quad T_1 \cos\theta_1 + T_1 \frac{\sin\theta_1}{\sin\theta_2} \cos\theta_2 - 3\,mg = 0$$

$$T_1 \left(\cos\theta_1 + \frac{\sin\theta_1}{\sin\theta_2} \cos\theta_2 \right) = 3\,mg$$

$$T_1 \left(\frac{\sin\theta_2 \cos\theta_1 + \sin\theta_1 \cos\theta_2}{\sin\theta_2} \right) = 3\,mg$$

$$T_1 \frac{\sin(\theta_1 + \theta_2)}{\sin\theta_2} = 3\,mg$$

$$T_1 = \frac{3\,mg \sin\theta_2}{\sin(\theta_1 + \theta_2)}$$

$$T_2 = T_1 \frac{\sin\theta_1}{\sin\theta_2}$$

$$= \frac{3\,mg \sin\theta_2}{\sin(\theta_1 + \theta_2)} \cdot \frac{\sin\theta_1}{\sin\theta_2}$$

$$= \frac{3\,mg \sin\theta_1}{\sin(\theta_1 + \theta_2)}$$

(b) ③ $\Rightarrow$ $\left(-T_1\cos\theta_1 + mg - T_2\cos\theta_2\right)x = \frac{L}{2}mg - T_2 L\cos\theta_2$

$$\left[-3mg\frac{\sin\theta_2}{\sin(\theta_1+\theta_2)}\cos\theta_1 + mg - 3mg\frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}\cos\theta_2\right]x$$

$$= \frac{L}{2}mg - 3mg\frac{\sin\theta_1}{\sin(\theta_1+\theta_2)}L\cos\theta_2$$

$$\left[\frac{-3mg\sin\theta_2\cos\theta_1 + mg\sin(\theta_1+\theta_2) - 3mg\sin\theta_1\cos\theta_2}{\sin(\theta_1+\theta_2)}\right]x$$

$$= \frac{\frac{L}{2}mg\sin(\theta_1+\theta_2) - 3mg\sin\theta_1 L\cos\theta_2}{\sin(\theta_1+\theta_2)}$$

$$\left[-3\sin\theta_2\cos\theta_1 + \sin(\theta_1+\theta_2) - 3\sin\theta_1\cos\theta_2\right]x$$

$$= \frac{L}{2}\sin(\theta_1+\theta_2) - 3L\sin\theta_1\cos\theta_2$$

$$\left[\sin(\theta_1+\theta_2) - 3\sin(\theta_1+\theta_2)\right]x = \frac{L}{2}\sin(\theta_1+\theta_2) - 3L\sin\theta_1\cos\theta_2$$

$$x = \frac{\frac{L}{2}\sin(\theta_1+\theta_2) - 3L\sin\theta_1\cos\theta_2}{-2\sin(\theta_1+\theta_2)}$$

$$= \frac{3L}{2}\frac{\sin\theta_1\cos\theta_2}{\sin(\theta_1+\theta_2)} - \frac{L}{4}$$

(c) $\theta_1 + \theta_2 = 90^\circ$

$$\Rightarrow \quad x = \frac{3L}{2}\sin\theta_1\cos\theta_2 - \frac{L}{4}$$

$$= \frac{3L}{2}\left[\frac{1}{2}\left(\sin(\theta_1-\theta_2) + \sin(\theta_1+\theta_2)\right)\right] - \frac{L}{4}$$

$$= \frac{3L}{4}\sin(\theta_1-\theta_2) + \frac{3L}{4} - \frac{L}{4}$$

$$= \frac{3L}{4}\sin(\theta_1-\theta_2) + \frac{L}{2}$$

(c) $x = \frac{L}{8}$

$$\Rightarrow \quad \frac{L}{8} = \frac{3L}{4} \sin(\theta_1 - \theta_2) + \frac{L}{2}$$

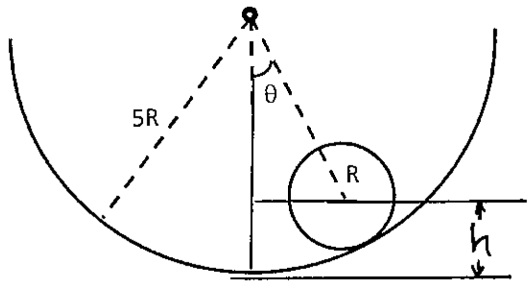
$$\sin(\theta_1 - \theta_2) = -\frac{1}{2}$$

$$\begin{cases} \theta_1 - \theta_2 = -30^\circ \\ \theta_1 + \theta_2 = 90^\circ \end{cases}$$

$$\Rightarrow \quad \theta_1 = 30^\circ$$
$$\theta_2 = 60^\circ$$

3.

(a) $KE = \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2$



For a displacement θ,
$h = 4R(1-\cos\theta)$

$PE = mgh = 4mgR(1-\cos\theta)$

(b) For small angle, $(1-\cos\theta) \approx \frac{\theta^2}{2}$

$\Rightarrow \quad PE \approx 4mgR\theta^2$

Total Energy = Constant = $E = \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 + 4mgR\theta^2$

$= \frac{1}{2}\left(\frac{112}{5} mR^2\right)\left(\frac{\Delta\theta}{\Delta t}\right)^2 + \frac{1}{2}(4mgR)\theta^2$

(c) $M_{eff} = \frac{112}{5} mR^2$

$k_{eff} = 4mgR$

$\omega = \sqrt{\frac{k_{eff}}{M_{eff}}} = \sqrt{\frac{4mgR}{112/5\, mR^2}}$

$= \sqrt{\frac{5}{28}\frac{g}{R}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{28R}{5g}}$

Alternative Approach

$\frac{dE}{dt} = \left(\frac{112}{5} mR^2\right)\ddot{\theta} + 4mgR\dot{\theta} = 0$

$\Rightarrow \quad \ddot{\theta} = -\frac{5}{28}\frac{g}{R}\dot{\theta}$

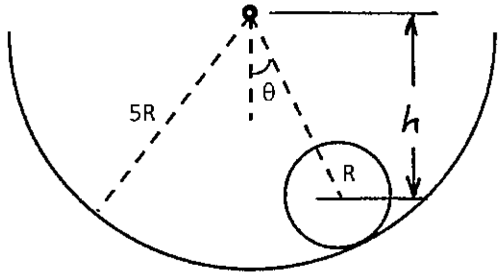
$\omega = \sqrt{\frac{5}{28}\frac{g}{R}}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{28}{5}\frac{R}{g}}$

3. Alternative Approach for (3a) and (3b)

(a) For students who use the circle center as the origin:

$$h = 4R\cos\theta$$



$$PE = mgh$$
$$= -4mgR\cos\theta$$

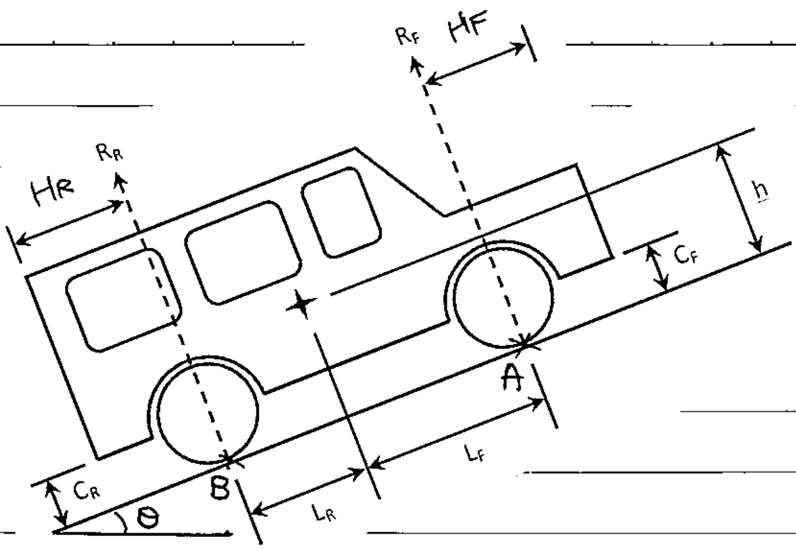
(b) For small angle, $\cos\theta \approx 1 - \frac{\theta^2}{2}$

$$PE + KE = \text{constant}$$

$$E = \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 - 4mgR\cos\theta$$
$$= \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 - 4mgR\left(1 - \frac{\theta^2}{2}\right)$$
$$= \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 - 4mgR + 2mgR\theta^2$$

$$\Rightarrow E + 4mgR = \text{constant} = \frac{56}{5} mR^2 \left(\frac{\Delta\theta}{\Delta t}\right)^2 + 2mgR\theta^2$$
$$= \frac{1}{2}\left(\frac{112}{5} mR^2\right)\left(\frac{\Delta\theta}{\Delta t}\right)^2 + \frac{1}{2}(4mgR)\theta^2$$

4.



(i) <u>Rear Wheel Drive</u>

Moment about A :

$$mg\cos\theta\, L_F + mg\sin\theta\, h - R_R L = 0 \quad \text{—— (1)}$$

Friction :

$$f = mg\sin\theta$$

Adhesion $\Rightarrow$ $f_{max} = \mu R_R = mg\sin\theta$

$$R_R = \frac{mg\sin\theta}{\mu} \quad \text{—— (2)}$$

(1) and (2) $\Rightarrow$ $mg\cos\theta\, L_F + mg\sin\theta\, h - \frac{mg\sin\theta}{\mu} L = 0$

$$L_F + h\tan\theta - L\frac{\tan\theta}{\mu} = 0$$

$$\tan\theta\left(h - \frac{L}{\mu}\right) = -L_F$$

$$\tan\theta = \frac{\mu L_F}{L - \mu h}$$

(ii) <u>Front Wheel Drive</u>

Moment about B :

$$-mg\cos\theta \, L_R + mg\sin\theta \, h + R_F L = 0 \quad \text{—— (3)}$$

Friction :

$$f = mg\sin\theta$$

Adhesion $\Rightarrow$ $f_{max} = \mu R_F = mg\sin\theta$

$$R_F = \frac{mg\sin\theta}{\mu} \quad \text{—— (4)}$$

(3) and (4)

$$\Rightarrow \quad -mg\cos\theta \, L_R + mg\sin\theta \, h + \frac{mg\sin\theta}{\mu} L = 0$$

$$-L_R + h\tan\theta + L\frac{\tan\theta}{\mu} = 0$$

$$\tan\theta \left(h + \frac{L}{\mu} \right) = L_R$$

$$\tan\theta = \frac{\mu L_R}{L + \mu h}$$

(iii) <u>Four Wheel Drive</u>

Friction : $f = mg\sin\theta$ ——— (5)

$f_{max} = \mu mg\cos\theta$ ——— (6)

(5) and (6) $\Rightarrow$ $mg\sin\theta = \mu mg\cos\theta$

$$\tan\theta = \mu$$

(iv) <u>Overturn</u>

Overturn $\Rightarrow$ $R_F = 0$

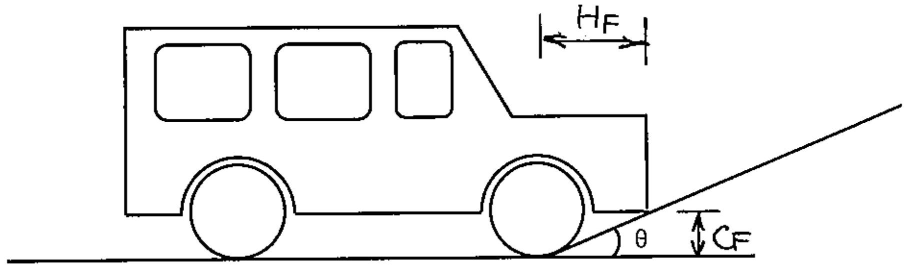
(3) $\Rightarrow$ $-mg\cos\theta\, L_R + mg\sin\theta\, h = 0$

$$\tan\theta = \frac{L_R}{h}$$

(V) <u>Clearance</u>

Front :

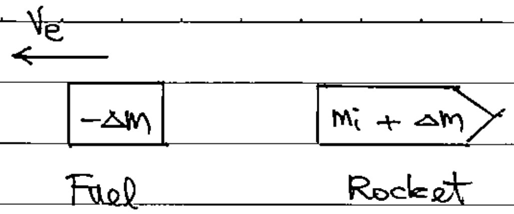
$$\tan\theta = \frac{C_F}{H_F}$$



Rear :

$$\tan\theta = \frac{C_R}{H_R}$$

5.



Remack: * At time $t + \Delta t$, rocket mass changes from m_i to $(m_i + \Delta m)$

* Δm is negative for decrease in rocket mass.

(a) Momentum: $$-\Delta m\, V_e = (m_i + \Delta m)\,\Delta V$$

$$-\Delta m\, V_e = m_i \Delta V + \cancel{\Delta m \Delta V}$$

$$\Delta V = -V_e \frac{\Delta m}{m_i}$$

(b) Acceleration $$a = \frac{\Delta V}{\Delta t} = -V_e \frac{\Delta m}{m_i}\left(\frac{1}{\Delta t}\right)$$

$$F = m_i a = -V_e \frac{\Delta m}{\Delta t}$$

$$\frac{\Delta m}{\Delta t} = \frac{-F}{V_e}$$

$$= \frac{-3.5 \times 10^7}{2500}$$

$$= -14000 \text{ kg/s}$$

Hong Kong Physics Olympiad 2016
2016 香港物理奧林匹克

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8 May, 2016
2016 年 5 月 8 日

Rules and Regulations 競賽規則

1. All questions are in bilingual versions. You can answer in either Chinese or English, but only ONE language should be used throughout the whole paper.

   所有題目均爲中英對照。你可選擇以中文或英文作答，惟全卷必須以單一語言作答。

2. The multiple-choice answer sheet will be collected 1.5 hours after the start of the contest. You can start answering the open-ended questions any time after you have completed the multiple-choice questions without waiting for announcements.

   選擇題的答題紙將於比賽開始後一小時三十分收回。若你在這之前已完成了選擇題，你即可開始作答開放式題目，而無須等候任何宣佈。

3. On the cover of the answer book and the multiple-choice answer sheet, please write your 8-digit Contestant Number, English Name, and Seat Number.

   在答題簿封面及選擇題答題紙上，請填上你的 8 位數字參賽者號碼，英文姓名，及座位號碼。

4. After you have made the choice in answering a multiple choice question, fill the corresponding circle on the multiple-choice answer sheet **fully** using a HB pencil.

   選定選擇題的答案後，請將選擇題答題紙上相應的圓圈用 HB 鉛筆完全塗黑。

5. The open problems are long. Please read the whole problem first before attempting to solve them. If there are parts that you cannot solve, you are allowed to treat the answer as a known answer to solve the following parts.

   開放式問答題較長，請將整題閱讀完後再著手解題。若某些部分不會做，也可把它們的答案當作已知來解答其他部分。

The following symbols and constants are used throughout the exam paper unless otherwise specified:
除非特別註明，否則本卷將使用下列符號和常數：

Useful Constants（常用常數）：

Astronomical Unit（**天文單位**）：1 AU = 1.496×10^8 km

Earth-Moon Distance（**地球-月球距離**）：d_M = 384,400 km

Mass of the Sun（**太陽質量**）：$M_S = 1.99\times10^{30}$ kg

Mass of the Earth（**地球質量**）：$M_E = 5.97\times10^{24}$ kg

Mass of the Moon（**月球質量**）：$M_M = 7.35\times10^{22}$ kg

Acceleration due to Gravity（**重力加速度**）：$g = 9.8$ ms^{-2}

Radius of the Sun（**太陽半徑**）：R_S = 696300 km

Radius of the Earth（**地球半徑**）：R_E = 6370 km

Radius of the Moon（**月球半徑**）：R_M = 1738 km

Gravitational Constant（**萬有引力常數**）：$G = 6.67\times10^{-11}$ m^3 kg^{-1} s^{-2}

Trigonometric Identities（三角學恆等式）：

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$

$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

$$\sin(2x)=2\sin(x)\cos(x)$$

$$\cos(2x)=\cos^2(x)-\sin^2(x)$$

$$\sin(x)\cos(y)=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\cos(y)=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$$

$$\sin(x)\sin(y)=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$$

Tayler Series（泰勒級數）：

$$\sin(x)\approx x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots$$

$$\cos(x)\approx 1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots$$

$$\tan(x)\approx x+\frac{x^3}{3}+\frac{2x^5}{15}+\cdots$$

$$\exp(x)\approx 1+x+\frac{x^2}{2}+\frac{x^3}{6}+\cdots$$

Series Summation（級數總和）：

$$\sum_{k=1}^{m}k=\frac{m(m+1)}{2}$$

$$\sum_{k=1}^{m}k^2=\frac{m(m+1)(2m+1)}{6}$$

$$\sum_{k=1}^{m}k^3=\left[\frac{m(m+1)}{2}\right]^2$$

$$\sum_{k=1}^{m}k^4=\frac{m(2m+1)(m+1)(3m^2+3m-1)}{30}$$

Hyperbolic Trigonometric Identities（雙曲三角恆等式）：

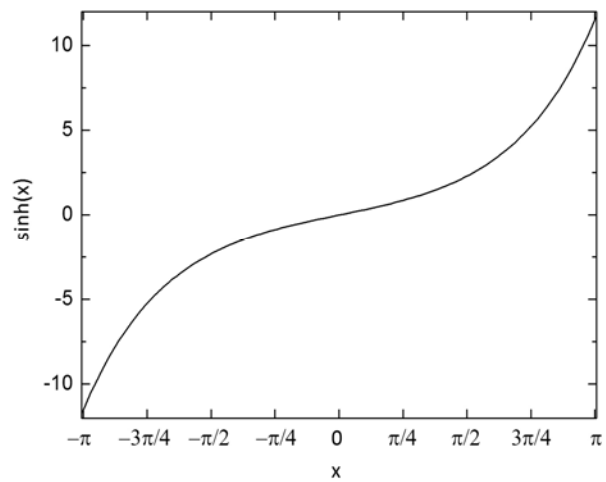
$$\sinh(x)=(e^x-e^{-x})/2$$

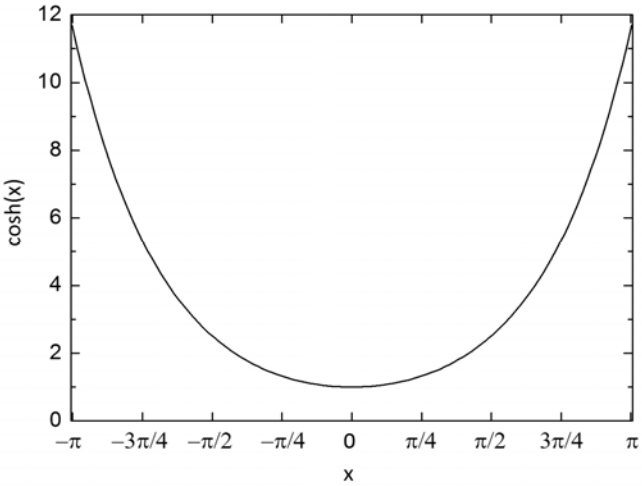
$$\cosh(x)=(e^x+e^{-x})/2$$

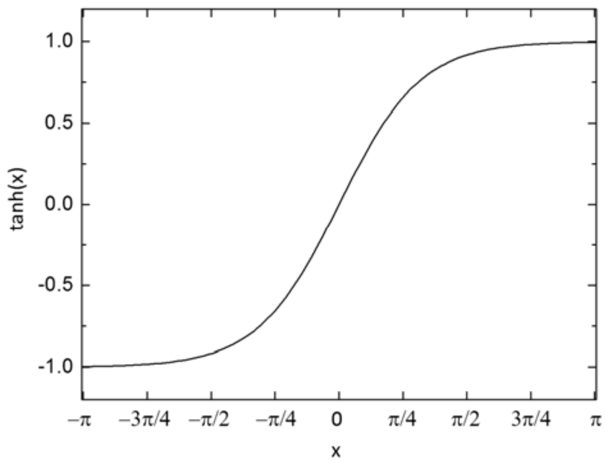
$$\tanh(x)=\sinh(x)/\cosh(x)$$

$$\cosh^2(x)-\sinh^2(x)=1$$

Sketch of Hyperbolic Functions（**雙曲函數示意圖**）：







Multiple Choice Questions

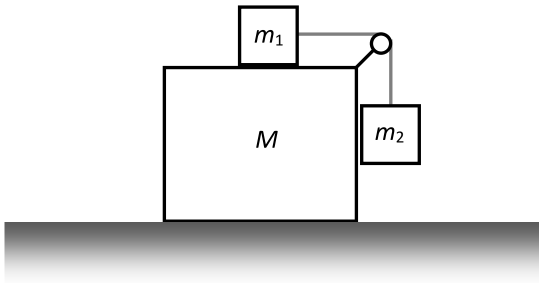
Select one answer in each question. For each question, 2 marks for correct answer, 0 mark for no answer, minus 0.25 mark for wrong answer, but the lowest mark of the multiple choice section is 0 mark.

選擇題

每題選擇一個答案，每題答對 2 分，不答 0 分，答錯扣 0.25 分，但全部選擇題最低為 0 分。

1. The following figure illustrates a pulley system with masses m_1 and m_2, and is supported by a block (mass M) on a horizontal ground. All frictions and inertia of the pulley are negligible, and the rope in the pulley system is massless. Find the horizontal force on block M such that there is no relative motion between m_1, m_2, and M.

   下圖示出了一個由質量 m_1 和 m_2 方塊所組成的滑輪系統。而這個滑輪系統是由另一個放在水平地面的方塊支撐。所有摩擦力和滑輪的轉動慣量可忽略不計，並在滑輪系統中的繩子是無質量。求作用在方塊 M 的水平力，使得 m_1，m_2，和 M 不存在相對運動。



   A. $\dfrac{m_1(M+m_1+m_2)g}{m_2}$

   B. $\dfrac{m_2(M+m_1+m_2)g}{m_1}$

   C. $\dfrac{m_1(M-m_1+m_2)g}{m_2}$

   D. $\dfrac{m_2(M-m_1+m_2)g}{m_1}$

   E. None of the above, because relative motion exists between m_1, m_2, and/or M.
   以上皆不是，因為相對運動存在於 m_1，m_2，與及/或 M 之間。

2. A man can generate a maximum propulsive power of 500 W during cycling. The air resistance has the force of $F = bv$ when the velocity of the bicycle is v, where b is a constant given by 5 N-s/m. Without wind, what is the maximum speed of the bicycle when cycling is conducted on a horizontal ground?

   一個人騎自行車時可以產生最大的推進力為 500 W。當自行車的速度是 v 時，空氣阻力是 $F = bv$，其中，常數 b 是 5 N-s/m。在沒有風的情況下，自行車在水平路面可達到的最高速度是甚麼？

   A. 1 ms^{-1}.
   B. 2 ms^{-1}.
   C. 5 ms^{-1}.
   D. 10 ms^{-1}.
   E. 20 ms^{-1}.

Questions 3 to 4. A spacecraft is orbiting in an elliptical orbit about the Sun with ~~perihelion~~ aphelion at the Earth's orbit, and ~~aphelion~~ perihelion at the Venus' orbit. The orbit equation with the Sun as the origin is given by $r = \lambda(1+\varepsilon)/(1+\varepsilon\cos\theta)$, where r is the distance in AU (Astronomical Unit) from the Sun, ε and λ are the orbital parameters. The mean distances of Venus and the Earth from the Sun are 0.73 AU and 1.0 AU, respectively.

選擇題 3 至 4. 一艘太空船正以橢圓形的軌道環繞太陽。該橢圓軌道之 ~~近日點~~ 遠日點 位於地球之軌道上，而 ~~遠日點~~ 近日點 位於金星之軌道上。以太陽為原點時該軌道的方程是 $r = \lambda(1+\varepsilon)/(1+\varepsilon\cos\theta)$，其中，$r$ 是從太陽起計之天文單位（AU）距離，ε 和 λ 則是軌道參數。金星和地球對太陽的平均距離分別是 0.73 AU 和 1.0 AU。

3. The orbital parameters ε and λ, respectively, are:

   所述的軌道參數 ε 和 λ 分別是：

   A. $\varepsilon = 0.156$, $\lambda = 0.73$ AU.
   B. $\varepsilon = 0.156$, $\lambda = 1.0$ AU.
   C. $\varepsilon = 0.312$, $\lambda = 0.73$ AU.
   D. $\varepsilon = 0.312$, $\lambda = 1.0$ AU.
   E. $\varepsilon = 0.312$, $\lambda = 1.73$ AU

4. What is the period of the spacecraft as mentioned above?

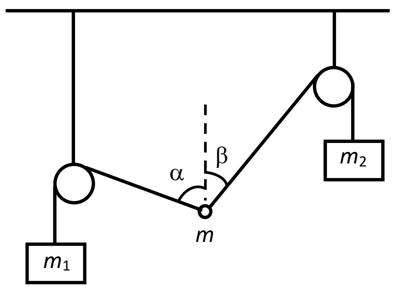
   太空船在上述軌道的週期是多少？

   A. 0.4 year (年) .
   B. 0.8 year (年) .
   C. 1.2 years (年) .
   D. 1.6 years (年) .
   E. 2.0 years (年) .

5. A point mass is hanged by 2 massless ropes, which make angles α and β to the vertical. The ropes are further connected to two masses m_1 and m_2 through frictionless and massless pulleys. Find the ratio m_1/m_2 such that the point mass hangs in equilibrium.

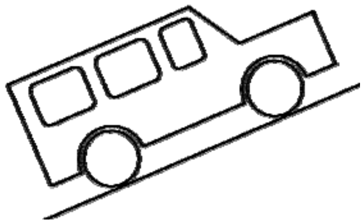
   一質點被 2 根無質量的繩子懸掛，並形成了以垂直計 α 和 β 的角度。繩子更進一步通過無摩擦且無質量的滑輪，並連接到 2 個質量為 m_1 和 m_2 的物體。在平衡狀態時，求 m_1/ m_2 之比值。

   A. $\sin(\alpha+\beta)$.
   B. $\cos(\alpha+\beta)$.
   C. $\sin(\alpha)\times\cos(\beta)$.
   D. $\sin(\alpha) / \cos(\beta)$.
   E. $\sin(\beta) / \sin(\alpha)$.



6. A 4-wheel car is moving uphill without slipping, as shown in the figure. If the car is driven by the rear wheels, in which direction(s) does friction(s) act on the rear and the front wheels?

如圖所示，一輛 4 輪汽車在不滑倒的情況下正向上行駛。假設汽車是由後輪驅動，問摩擦力作用在後輪和前輪是何方向？



A. Rear wheels: uphill, front wheels: uphill.
B. Rear wheels: downhill, front wheels: uphill.
C. Rear wheels: uphill, front wheels: downhill.
D. Rear wheels: downhill, front wheels: downhill.
E. Frictions do not act on the wheels because the car is moving at constant speed.

A. 後輪：向上斜坡，前輪：向上斜坡。
B. 後輪：向下斜坡，前輪：向上斜坡。
C. 後輪：向上斜坡，前輪：向下斜坡。
D. 後輪：向下斜坡，前輪：向下斜坡。
E. 汽車以恆定速度移動，因此摩擦力不會存在。

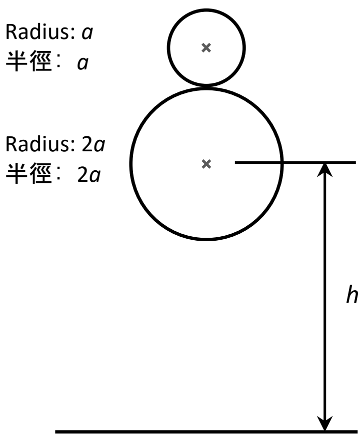
(Remark: Question 7 is cancelled)

7. Two rigid spheres (uniform masses, zero initial speed, radii a and $2a$) are dropped from a height h (measured from ground to center of the lower sphere) under the influence of gravity. What is the maximum height (measured from ground to sphere center) that the upper sphere will reach? Assume all the collisions are instantaneous.

Hints: i) Air resistance is negligible; ii) Collisions are elastic; iii) Centers of the spheres (marked as "x") always lie on the same vertical line; iv) Lower sphere collides with the ground before it collides with the upper sphere.

在重力作用的影響下，兩個剛體球（均勻質量，起始速度為零，半徑 a 和 $2a$）從高處 h（由地面至較低球中心計）掉下。從地面至球中心，較上球可達到的最高高度為多少？假設所有碰撞是瞬間完成。

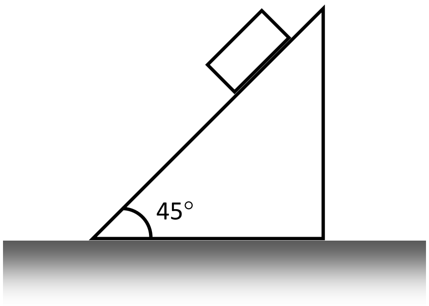
提示：i）空氣阻力可忽略不計；ii）碰撞是彈性的；iii）球體的中心處（以"x"為標記）總是在同一垂直線上；iv）較低球先碰撞地面，然後碰撞較上球。



A. $\frac{286}{81}(h-a)$
B. $\frac{286}{81}(h-2a)$
C. $\frac{529}{81}(h-a)$
D. $\frac{529}{81}(h-2a)$
E. $\frac{610}{81}(h-3a)$

8. A block (mass m) slides on an inclined plane (mass M, angle 45° to the horizontal). Ignoring all the frictions between all the surfaces, find the acceleration of the inclined plane when the block is sliding.

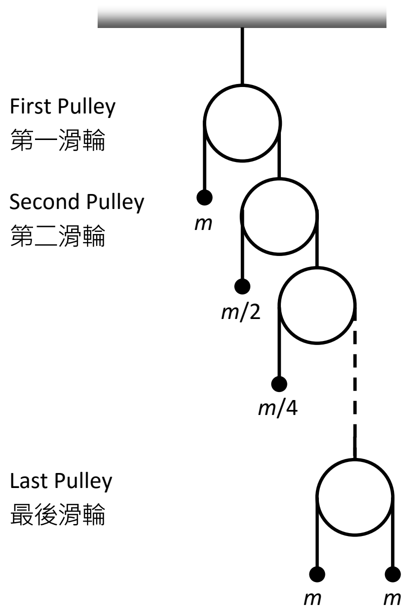
一方塊（質量 m）在一斜面（質量 M，從水平計角度為 45°）上滑行。在所有表面之間都沒有摩擦力的情況下，求方塊滑行時斜面的加速度。



A. $\left(\dfrac{m/2}{m+M}\right)g$.
B. $\left(\dfrac{m}{m+M}\right)g$.
C. $\left(\dfrac{2m}{m+M}\right)g$.
D. $\left(\dfrac{m}{2m+M}\right)g$.
E. $\left(\dfrac{m}{m+2M}\right)g$.

9. An Atwood machine consists of n masses (masses m, $m/2$, $m/4$, ……, respectively attached on the first, second, third, ……, pulley) and (n – 1) pulleys (massless, frictionless, and negligible inertia), where n is an integer with $n \geq 100$. At the last pulley, it is attached 2 equal masses (mass m). What is the acceleration of the mass attached to the first pulley?

一阿特伍德機由 n 個質量物（質量為 m，$m/2$，$m/4$，......，分別安裝在第一，第二，第三，......，滑輪上）和（n – 1）滑輪（無質量，無摩擦，及轉動慣量可忽略不計）所組成，其中 n 是整數且 $n \geq 100$。在最後的滑輪上，是與 2 個相等質量物（質量 m）連接在一起。附在第一滑輪的質量物之加速度為何值？

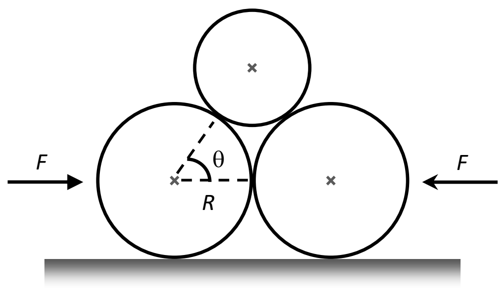


A. Zero acceleration.
B. g / 4.
C. g / 3.
D. g / 2.
E. Greater than or equal to 2^{100} g.

A. 零加速。
B. g / 4。
C. g / 3。
D. g / 2。
E. 大於或等於 2^{100} g。

10. Consider the following 3 planar circles with identical mass density per unit area of σ. One of the circles is put between and above the other 2 identical circles (radius R). The radii of contact points between the upper circle and the lower circles make an angle θ with the horizontal. Ignoring all the frictions, what is the horizontal force, F, on each side of the lower circle in order to keep them stationary?

以下是 3 個相同質量密度的平面圓，其每單位面積的質量密度是 σ。其中一個圓圈被置於另外 2 個相同的圓圈之上面（半徑 R）。上圓和下圓的半徑接觸點與水平線之間形成一個 θ 的角度。在沒有摩擦力的影響下，為了保持平面圓靜止不動，兩邊在下圓的水平力，F，應該是多少？



A. $\dfrac{\sigma\pi R^2}{2\sin\theta}\dfrac{(1-\sin\theta)^2}{\cos\theta}g$.

B. $\dfrac{\sigma\pi R^2}{2\sin\theta}\dfrac{(1-\cos\theta)^2}{\cos\theta}g$.

C. $\dfrac{\sigma\pi R^2}{2\cos\theta}\dfrac{(1-\sin\theta)^2}{\sin\theta}g$.

D. Revised to: $\dfrac{\sigma\pi R^2}{2\cos\theta}\dfrac{(1-\cos\theta)^2}{\tan\theta}g$

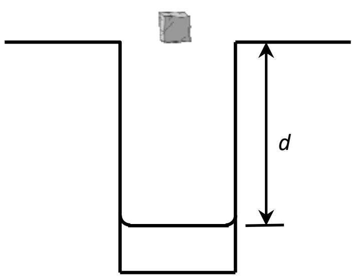
E. $\dfrac{\sigma\pi R^2}{2(\sin\theta+\cos\theta)}(1-\tan\theta)^2 g$.

(Remark: Question 11 is cancelled)

11. A stone is dropped from rest into a deep well. After three seconds, a sound is heard when the stone hits the well water. Assume that the speed of sound in air is 340 m/s, and air resistance is negligible; what is the approximate depth (from the top of the well to water surface), d, of the well?

一塊石頭從靜止狀態中被掉入一個深井。三秒之後，當石擊中井水時便聽到聲音。假定聲音在空氣中的傳播速度是 340 m/s，及空氣阻力可忽略不計；井的深度（從井的頂部至水的表面），d，大概是多少？

A. 15 m.
B. 20 m.
C. 25 m.
D. 30 m.
E. 35 m.



12. A sphere (mass m and radius r) is hanged vertically with a massless rope (length L). A wind is exerting constant horizontal force density σ (force per unit area), on the sphere. Without energy loss, what is the maximum height H that the sphere can rise?

一球體（質量為 m，半徑為 r）被一根無質量的繩子（長度 L）懸掛。一陣風在球體上施加了恆定且水平的力密度 σ（即每單位面積之力）。在沒有能量損耗的情況下，該球體可以上升的最高高度 H 是何值？

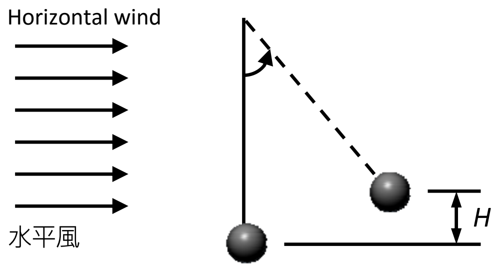
A. $\dfrac{2L\sigma^2\pi^2r^4}{m^2g^2+\sigma^2\pi^2r^4}$.

B. $\dfrac{32L\sigma^2\pi^2r^4}{9m^2g^2+16\sigma^2\pi^2r^4}$.

C. $\dfrac{2L\sigma\pi r^2}{9m^2g^2+16\sigma\pi r^2}$.

D. $\dfrac{32L\sigma\pi r^2}{9m^2g^2+16\sigma\pi r^2}$.

E. $\dfrac{2L^2\sigma^2\pi^2r^4}{m^2g^2+\sigma^2\pi^2r^4}$



13. A rope (mass m) is hanged between two supports (separation $2d$) at the same level. The shape of the hanging rope can be approximated to the function $y(x)=(1/\alpha)\cosh(\alpha x)$ in an x-y coordinate system, where α is a constant. The slope of the function is $\sinh(\alpha x)$. What is the magnitude of force F at each of the support?

一根繩子（質量 m）在兩個同一水平的支架點（距離 $2d$）之間懸掛。該懸掛繩子的形狀可以被近似地描述為以 x-y 坐標系統中的函數 $y(x)=(1/\alpha)\cosh(\alpha x)$，其中，$\alpha$ 是一個常數。該函數的斜率是 $\sinh(\alpha x)$。問在每一個支架點的力 F 是何值？

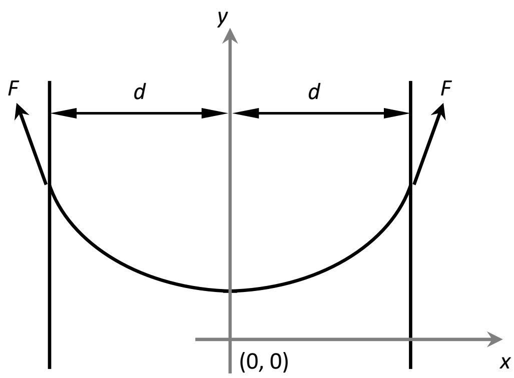
A. $\dfrac{mg}{2\sinh(\alpha d)}$.

B. $\dfrac{mg}{2\cosh(\alpha d)}$.

C. $\dfrac{mg}{2\tanh(\alpha d)}$.

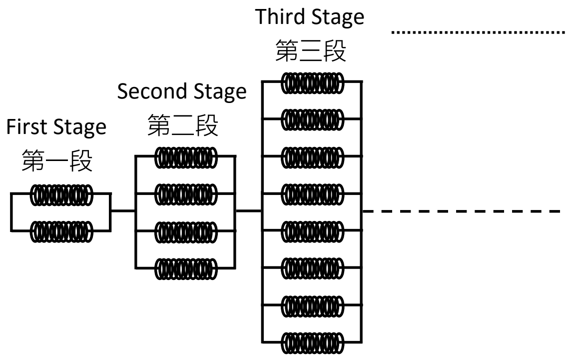
D. $\dfrac{mg\sinh(\alpha d)}{2}$.

E. $\dfrac{mg\cosh(\alpha d)}{2}$.



Questions 14 to 15. A spring system consists of *n* stages, where *n* tends to infinity. The first stage contains 2 springs, and the number of springs in the next stages are a geometric sequence, that is, the second stage contains 4 springs, the third stage contains 8 springs, and so on. All the springs are massless and identical having the same spring constant of *k*.

選擇題 14 至 15. 一個彈簧系統由 *n* 段彈簧所組成，其中 *n* 趨於無窮。第一段包含 2 個彈簧，而在下一段的彈簧數量是以幾何序列，亦即第二段有 4 個彈簧，第三段有 8 個彈簧，等等。所有彈簧均是無質量的，並且具有相同的彈簧常數 *k*。



14. The equivalent spring constant of this spring system is:

    所述的彈簧系統之當量彈簧常數是：

    A. *k*.
    B. 2*k*.
    C. 4*k*.
    D. 8*k*.
    E. 16*k*.

15. If the total elongation of the spring system is 10 cm, what is the elongation in the fourth stage, that is, the stage that contains 16 springs?

    如果彈簧系統的總伸長為 10 cm，問第四段，亦即該段有 16 個彈簧的伸長是何值？

    A. 7.5 cm.
    B. 5.0 cm.
    C. 2.5 cm.
    D. 1.25 cm.
    E. 0.625 cm.

16. Two identical balls are projected up an incline (angle ϕ) with initial speed v_1 at angle θ_1 and v_2 at angle θ_2 with respect to the horizontal, where θ_1 and $\theta_2 > \phi$. The two balls eventually reach the same distance on the incline. The relationship between θ_1, θ_2, v_1, and v_2 is:

兩個相同的球在一個斜坡（角度 ϕ）上被投射。它們被投射的初始條件是當角度為 θ_1 時，速度是 v_1，而當角度為 θ_2 時，速度是 v_2，其中，θ_1 和 $\theta_2 > \phi$。角度都是以水平線起計。這兩個球最終被投射到在斜坡上的相同距離。θ_1，θ_2，v_1，和 v_2 之間的關係式是：

A. $\left(\frac{v_1}{v_2}\right)^2 = \frac{\cos\theta_2 \sin(\theta_2 - \phi)}{\cos\theta_1 \sin(\theta_1 - \phi)}$.
B. $\left(\frac{v_1}{v_2}\right)^2 = \frac{\cos\theta_1 \sin(\theta_1 - \phi)}{\cos\theta_2 \sin(\theta_2 - \phi)}$.
C. $\frac{v_1}{v_2} = \frac{\cos\theta_2 \sin(\theta_2 - \phi)}{\cos\theta_1 \sin(\theta_1 - \phi)}$.
D. $\frac{v_1}{v_2} = \frac{\cos\theta_1 \sin(\theta_1 - \phi)}{\cos\theta_2 \sin(\theta_2 - \phi)}$.
E. $\frac{v_1}{v_2} = \cos\theta_1 \cos\theta_2 \sin(\theta_1 - \phi)\sin(\theta_2 - \phi)$.

17. A helium-filled balloon is held on a horizontal floor of a MTR train by a massless string (length 0.5 m). The train is travelling at constant speed of 105 km/hour and turning a circular path of radius 1000 m. The densities of air and helium gases are 1.2 kg/m^3 and 0.18 kg/m^3, respectively. The angle of the balloon string making with the vertical is:

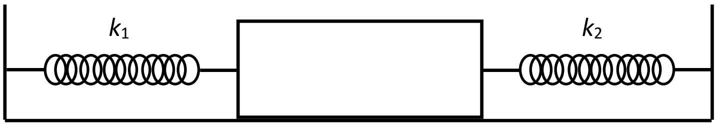
一個氦氣球被一根無質量的繩子（長度 0.5 m）綁緊在地鐵列車的水平地板上。列車以每小時 105 公里的恆定速度行駛在一條半徑為 1000 m 的圓形軌道。空氣和氦氣的密度分別是 1.2 kg/m^3 和 0.18 kg/m^3。氣球繩子與垂直線的角度是：

A. 1°.
B. 5°.
C. 10°.
D. 15°.
E. 18°.

18. A wooden block (mass 1 kg) is connected to 2 massless springs (Spring 1 with spring constant k_1 = 5 N/m; and Spring 2 with spring constant k_2 = 4 N/m), and is oscillating on a frictionless horizontal table with small amplitude. At the equilibrium position (time t = 0 second), the block is moving at 2 m/s towards the Spring 2. The time that the block will take when the Spring 1 has maximum compression is:

一木塊（質量 1 kg）被連接到 2 條無質量的彈簧（彈簧 1 具有彈簧常數 k_1 = 5 N/m; 而彈簧 2 具有彈簧常數 k_2 = 4 N/m），並以小幅度的振幅在一水平及無摩擦力的桌子上擺動。處於平衡位置時（時間 t = 0 秒），該木塊以 2 m/s 移向彈簧 2。當彈簧 1 處於最大壓縮時，木塊所需要的時間是：

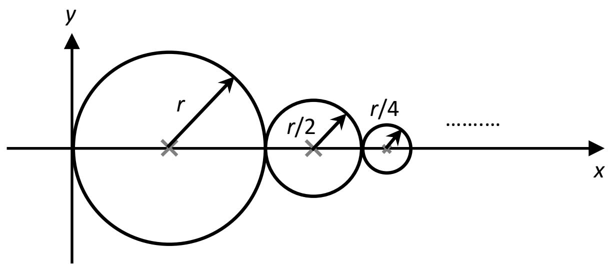
A. π seconds （ 秒 ）.
B. 3π/4 seconds （ 秒 ）.
C. π/2 seconds （ 秒 ）.
D. π/4 second （ 秒 ）.
E. π/8 second （ 秒 ）.



19. Find the center of gravity of the integrated shape in the *x*-*y* coordinate system. The integrated shape is an infinite series of circular discs with radii of *r*, *r*/2, *r*/4, *r*/8, ………., and their centers are on the *x*-axis. The masses of the circular discs are proportional to their areas.

尋找集成形狀在 *x*-*y* 坐標系統中的重心。該集成形狀是由無窮級數的圓盤所組成，半徑為 *r*， *r*/2， *r*/4， *r*/8， ……….，並且它們的中心位於在 *x* 軸上。圓盤的質量與其面積成正比。

Hints 提示： $\sum_{n=1}^{\infty} 4^{-n} = 1/3$；$\sum_{n=1}^{\infty} 8^{1-n}\left(2^{n+1}-3\right) = 40/21$.

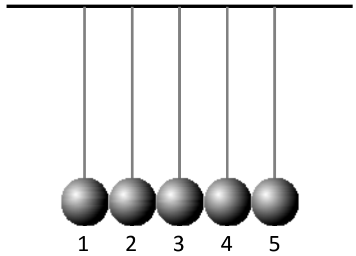


A. $x = (40/7)\,r$, $y = 0$.
B. $x = (80/21)\,r$, $y = 0$.
C. $x = (40/21)\,r$, $y = 0$.
D. $x = (10/7)\,r$, $y = 0$.
E. $x = (40/63)\,r$, $y = 0$.

20. A Newton kinetic ball stress reliever gadget contains 5 identical hard balls supported by identical strings (negligible mass) of equal lengths. From left to right, the hard balls are labelled as ball 1, ball 2, ball 3, ball 4 and ball 5. Balls 4 and 5 are glued together (glue is massless). When Ball 1 is slightly pulled out and released, velocity of Ball 1 just before collision with Ball 2 is $+v$ (positive velocity represents moving to the right). Assuming all the collisions are elastic, the velocities of Ball 1 (v_{1f}), and Balls 4, 5 (v_{45f}) after collision are:

牛頓動力球應力消除小擺設由 5 個相同的硬球組成，硬球被 5 根相同且長度相等的繩子（質量可忽略不計）懸掛。從左到右，硬球被標記為球 1，球 2，球 3，球 4，和球 5。球 4 和 5 被緊緊地膠粘在一起（膠是無質量的）。當球 1 被稍微拉出並釋放，球 1 剛碰撞到球 2 前的速度是 $+v$（正速度表示向右移動）。假設所有碰撞都是彈性碰撞，球 1（v_{1f}），和球 4，5（v_{45f}）在碰撞後的速度是：

A. $v_{1f} = 0$；$v_{45f} = +(1/2)v$.
B. $v_{1f} = -v$；$v_{45f} = 0$.
C. $v_{1f} = +v$；$v_{45f} = 0$.
D. $v_{1f} = +(1/3)v$；$v_{45f} = -(2/3)v$.
E. $v_{1f} = -(1/3)v$；$v_{45f} = +(2/3)v$.



--- END OF MULTIPLE CHOICE SESSION 選擇題完 ---

Open Problems 開放題

Total 5 Problems 共 5 題

1. Faster track (9 Marks)

A school student designed a frictionless track to allow a ball (negligible radius) to move on a track, which consisted of a downward slope and a horizontal path (Fig. 1, Track 1). At time t = 0 second, a ball is released from rest to move down the slope (angle θ_1, height h). After the slope, it is a horizontal path (length d). The time duration to complete this track (Track 1) is t_{T1}. However, the student did not satisfy the time duration. He/she would like to design a faster track with shorter completion time.

Another faster track (Fig. 1, Track 2) is designed, which includes 2 downward slopes (angles θ_1 and θ_2, height h), 1 upward slope (angle θ_2, height h), and 3 horizontal paths. The length d_1, d_2, d_3, d_4, d_5, and d_6 are the horizontal lengths.

You may assume that the ball speeds on the 2 tracks are low enough such that the balls are always moving on the tracks without projectile motion.

(a) Express d in terms of h, θ_2, d_2, d_4, and d_6.

(b) Find the time durations to complete Track 1 (t_{T1}) and Track 2 (t_{T2}). Express your answers in terms of θ_1, θ_2, g (acceleration due to gravity), d, h, d_2, d_4, and d_6.

(c) Prove that Track 2 is always faster than Track 1, that is, $t_{T1} > t_{T2}$. Find the time difference (Δt) between the 2 tracks. Express your answer in terms of g, h, d_4, and θ_2.

更快的賽道（9 分）

一名學生設計了一條無摩擦力的軌道好讓一珠子（半徑很小可忽略不計）在軌道上移動。此軌道是由一塊向下斜面和一個水平路徑所組成（圖 1，軌道 1）。在時間 t = 0 秒時，該珠子從靜止位置釋放，並沿斜面（角度 θ_1，高度 h）向下。在斜面之後是一條水平路徑（長度 d）。完成此軌道（軌道 1）所需的時間是 t_{T1}。然而，該學生並不滿足此時間。他/她想設計另一條更快且時間更短的軌道。

另一條更快的軌道（圖 1，軌道 2）已經設計好了。新的軌道包括 2 塊向下斜面（角度 θ_1 和 θ_2，高度 h），1 塊向上斜面（角度 θ_2，高度 h），與及 3 條水平路徑。長度 d_1，d_2，d_3，d_4，d_5 和 d_6 則是水平之長度。

你可假設珠子在軌道上移動時的速度比較慢，使得珠子總是在軌道上移動，並無拋物運動。

(a) 以 h，θ_2，d_2，d_4，和 d_6 作表達，計算 d。

(b) 以 θ_1，θ_2，g（重力加速度），d，h，d_2，d_4，和 d_6 作表達，計算完成軌道 1（t_{T1}）和軌道 2（t_{T2}）所需的時間。

(c) 證明軌道 2 總是比軌道 1 快。以 g，h，d_4，和 θ_2 作表達，計算 2 條軌道的時間差異。

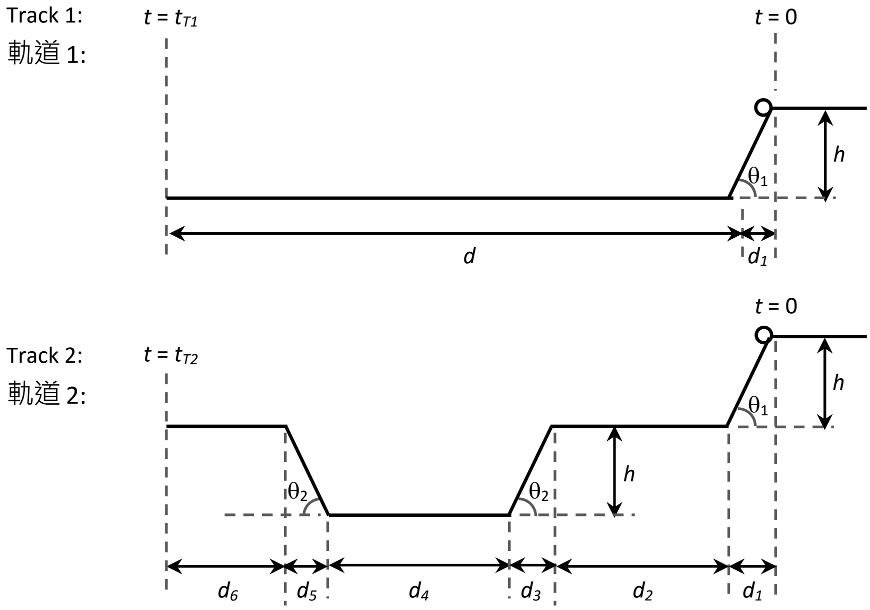


Fig 1. Faster track.
圖 1. 更快的賽道.

2. Rod in semispherical bowl (9 Marks)

A uniform rod (mass m, length L) is placed partially within and partially outside a fixed hemispherical bowl (radius R, where $L > R$). The bowl rests on a horizontal table. When the rod is in equilibrium, the rod makes an angle θ with the horizontal. AB is the length of the rod partially within the bowl. Ignore all the frictions.

(a) Derive the angle θ and the length AB. Express your answers in terms of L and R.

(b) Give the numerical value θ for the special case of $L = 4R$.

半球形碗上的棒（9 分）

一均勻質量的棒（質量 m，長度 L），部分被放置於半球形碗的內部，而部分則處於碗（半徑 R，其中 $L > R$）的外部。該碗被放置在水平的桌子上。當棒子處於平衡狀態時，棒子形成了以水平計 θ 的角度。AB 是棒子在碗內之長度。所有摩擦力可忽略不計。

（a）以 L 和 R 作表達，推導角度 θ 和 AB 的長度。

（b）對於 $L = 4R$ 的特殊情況下，計算 θ 的數值。

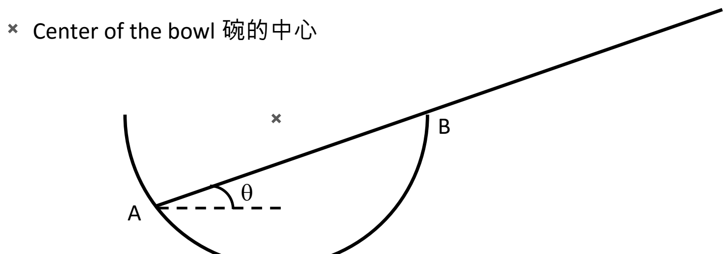


Fig 2. Rod in semispherical bowl.
圖 2. 半球形碗上的棒.

3. Stacked spheres (24 Marks)

Two spheres are stacked such that an upper sphere (radius r, mass m) is put on the top of a stationary lower sphere (radius R; where $R > r$). Eventually, under the influence of gravity, the upper sphere rolls without sliding on the surface of the lower stationary sphere. α ($0° \le \alpha \le 90°$) is the angle between the centers of the upper sphere and the lower sphere, whereas β is the angle that the upper sphere rotates. Initially, $\alpha = 0°$ and $\beta = 0°$.

(a) Find the angular velocities of α and β. Also obtain the velocity of the upper sphere as a function of α. Express your answers in terms of r, R, g (acceleration due to gravity), and α.

(b) Find the angle at which the upper sphere will detach from the lower sphere.

(c) Suppose that coefficient of static friction between the two spheres is μ. Find the minimum value of α that the upper sphere starts to slide. Express your answer in terms of μ.

(d) By considering the coefficient of static friction, find the minimum and maximum values of α.

Hints: i) Kinetic energy of the upper sphere at an angular velocity $\dot{\beta}$ is given by [Revised to: $\frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\dot{\beta}\right)^2$].

ii) In case of existence of friction, the frictional force, f, at an angle α is: $f = \frac{2}{7}mg\sin\alpha$.

疊球（24 分）

兩個球體堆疊在一起。上層球體（半徑 r，質量 m）被放置在靜止的下層球體（半徑 R，其中 $R > r$）的頂部。最終，在重力的影響下，上層球體在不滑動的情況下，沿著靜止的下層球體表面滾下。α（$0° \le \alpha \le 90°$）是上層球體和下層球體的中心角度，而 β 是上層球體旋轉角度。最初，$\alpha = 0°$ 和 $\beta = 0°$。

（a）計算 α 和 β 的角速度。以 α 作為函數，計算上層球體的速度。答案請以 r，R，g（重力加速度），和 α 作表達。

（b）計算上層球體與下層球體分離時的角度。

（c）假設兩個球之間的靜摩擦係數為 μ，計算上層球體開始滑動時的最小 α 值。

（d）通過考慮靜摩擦係數，計算 α 的最小值和最大值。

提示： i）上層球體速度為 $\dot{\beta}$ 時，其動能是 [Revised to: $\frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\dot{\beta}\right)^2$]。

ii）在摩擦存在的情況下，其摩擦力，f，於角度 α 時是 $f = \frac{2}{7}mg\sin\alpha$。

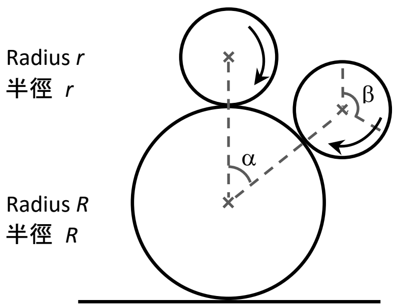


Fig 3. Stacked sphere.
圖 3. 疊球.

4. A '8' shape oscillator (11 Marks)

Two uniform, thin, and hollow cylinders (radii R_1 and R_2, masses m_1 and m_2, respectively) are stacked to form an "8" shape such that their diameters are aligned in a straight line. The stacked cylinders is free to oscillate about its axis of oscillation (Fig. 4). The kinetic energy of the stacked cylinders is $KE = \frac{1}{2}\left(2m_1R_1^{\,2} + 2m_2R_2^{\,2}\right)\omega^2$, where ω is the instantaneous angular velocity about the oscillation axis. A small angle θ is made to displace the stacked cylinders from the equilibrium position. Figure 4 shows the equilibrium position of the stacked cylinders. At equilibrium position, $\theta = 0°$.

(a) Find the potential energy of the stacked cylinders in terms of m_1, m_2, R_1, R_2, g (acceleration due to gravity) and θ. Take zero potential energy at the level of oscillation axis.

(b) Find the total energy of the oscillator of the stacked cylinders. Express your answer in the form of energy transformation in a simple harmonic mass-spring system, that is, Total Energy $E = \frac{1}{2}mv_x^{\,2} + \frac{1}{2}kx^2$, where you may consider x as an angular displacement.

(c) Determine the effective mass (m_{eff}), effective spring constant (k_{eff}), and period of the oscillation.

(d) According to the above results, estimate the oscillation period of a single hollow cylinder about the same oscillation axis, that is, without the upper hollow cylinder.

一個 "8" 字形的振盪（11 分）

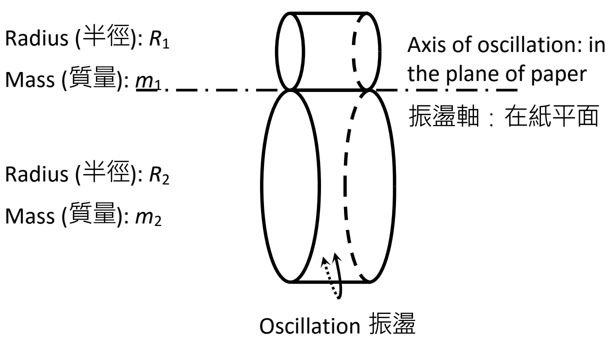
兩個均勻、薄、及中空的圓柱體（半徑 R_1 和 R_2，質量分別為 m_1 和 m_2）被堆疊成一個 "8" 字的形狀，而它們的直徑都在同一條直線上對齊。該堆疊圓柱體可以在其軸線上自由擺動（圖 4）。該堆疊圓柱體的動能是 $KE = \frac{1}{2}\left(2m_1R_1^{\,2} + 2m_2R_2^{\,2}\right)\omega^2$，而 ω 是軸線上擺動的瞬時角速度。該堆疊圓柱體被弄至偏離其平衡點計的細小角度。圖 4 示出了堆疊圓柱的平衡位置。在平衡位置時，$\theta = 0°$。

（a） 計算堆疊圓柱體的勢能。答案請以 m_1，m_2，R_1，R_2，g（重力加速度）和 θ 作表達。設勢能在振盪軸的水平位置為零。

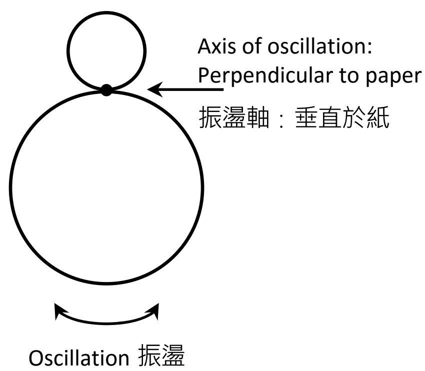
（b） 計算堆疊圓柱體擺動系統的總能量。答案請以質量與彈簧的簡諧運動能量轉換形式表達，即是總能量 $E = \frac{1}{2}mv_x^{\,2} + \frac{1}{2}kx^2$，其中你可考慮 x 為角位移。

（c） 計算該系統的有效質量（m_{eff}），有效彈簧常數（k_{eff}），與及擺動週期。

（d） 根據以上的結果，計算單一中空圓柱體在同一擺動軸的擺動週期，亦即沒有上部中空圓柱體時的擺動週期。



Three dimensional view
三維視圖



Two dimensional view
二維視圖

Fig 4. A '8' shape oscillator.
圖 4. 一個 "8" 字形的振盪.

5. Energy to break up the Sun completely (7 Marks)

By considering the gravitational potential energy to separate all the solar mass to infinity, estimate the energy required to break up the Sun completely. Express your answer in terms of M_S (mass of the Sun), R_S (radius of the Sun), and G (Gravitational Constant). You may assume i) the Sun is spherical, and ii) the Sun density is uniform.

Hint: Other than solving integral, the following summation formula may help:

$$\sum_{k=1}^{m} k^4 = \frac{m(2m+1)(m+1)(3m^2+3m-1)}{30}$$, where *k* and *m* are integers.

完全打散太陽所需的能量（7 分）

通過考慮引力勢能來分隔所有太陽的質量至無窮遠，估計要完全打破太陽所需的能量。答案請以 M_S（太陽質量），R_S（太陽半徑），和 G（萬有引力常數）作表達。你可假設 i）太陽是球形，和 ii）太陽密度是均勻。

提示：求積分以外，下列總和公式或可幫助計算：

$$\sum_{k=1}^{m} k^4 = \frac{m(2m+1)(m+1)(3m^2+3m-1)}{30}$$，其中，*k* 和 *m* 為整數。

--- END OF EXAM PAPER 全卷完 ---

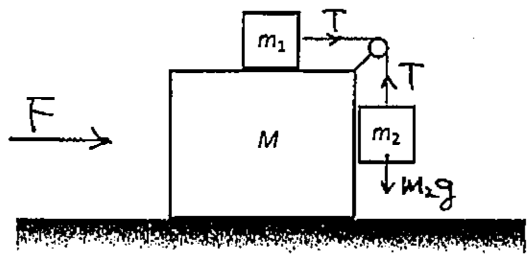
MC Key

1. B
2. D
3. A
4. B
5. E
6. C
7. ~~D~~ (Cancelled)
8. E
9. D
10. B
11. ~~C~~ (Cancelled)
12. A
13. C
14. A
15. E
16. A
17. B
18. C
19. D
20. E

1.

Without relative motion,

$\Rightarrow$ m_1, m_2, and M have the same acceleration



Along x-axis:

$$(M + m_1 + m_2)\ddot{x} = F \quad — \quad ①$$

$$m_1 \ddot{x} = T \quad — \quad ②$$

Along y-axis: $T = m_2 g$ — ③

①, ② and ③ $\Rightarrow$ $(M + m_1 + m_2)\dfrac{m_2 g}{m_1} = F$

2.

$$\begin{cases} F = bv \\ Fv = 500\,W \end{cases}$$

$$\begin{cases} F = (5\ Ns/m)(v) & \text{———} \quad 1 \\ Fv = 500\,W & \text{———} \quad 2 \end{cases}$$

$$\frac{2}{1} \Rightarrow \qquad v = \frac{500W}{5\ Ns/m\,(v)}$$

$$v^2 = \frac{500\ N\,ms^{-1}}{5\ N\,s/m}$$

$$v = 10\ ms^{-1}$$

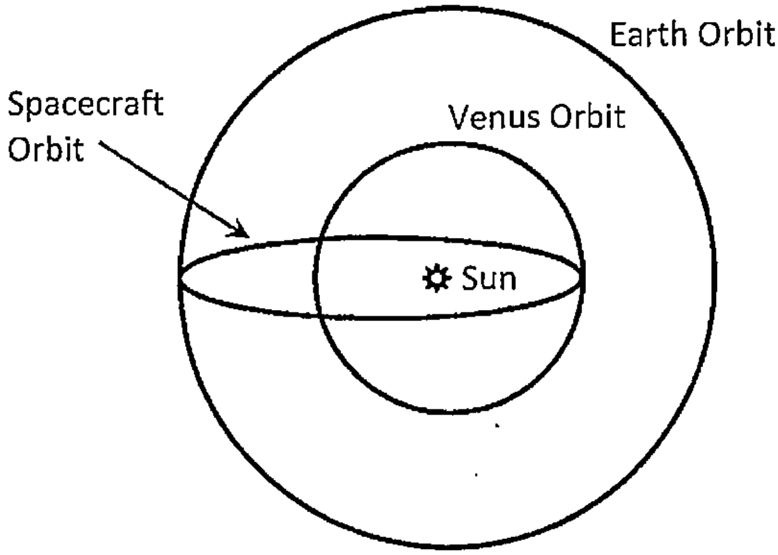
3.

$$\begin{cases} 0.73\,AU = \dfrac{\lambda(1+\varepsilon)}{1+\varepsilon} = \lambda \\ 1\,AU = \dfrac{\lambda(1+\varepsilon)}{1-\varepsilon} \end{cases}$$

$$1\,AU = \frac{0.73\,AU\,(1+\varepsilon)}{1-\varepsilon}$$

$$0.27\,AU = 1.73\,AU \times \varepsilon$$

$$\Rightarrow \varepsilon = 0.156$$



4.

$$\frac{T_{Earth}^2}{1\,AU^3} = \frac{T_{spacecraft}^2}{\left(\frac{1\,AU+0.73\,AU}{2}\right)^3}$$

$$\Rightarrow T = 0.8 \text{ year}$$

~~⇒ Time Duration = 0.4 year~~

5.

$$\begin{cases} T_1 \cos\alpha + T_2 \cos\beta - mg = 0 \\ T_1 \sin\alpha - T_2 \sin\beta = 0 \end{cases} \Rightarrow T_1 = T_2 \frac{\sin\beta}{\sin\alpha}$$

$$T_2 \frac{\sin\beta}{\sin\alpha} \cos\alpha + T_2 \cos\beta - mg = 0$$

$$T_2 (\sin\beta \cos\alpha + \sin\alpha \cos\beta) = mg \sin\alpha$$

$$T_2 \sin(\alpha + \beta) = mg \sin\alpha$$

$$T_2 = \frac{mg \sin\alpha}{\sin(\alpha+\beta)}$$

$$T_1 = \frac{mg \sin\alpha}{\sin(\alpha+\beta)} \frac{\sin\beta}{\sin\alpha}$$

$$= mg \frac{\sin\beta}{\sin(\alpha+\beta)}$$

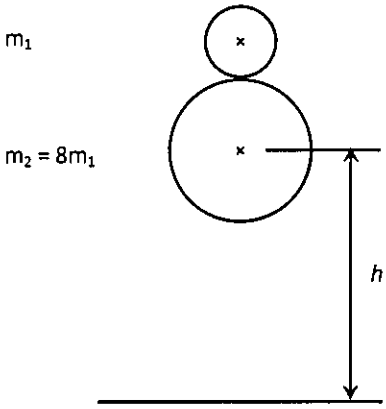
$$\begin{cases} T_1 = m_1 g \\ T_2 = m_2 g \end{cases}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{T_1}{T_2} = \frac{mg \sin\beta / \sin(\alpha+\beta)}{mg \sin\alpha / \sin(\alpha+\beta)}$$

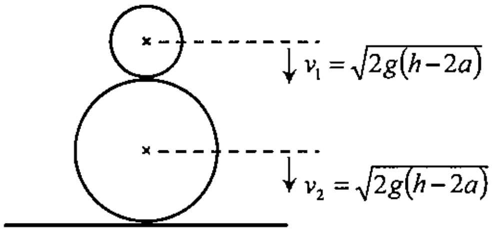
$$= \frac{\sin\beta}{\sin\alpha}$$

7.

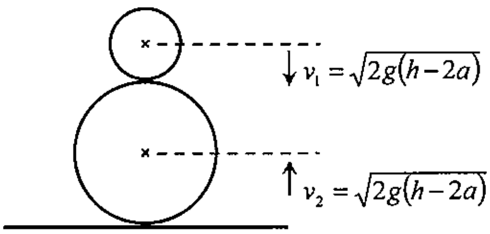
(i) Initial condition



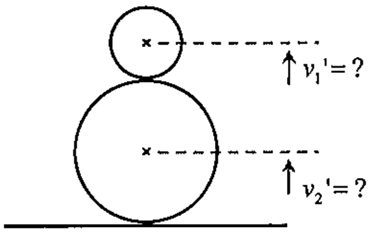
(ii) Just before collision with the ground



(iii) After lower sphere collision from the ground



(iv) Just after elastic collision between spheres



7.

Momentum: $m_2V_2 - m_1V_1 = m_2V_2' + m_1V_1'$ ↑ +ve

$8m_1V_2 - m_1V_1 = 8m_1V_2' + m_1V_1'$

$8V_2 - V_1 = 8V_2' + V_1'$

$V_1' = 8V_2 - V_1 - 8V_2'$ ↓ −ve

$= 7V_1 - 8V_2'$

Energy: $\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{1}{2}m_2V_2'^2 + \frac{1}{2}m_1V_1'^2$

$m_1V_1^2 + 8m_1V_2^2 = 8m_1V_2'^2 + m_1V_1'^2$

$V_1^2 + 8V_2^2 = 8V_2'^2 + V_1'^2$

$9V_1^2 = 8V_2'^2 + (7V_1 - 8V_2')^2$

$72V_2'^2 - 112V_1V_2' + 40V_1^2 = 0$

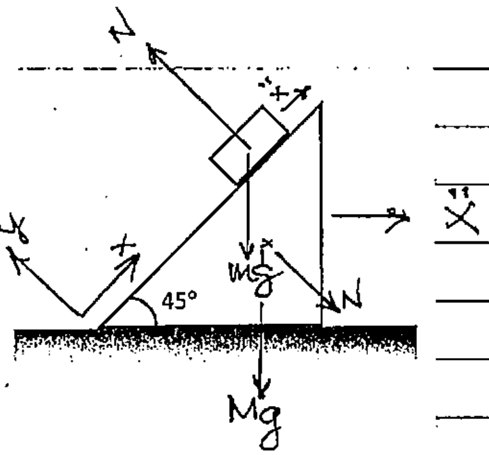
$\Rightarrow \quad V_2' = \frac{5}{9}V_1 = \frac{5}{9}\sqrt{2g(h-2a)}$ or $V_2' = V_1$ (rejected)

$V_1' = \frac{23}{9}V_1 = \frac{23}{9}\sqrt{2g(h-2a)}$

Maximum Height $= 5a + \frac{V_1'^2}{2g} = 5a + \frac{529}{81}(h-2a)$

8.

$$\begin{cases} M\ddot{x} = N\sin\theta & \text{— (1)} \\ m(\ddot{x} + \ddot{X}\cos\theta) = -mg\sin\theta & \text{— (2)} \\ -m\ddot{X}\sin\theta = N - mg\cos\theta & \text{— (3)} \end{cases}$$



$$(3) \Rightarrow \quad -m\ddot{X}\frac{\sqrt{2}}{2} = N - mg\frac{\sqrt{2}}{2}$$

$$N = mg\frac{\sqrt{2}}{2} - m\ddot{X}\frac{\sqrt{2}}{2}$$

Sub. into (1),

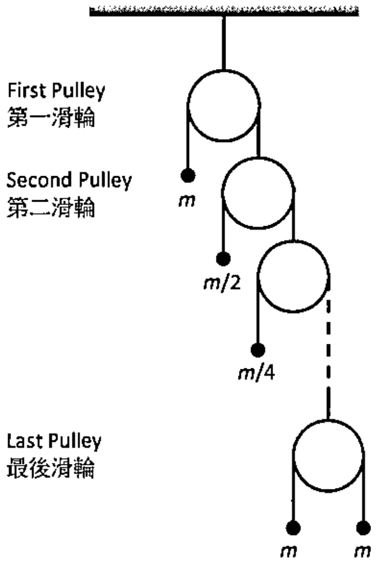
$$M\ddot{X} = N\frac{\sqrt{2}}{2}$$

$$= \left(mg\frac{\sqrt{2}}{2} - m\ddot{X}\frac{\sqrt{2}}{2}\right)\frac{\sqrt{2}}{2}$$

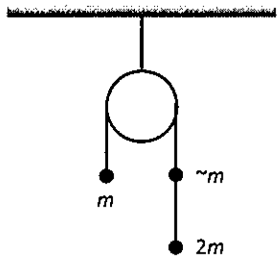
$$= \frac{mg}{2} - \frac{m\ddot{X}}{2}$$

$$\Rightarrow \ddot{X} = \frac{mg}{m + 2M}$$

9.



$$\frac{m}{2}+\frac{m}{4}+\frac{m}{8}\cdots\cdots\approx m$$

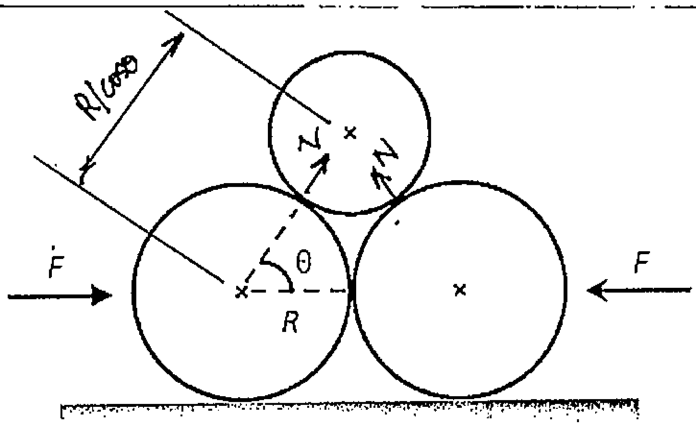


$$a=\left(\frac{3m-m}{4m}\right)g$$
$$=\frac{g}{2}$$

10.

Radius of the upper circle $= \frac{R}{\cos\theta} - R$

$= R\left(\frac{1}{\cos\theta} - 1\right)$



Mass of the upper circle

$= \sigma\pi R^2\left(\frac{1}{\cos\theta} - 1\right)^2$

Keeping stationary :

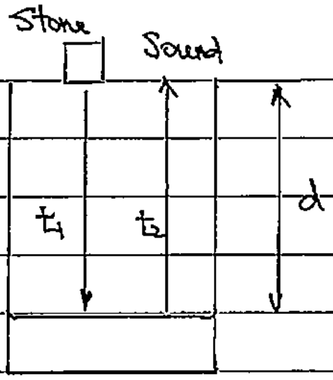
$$2N\sin\theta = \sigma\pi R^2\left(\frac{1}{\cos\theta} - 1\right)^2 g \cancel{\cos\theta}$$

$$N = \frac{\sigma\pi R^2}{2\sin\theta}\left(\frac{1}{\cos\theta} - 1\right)^2 g \cancel{\cos\theta}$$

~~F = N sin~~

$$F = N\cos\theta = \frac{\sigma\pi R^2}{2\sin\theta}\,\frac{(1-\cos\theta)^2}{\cos\text{(theta)}}\,g$$

11.



$d = \frac{1}{2}(9.8)t_1^2 \Rightarrow d = 4.9\, t_1^2$ — ①

$t_1 + t_2 = 3s \Rightarrow t_2 = 3 - t_1$ — ②

$d = 340\, t_2$ — ③

Sub. ② into ③,

$d = 340(3 - t_1)$

$= 1020 - 340\, t_1$

$= 1020 - 340\sqrt{\frac{d}{4.9}}$ (from ①)

Rearranging :

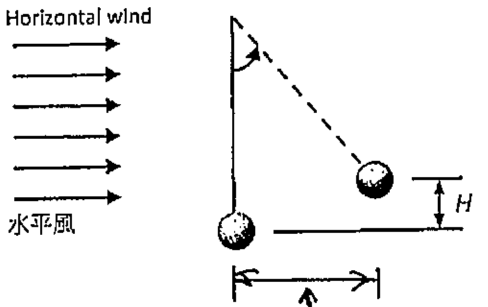
$\cancel{d^2 - 25632d + 1020^2 = 0}$

$\cancel{d = -40.5\,m \text{ or } d = 25.7\,m}$

$\cancel{\Rightarrow d = 25.7\,m}$

$d = 40.65\,m$

12.



$$\sqrt{L^2-(L-H)^2} = \sqrt{2LH - H^2}$$

$$F = \sigma \times \pi r^2$$

$$\text{Work Done by Wind} = \int F \cdot ds$$

$$= F \times \sqrt{2LH - H^2}$$

$$= \sigma \pi r^2 \sqrt{2LH - H^2}$$

Energy Conservation:

$$\sigma \pi r^2 \sqrt{2LH - H^2} = mgH$$

$$\sigma^2 \pi^2 r^4 (2LH - H^2) = m^2 g^2 H^2$$

$$(m^2 g^2 + \sigma^2 \pi^2 r^4) H = 2L\sigma^2 \pi^2 r^4$$

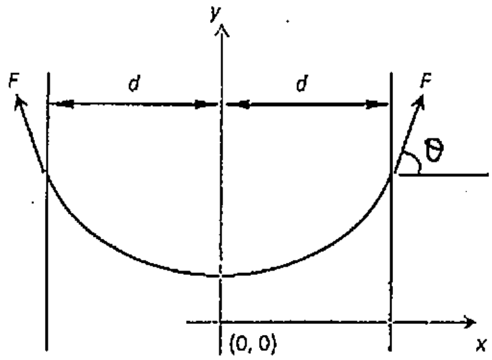
$$H = \frac{2L\sigma^2\pi^2 r^4}{m^2 g^2 + \sigma^2 \pi^2 r^4}$$

13.

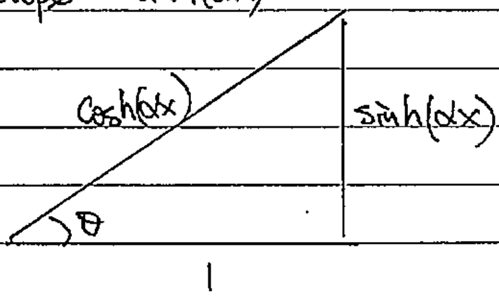
$$2F\sin\theta = mg$$

$$2F\tanh(\alpha d) = mg$$

$$F = \frac{mg}{2\tanh(\alpha d)}$$

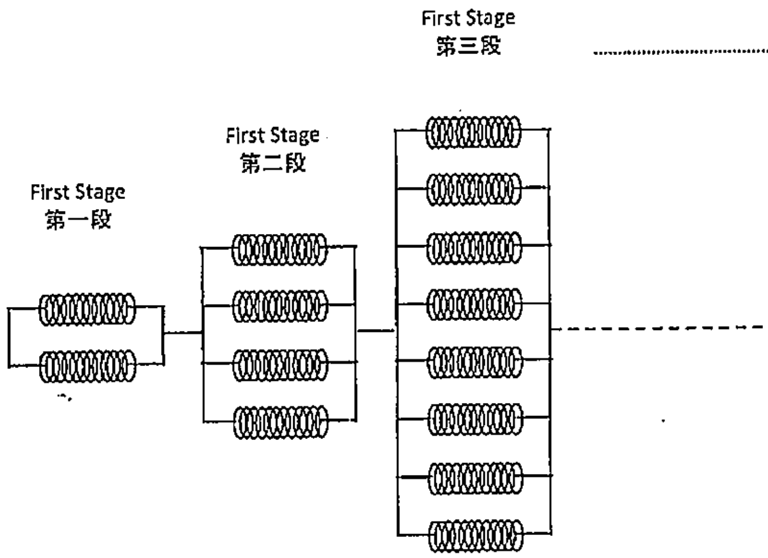


Slope = $\sinh(\alpha x)$



$$\Rightarrow \quad \sin\theta = \frac{\sinh(\alpha x)}{\cosh(\alpha x)}$$

$$= \tanh(\alpha x)$$



Spring Constant	$2k$	$4k$	$8k$	......
Elongation	x	$\frac{x}{2}$	$\frac{x}{4}$	......

14.

$$\frac{1}{k_{eq}} = \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \cdots\cdots$$

$$= \frac{1}{k}\left(\frac{\frac{1}{2}}{1-\frac{1}{2}}\right)$$

$$k_{eq} = k$$

15.

$$x + \frac{x}{2} + \frac{x}{4} + \cdots\cdots = 10\text{cm}$$

$$x\left(\frac{1}{1-\frac{1}{2}}\right) = 10\text{cm}$$

$$\Rightarrow \quad x = 5\text{cm}$$

Fourth Stage Elongation $= \frac{5\text{cm}}{8} = 0.625\text{ cm}$

Trajectory Equation:

$$y = (\tan\theta_1)(x) - \frac{g}{2V_1^2\cos^2\theta_1}(x^2).$$

$$\begin{cases} x = d\cos\phi \\ y = d\sin\phi \end{cases}$$

$$\Rightarrow \quad d\sin\phi = (\tan\theta_1)(d\cos\phi) - \frac{g}{2V_1^2\cos^2\theta_1}(d^2\cos^2\phi)$$

$$\sin\phi\cos^2\theta_1 = \sin\theta_1\cos\theta_1\cos\phi - \frac{gd\cos^2\phi}{2V_1^2}$$

$$d_1 = \frac{2V_1^2}{g\cos^2\phi}\cos\theta_1(\sin\theta_1\cos\phi - \sin\phi\cos\theta_1)$$

$$= \frac{2V_1^2}{g\cos^2\phi}\cos\theta_1\sin(\theta_1 - \phi)$$

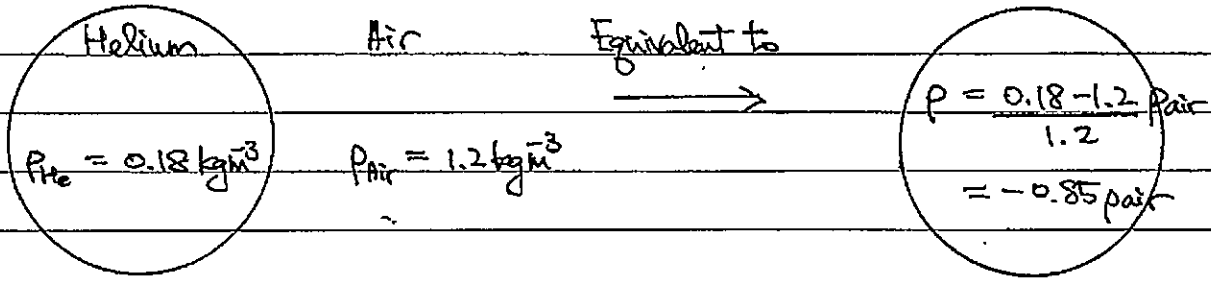
Similarly, $\quad d_2 = \dfrac{2V_2^2}{g\cos^2\phi}\cos\theta_2\sin(\theta_2 - \phi)$

$d_1 = d_2, \quad \Rightarrow \quad 2V_1^2\cos\theta_1\sin(\theta_1 - \phi) = 2V_2^2\cos\theta_2\sin(\theta_2 - \phi)$

$$\left(\frac{V_1}{V_2}\right)^2 = \frac{\cos\theta_2\sin(\theta_2 - \phi)}{\cos\theta_1\sin(\theta_1 - \phi)}$$

17.

Consider an equivalent case that the density of the balloon is $-0.85\rho_{air}$ (Density of air), and neglect the effect of air.



Effective gravitational acceleration $= -0.85g$

Effective centrifugal acceleration $= -0.85a$

$$a = \frac{v^2}{R} = \frac{(105\,km/hour)^2}{1000\,m} = 0.85\,ms^{-2}$$

$$\Rightarrow \tan\theta = \frac{-0.85}{-0.85}\left(\frac{a}{g}\right)$$

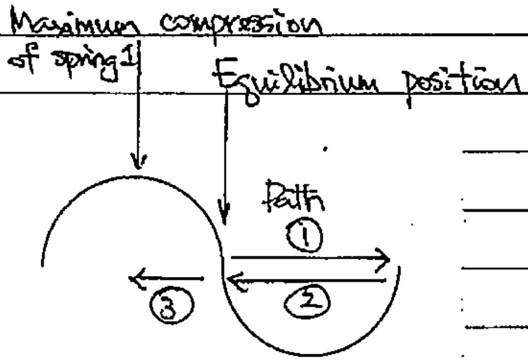
$$= \frac{0.85}{9.8}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{0.85}{9.8}\right)$$

$$= 5^{\circ}$$

18.

Effective spring constant, $k = k_1 + k_2 = 9$ N/m.



Maximum compression of Spring 1 occurs after $\frac{3}{4}$ period,

$\Rightarrow$ The time that the block will take

$$= \frac{3T}{4}$$

$$= \frac{3}{4}(2\pi)\sqrt{\frac{m}{k}}$$

$$= \frac{3}{4}(2\pi)\sqrt{\frac{1}{9}}$$

$$= \frac{\pi}{2} \text{ seconds}$$

19.

$$\text{Center of Mass} = \frac{\sum m_i x_i}{M}$$

$$\begin{aligned}
\sum m_i x_i &= \pi r^2 (r) + \pi\left(\frac{r}{2}\right)^2 \left(2r + \frac{r}{2}\right) \\
&\quad + \pi\left(\frac{r}{4}\right)^2 \left[2r + 2\left(\frac{r}{2}\right) + \frac{r}{4}\right] \\
&\quad + \pi\left(\frac{r}{8}\right)^2 \left[2r + 2\left(\frac{r}{2}\right) + 2\left(\frac{r}{4}\right) + \frac{r}{8}\right] \\
&\quad + \pi\left(\frac{r}{16}\right)^2 \left[2r + 2\left(\frac{r}{2}\right) + 2\left(\frac{r}{4}\right) + 2\left(\frac{r}{8}\right) + \frac{r}{16}\right] \\
&\quad + \cdots\cdots \\
&= \pi r^3 \left[1 + \frac{5}{8} + \frac{13}{64} + \frac{29}{512} + \frac{61}{4096} + \cdots\cdots\right] \\
&= \pi r^3 \sum_{n=1}^{\infty} 8^{1-n}\left(2^{n+1} - 3\right) \\
&= \frac{40}{21}\pi r^3
\end{aligned}$$

$$\begin{aligned}
M &= \pi r^2 + \pi\left(\frac{r}{2}\right)^2 + \pi\left(\frac{r}{4}\right)^2 + \pi\left(\frac{r}{8}\right)^2 + \cdots\cdots \\
&= \pi r^2 \left(1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots\cdots\right) \\
&= \pi r^2 \left(1 + \sum_{n=1}^{\infty} 4^{-n}\right) \\
&= \pi r^2 \left(1 + \frac{1}{3}\right) \\
&= \pi r^2 \left(\frac{4}{3}\right)
\end{aligned}$$

Note: The mass proportional constant is omitted. It will be cancelled finally.

$$\Rightarrow \text{Center of Mass} = \frac{\frac{40}{21}\pi r^3}{\frac{4}{3}\pi r^2} = \frac{10}{7} r$$

20.

$$\begin{cases} mv = mv_{1f} + 2m v_{45f} \\ \frac{1}{2} mv^2 = \frac{1}{2} m v_{1f}^2 + \frac{1}{2}(2m) v_{45f}^2 \end{cases}$$

Solutions give :

$$\begin{cases} v_{1f} = -\frac{1}{3} v \\ v_{45f} = \frac{2}{3} v \end{cases}$$

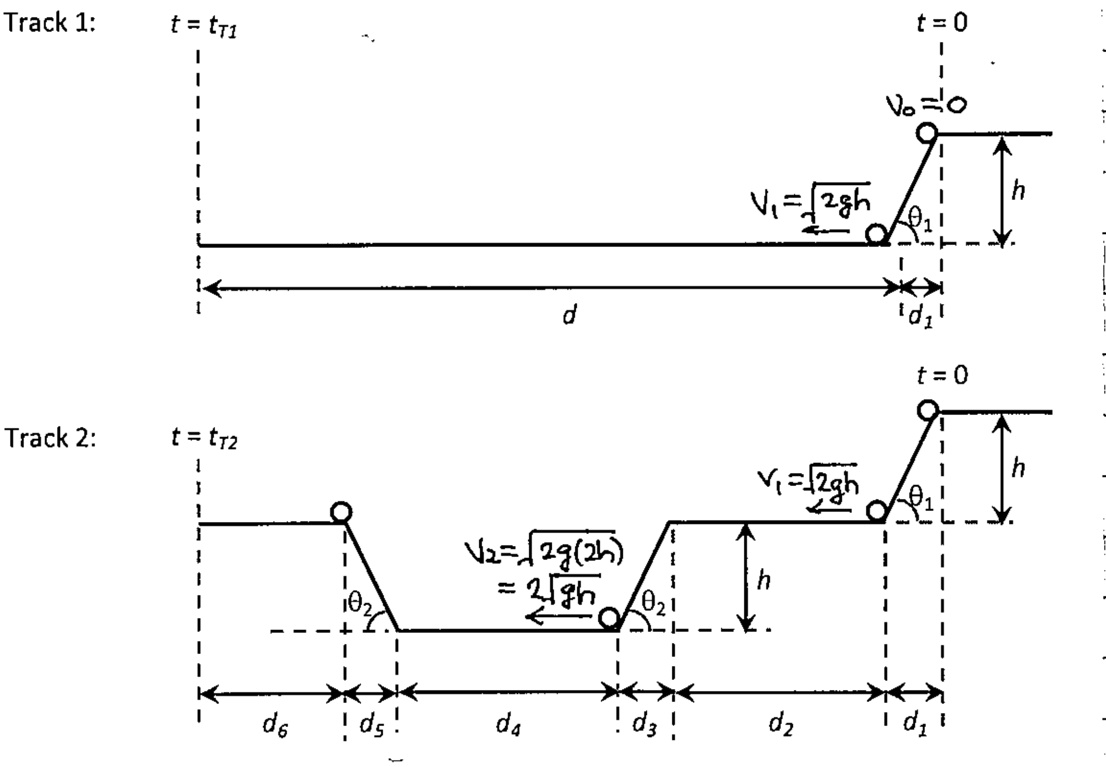
1.

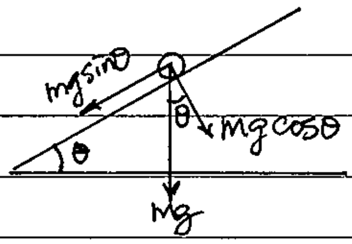
(a) $d = d_2 + d_3 + d_4 + d_5 + d_6$

$$= d_2 + \frac{h}{\sin\theta_2} + d_4 + \frac{h}{\sin\theta_2} + d_6$$

$$= d_2 + d_4 + d_6 + \frac{2h}{\sin\theta_2}$$

(b)





$$t_{d_1} = \frac{\sqrt{2gh}}{g\sin\theta_1} \qquad t_{d_3} = \frac{\sqrt{gh}\,(2-\sqrt{2})}{g\sin\theta_2}$$

$$t_{d_4} = \frac{d_4}{2\sqrt{gh}}$$

$$t_d = \frac{d}{\sqrt{2gh}} \qquad t_{d_5} = t_{d_3} = \frac{\sqrt{gh}\,(2-\sqrt{2})}{g\sin\theta_2}$$

$$t_{d_2} = \frac{d_2}{\sqrt{2gh}} \qquad t_{d_6} = \frac{d_6}{\sqrt{2gh}}$$

1.

(b) $t_{T_1} = t_{d_1} + t_d$

$$= \frac{\sqrt{2gh}}{g\sin\theta_1} + \frac{d}{\sqrt{2gh}}$$

$t_{T_2} = t_{d_1} + t_{d_2} + \cdots + t_{d_6}$

$$= \frac{\sqrt{2gh}}{g\sin\theta_1} + \frac{d_2}{\sqrt{2gh}} + \frac{\sqrt{gh}\,(2-\sqrt{2})}{g\sin\theta_2} + \frac{d_4}{2\sqrt{gh}} + \frac{\sqrt{gh}\,(2-\sqrt{2})}{g\sin\theta_2} + \frac{d_6}{\sqrt{2gh}}$$

$$= \frac{\sqrt{2gh}}{g\sin\theta_1} + \frac{d_2+d_6}{\sqrt{2gh}} + \frac{d_4}{2\sqrt{gh}} + \frac{2\sqrt{gh}\,(2-\sqrt{2})}{g\sin\theta_2}$$

(c) $\Delta t = \frac{d}{\sqrt{2gh}} - \frac{d_2+d_6}{\sqrt{2gh}} - \frac{d_4}{2\sqrt{gh}} - \frac{2\sqrt{gh}\,(2-\sqrt{2})}{g\sin\theta_2}$

$$= \frac{1}{\sqrt{2gh}}\left(d - d_2 - d_4 + d_4 - \frac{d_4}{\sqrt{2}} - d_6\right) - \frac{2\sqrt{gh}\,(2-\sqrt{2})}{g\sin\theta_2}$$

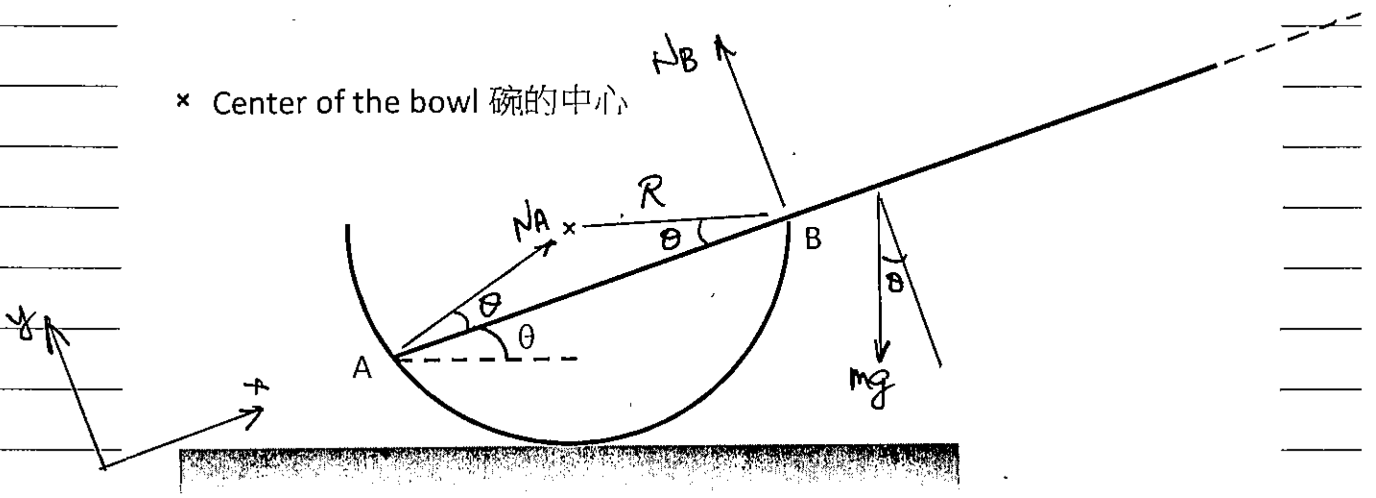
$$= \frac{1}{\sqrt{2gh}}\left(\frac{2h}{\sin\theta_2} + \frac{\sqrt{2}-1}{\sqrt{2}}\,d_4\right) - \frac{2\sqrt{gh}\,(2-\sqrt{2})}{g\sin\theta_2}$$

$$= \frac{\sqrt{2h}}{\sqrt{g}\sin\theta_2} - \frac{2\sqrt{h}\,(2-\sqrt{2})}{\sqrt{g}\sin\theta_2} + \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\frac{1}{\sqrt{2gh}}\,d_4$$

$$= \frac{\sqrt{h}\,(3\sqrt{2}-4)}{\sqrt{g}\sin\theta_2} + \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\frac{d_4}{\sqrt{2gh}}$$

2.

(a)



$$\Sigma F_x = 0, \quad N_A \cos\theta = mg\sin\theta \quad \text{—— (1)}$$

$$\Sigma F_y = 0, \quad N_A \sin\theta + N_B = mg\cos\theta \quad \text{—— (2)}$$

$\Sigma M_A = 0,$ Moment about A :

Length of AB

$$N_B(2R\cos\theta) - \frac{L}{2} mg\cos\theta = 0$$

$$\Rightarrow N_B = \frac{Lmg}{4R}$$

Sub. into (2),

$$N_A \sin\theta + \frac{Lmg}{4R} = mg\cos\theta$$

$$N_A = \left(mg\cos\theta - \frac{Lmg}{4R}\right)\frac{1}{\sin\theta} \quad \text{—— (3)}$$

Sub. into (1),

$$\left(mg\cos\theta - \frac{Lmg}{4R}\right)\frac{1}{\sin\theta}\cos\theta = mg\sin\theta$$

$$\frac{4R\cos^2\theta - L\cos\theta}{4R} = \sin^2\theta$$

2.

(a) $4R\cos^2\theta - L\cos\theta = 4R(1-\cos^2\theta)$

$$8R\cos^2\theta - L\cos\theta - 4R = 0$$

$$\cos\theta = \frac{L \pm \sqrt{L^2 + 128R^2}}{16R}$$

$$\Rightarrow \cos\theta = \frac{L + \sqrt{L^2 + 128R^2}}{16R}$$

$$AB = 2R\left(\frac{L + \sqrt{L^2 + 128R^2}}{16R}\right)$$

(b) When $L = 4R$, $\cos\theta = 1$

$$\Rightarrow \theta = 0^\circ$$

3.

(a) Consider the upper sphere, energy conservation gives

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\dot{\beta}^2 + mg(R+r)\cos\alpha = mg(R+r)$$

$$v = \dot{\beta}r = (R+r)\dot{\alpha}$$

$$\dot{\alpha}^2 = \dot{\beta}\,\frac{r^2}{(R+r)^2}$$

$$\frac{v^2}{2} + \frac{1}{5}r^2\dot{\beta}^2 + g(R+r)\cos\alpha = g(R+r)$$

$$\frac{\dot{\beta}^2 r^2}{2} + \frac{r^2\dot{\beta}^2}{5} + g(R+r)\cos\alpha = g(R+r)$$

$$\frac{7r^2\dot{\beta}^2}{10} = g(R+r)(1-\cos\alpha)$$

$$r^2\dot{\beta}^2 = \frac{10}{7}g(R+r)(1-\cos\alpha)$$

$$\dot{\beta}^2 = \frac{10}{7r^2}g(R+r)(1-\cos\alpha)$$

$$\dot{\alpha}^2 = \frac{10}{7}g\,\frac{(1-\cos\alpha)}{R+r}$$

$$v = (R+r)\dot{\alpha}$$

$$= (R+r)\sqrt{\frac{10}{7}g\,\frac{(1-\cos\alpha)}{R+r}}$$

$$= \sqrt{\frac{10}{7}g(R+r)(1-\cos\alpha)}$$

3.

(b) $$mg\cos\alpha - N = \frac{mv^2}{R+r}$$

Upper sphere will detach from the lower sphere when $N=0$.

$$g\cos\alpha = \frac{v^2}{R+r}$$

$$g\cos\alpha = \frac{10}{7}g(R+r)(1-\cos\alpha) \times \frac{1}{R+r}$$

$$\cos\alpha = \frac{10}{7}(1-\cos\alpha)$$

$$\Rightarrow \cos\alpha = \frac{10}{17}$$

$$\alpha \approx 54^\circ$$

(c) $$\begin{cases} mg\sin\alpha - f = m\dot{v} & \Rightarrow \dot{v} = g\sin\alpha - \dfrac{f}{m} \\ fr = \left(\dfrac{2}{5}mr^2\right)\ddot{\beta} & \\ v = \dot{\beta}r = (R+r)\dot{\alpha} & \Rightarrow \dot{v} = \ddot{\beta}r = (R+r)\ddot{\alpha} \end{cases}$$

$$fr = \left(\frac{2}{5}mr^2\right)\ddot{\beta}$$

$$= \left(\frac{2}{5}mr^2\right)\frac{\dot{v}}{r}$$

$$= \left(\frac{2}{5}mr^2\right)\frac{1}{r}\left(g\sin\alpha - \frac{f}{m}\right)$$

$$f = \left(\frac{2}{5}m\right)\left(g\sin\alpha - \frac{f}{m}\right)$$

$$= \frac{2}{5}mg\sin\alpha - \frac{2}{5}f$$

3.

(c) $\Rightarrow \quad f = \frac{2}{7} mg \sin\alpha \quad$ (Given)

Condition to slide: $\quad f = \mu N$

$$\Rightarrow \quad \frac{2}{7} mg \sin\alpha = \mu N$$

$$\frac{2}{7} mg \sin\alpha = \mu\left(mg\cos\alpha - \frac{mv^2}{R+r} \right)$$

Substitute v using the results from (a),

$$\frac{2}{7} g \sin\alpha = \mu\left[g\cos\alpha - \frac{10}{7} g (1-\cos\alpha) \right]$$

$$\frac{2}{7} \sin\alpha = \mu\cos\alpha - \frac{10}{7}\mu + \frac{10}{7}\mu\cos\alpha$$

$$2\sin\alpha = 17\mu\cos\alpha - 10\mu$$

$$2\left(\sqrt{1-\cos^2\alpha}\right) = 17\mu\cos\alpha - 10\mu$$

$$4(1-\cos^2\alpha) = 289\mu^2\cos^2\alpha + 100\mu^2 - 340\mu^2\cos\alpha$$

$$\cos^2\alpha\,(289\mu^2+4) - \cos\alpha\,(340\mu^2) + 100\mu^2 - 4 = 0$$

$$\cos\alpha = \frac{340\mu^2 \pm \sqrt{340^2\mu^4 - 4(289\mu^2+4)(100\mu^2-4)}}{2(289\mu^2+4)}$$

$$= \frac{340\mu^2 \pm \sqrt{16(189\mu^2+4)}}{2(289\mu^2+4)}$$

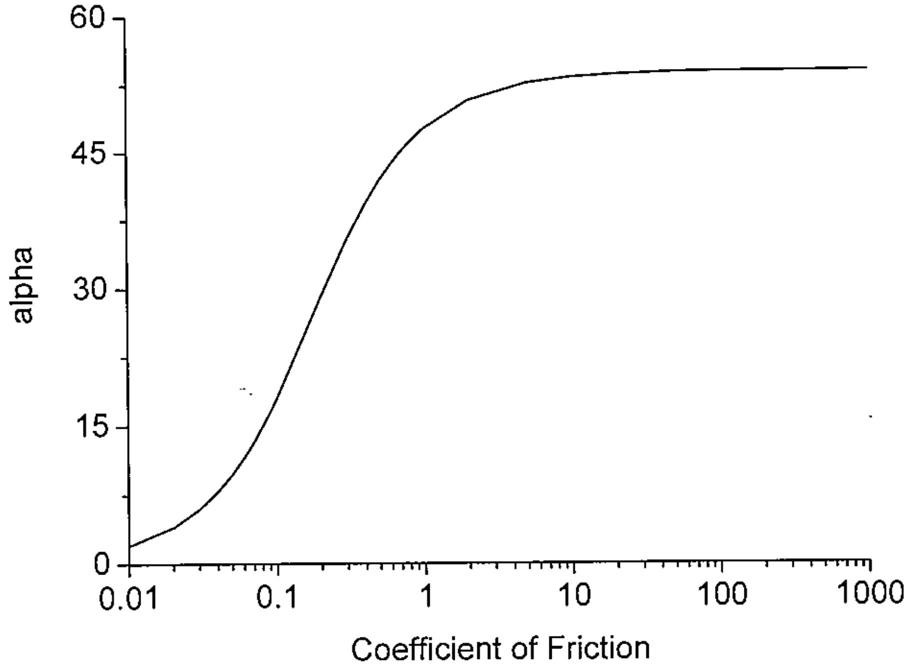
$$= \frac{170\mu^2 \pm 2\sqrt{189\mu^2+4}}{289\mu^2+4}$$

Since $0^\circ \le \alpha \le 90^\circ$,

$$\Rightarrow \quad \cos\alpha = \frac{170\mu^2 + 2\sqrt{189\mu^2+4}}{289\mu^2+4}$$

3.

(c) For reference :



(d) When $\mu = 0$, $\cos\alpha = 1$ and $\alpha = 0^\circ$

Remark: without friction, the upper sphere slides immediately.

When $\mu \to \infty$, $\cos\alpha = \dfrac{170 + 2\sqrt{189/\mu^2 + 4/\mu^4}}{289 + 4/\mu^4}$

$$= \frac{170}{289}$$

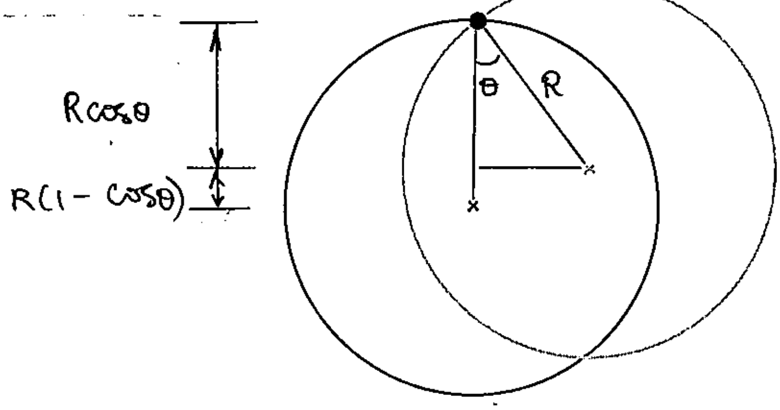
$$= \frac{10}{17}$$

$$\Rightarrow \quad \alpha \approx 54^\circ$$

4.

$$KE = \frac{1}{2}\left(2M_1R_1^2\right)\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}\left(2M_2R_2^2\right)\left(\frac{d\theta}{dt}\right)^2$$

$$= \left(M_1R_1^2 + M_2R_2^2\right)\left(\frac{d\theta}{dt}\right)^2$$



(a)

$$PE = -M_1 g R_1(1-\cos\theta) + M_2 g R_2 (1-\cos\theta)$$

(b) $KE + PE = \text{constant}$

$$\left(M_1R_1^2 + M_2R_2^2\right)\left(\frac{d\theta}{dt}\right)^2 - M_1 g R_1(1-\cos\theta) + M_2 g R_2(1-\cos\theta) = \text{constant}$$

$$\left(M_1R_1^2 + M_2R_2^2\right)\left(\frac{d\theta}{dt}\right)^2 - M_1 g R_1\left(2\sin^2\frac{\theta}{2}\right) + M_2 g R_2\left(2\sin^2\frac{\theta}{2}\right) = \text{constant}$$

For small θ, $\sin^2\left(\frac{\theta}{2}\right) \approx \frac{\theta^2}{4}$.

$$\left(M_1R_1^2 + M_2R_2^2\right)\left(\frac{d\theta}{dt}\right)^2 + \frac{\theta^2}{2} g\left(M_2R_2 - M_1R_1\right) = \text{constant}$$

$$\frac{1}{2}(2)\left(M_1R_1^2 + M_2R_2^2\right)\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2} g\left(M_2R_2 - M_1R_1\right)\theta^2 = \text{constant}$$

(c) $k_{eff} = g\left(M_2R_2 - M_1R_1\right)$

$$m_{eff} = 2\left(M_1R_1^2 + M_2R_2^2\right)$$

$$\omega = \sqrt{\frac{k_{eff}}{m_{eff}}} = \sqrt{\frac{g\left(M_2R_2 - M_1R_1\right)}{2\left(M_1R_1^2 + M_2R_2^2\right)}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2\left(M_1R_1^2 + M_2R_2^2\right)}{g\left(M_2R_2 - M_1R_1\right)}}$$

Alternative Method to (b) and (c):

$$(m_1R_1^2 + m_2R_2^2)\left(\frac{d\theta}{dt}\right)^2 - m_1gR_1(1-\cos\theta) + m_2gR_2(1-\cos\theta) = \text{constant}$$

$$(m_1R_1^2 + m_2R_2^2)(2)\left(\frac{d\theta}{dt}\right)\left(\frac{d^2\theta}{dt^2}\right) - m_1gR_1\sin\theta\frac{d\theta}{dt} + m_2gR_2\sin\theta\frac{d\theta}{dt} = 0$$

$$2(m_1R_1^2 + m_2R_2^2)\left(\frac{d^2\theta}{dt^2}\right) + g(m_2R_2 - m_1R_1)\sin\theta = 0$$

$$\frac{d^2\theta}{dt^2} \approx -\frac{g(m_2R_2 - m_1R_1)\,\theta}{2(m_1R_1^2 + m_2R_2^2)}$$

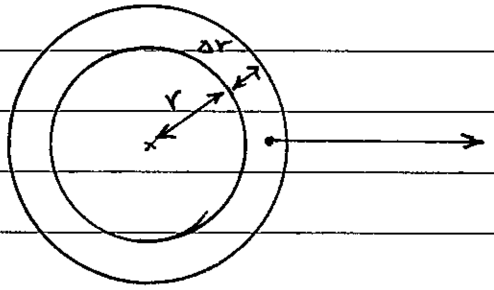
$$\Rightarrow \omega = \frac{g(m_2R_2 - m_1R_1)}{2(m_1R_1^2 + m_2R_2^2)}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2(m_1R_1^2 + m_2R_2^2)}{g(m_2R_2 - m_1R_1)}}$$

(d) When $m_1R_1 \to 0$,

$$T = 2\pi\sqrt{\frac{2m_2R_2^2}{gm_2R_2}} = 2\pi\sqrt{\frac{2R_2}{g}}$$

5.



Energy to remove a thin shell at radius r :

$$E = -\frac{G M_r m_r}{r}$$

where M_r = mass within the radius $r = \frac{4}{3}\pi r^3 \rho$

m_r = Mass of thin shell at radius r and thickness Δr.

$= -4\pi r^2 \rho \, \Delta r$ (ρ = Density of the Sun)

Energy to remove the ith thin shell :

$$E_i = -\frac{G}{r}\left(\frac{4\pi}{3} r_i^3 \rho\right)\left(-4\pi r_i^2 \rho \, \Delta r\right)$$

$$= G \frac{(4\pi)^2}{3} \rho^2 r_i^4 \, \Delta r$$

$$E_{total} = \sum_i E_i$$

$$= \frac{(4\pi)^2}{3} G \rho^2 \sum_i \left(\frac{R_s}{N} i\right)^4 \left(\frac{R_s}{N}\right)$$

[Suppose the Sun is divided into N thin shells, where $N \to \infty$]

5.

$$E_{Total} = \frac{(4\pi)^2}{3} \frac{G\rho^2}{N^5} R_S^5 \sum_i i^4$$

$$= \frac{(4\pi)^2}{3} \frac{G\rho^2}{N^5} R_S^5 \left(\frac{6N^5}{30} \right)$$

$$= \frac{(4\pi)^2}{3} G\rho^2 \frac{R_S^5}{5}$$

$$= \frac{(4\pi)^2}{3} G \left(\frac{M_S}{4/3\, \pi R_S^3} \right)^2 \frac{R_S^5}{5}$$

$$= \frac{3}{5} G \frac{M_S^2}{R_S}$$